Should We Use Linearized Models To Calculate Fiscal Multipliers?

Jesper Lindé\textsuperscript{y} Sveriges Riksbank and CEPR
Mathias Trabandt\textsuperscript{z} Freie Universität Berlin

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Abstract

Some recent influential papers have shown that the government spending multiplier can be substantially elevated in a liquidity trap. However, a potentially serious drawback of this literature is the use of linearized models, apart from the zero lower bound on interest rates. Boneva, Braun, and Waki (2016) and others claim that in a liquidity trap, a model can behave different depending on whether it has been linearized or not. We examine their claim with an eye on the government debt implications of changes in fiscal spending. Specifically, we compare the effects changes in fiscal spending have on output and government debt in linearized and nonlinear general equilibrium models. We start with a variant of the simple benchmark model in Woodford (2003), which allows us to carefully parse out the differences between the linear and nonlinear solutions. Finally, we examine the robustness of our results in the workhorse model of Christiano, Eichenbaum and Evans (2005) augmented with a financial accelerator mechanism.

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\textsuperscript{y}Research Division, Sveriges Riksbank, SE-103 37 Stockholm, Sweden, E-mail: jesper.linde@riksbank.se.
\textsuperscript{z}Freie Universität Berlin, School of Business and Economics, Chair of Macroeconomics, Boltzmannstrasse 20, 14195 Berlin, Germany, E-mail: mathias.trabandt@gmail.com.
1. Introduction

The magnitude of the fiscal spending multiplier is a classic subject in macroeconomics. To calculate the magnitude of the multiplier, economists typically employ a linearized version of their actual nonlinear model. Does linearizing the nonlinear model matter for the conclusions about the multiplier? We document this may be the case in long-lived liquidity traps. When interest rates are expected to be constrained by the zero (or effective) lower bound for a protracted time period, the actual nonlinear model suggests a much smaller multiplier than the linearized version of the same model. Our results have important implications for the self-financing of fiscal stimulus and self-defeating of fiscal consolidations.

The financial crisis and Great Recession have revived interest in the magnitude of the fiscal spending multiplier. A quickly growing literature suggests that the fiscal spending multiplier can be very large at the zero lower bound (ZLB) of nominal interest rates, see e.g. Eggertsson (2010), Davig and Leeper (2011), Christiano, Eichenbaum and Rebelo (2011), Woodford (2011), Coenen et al. (2012) and Leeper, Traum and Walker (2015). Erceg and Linde (2014) show that in a long-lived liquidity trap fiscal stimulus can be self-financing. Conversely, the results of the above literature suggest that at the ZLB it is hard if not impossible to reduce government debt in the short-run through aggressive government spending cuts: fiscal consolidation can be self-defeating.

Importantly, the bulk of the existing literature analyzes fiscal multipliers in models where all equilibrium equation have been linearized around the steady state, except for the ZLB constraint on the monetary policy rule. Implicit in the linearization procedure is the assumption that the linearized solution is accurate even far away from the steady state. Recent work by Boneva, Braun, and Waki (2016) suggests that linearization produces severely misleading results at the zero lower bound. Essentially, Braun et al. argue that extrapolating decision rules far away from the steady state is invalid.

Our paper provides a positive analysis of the effect of spending-based fiscal stimulus on output and government debt using a fully nonlinear model. We compare the magnitude of the fiscal spending multipliers for output and government debt of the nonlinear model and the linearized version of the same model. In our analysis we pin down key features that account for the differences in the resulting multipliers between the fully nonlinear and linearized solutions of the model.

The modeling starting point is a variant of the workhorse New Keynesian DSGE model of Woodford (2003). This model features monopolistic competition and Calvo sticky prices and the
central bank follows a Taylor rule subject to the ZLB constraint on nominal rates. We rule out the well-known problems associated with steady state multiplicity emphasized by Benhabib, Schmitt-Grohe and Uribe (2001) by restricting our attention to the steady state with a positive inflation rate. We document and analyze the key differences between the linearized and fully nonlinear solutions of this model.

Next, we examine the differences in multiplier schedules in an empirically plausible model developed by Christiano, Eichenbaum and Evans (2005) which we augmented with the Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism. Our analysis allows us to study potential fiscal free lunches in a liquidity trap in a model which has a spending multiplier in the mid-range of the VAR evidence when monetary policy is unconstrained.\footnote{A large empirical literature has examined the effects of government spending shocks, mainly focusing on the post-WWII pre-financial crisis period when monetary policy had latitude to adjust interest rates. The bulk of this research suggests a government spending multiplier in the range of 0.5 to somewhat above unity (1.5). See e.g. Hall (2009), Ramey (2011), Blanchard, Erceg and Lindé (2016) and the references therein.}

In our analysis, we compare fiscal spending multipliers on for output and debt in nonlinear and linearized representations of the model, focusing on features which account for the discrepancies between the nonlinear and linearized solution. A distinct difference to the existing literature which uses variations of the standard New Keynesian model, for instance the recent work by Boneva, Braun and Waki (2016), Christiano, Eichenbaum and Johanssen (2016), and Fernandez-Villaverde et al. (2015) and Eggertsson and Singh (2016), is that we consider a framework with real rigidities.\footnote{There is also a recent literature which studies models where the effects of government spending is state dependent (i.e. differs in booms and recessions even absent zero lower bound considerations due to labor market), see e.g. Michaillat (2014), Rendahl (2016) and Roulleau-Pasdeloup (2016). This paper does not address this literature.}

In particular, we introduce real rigidities through the Kimball (1995) state-dependent demand elasticity which allows our model to simultaneously account for the macroeconomic evidence of a low linearized Phillips curve slope (0.01) and the microeconomic evidence of frequent price re-optimization (3-4 quarters).

Our analysis points toward important quantitative differences between output and debt multipliers in linearized and nonlinear DSGE models when the model is calibrated to reflect microeconomic evidence on the frequency of price changes only. In this version of the model, movements in the price distortion are key to explain the differences between the loglinearized and nonlinear models. However, when the model is calibrated to account for macroeconomic evidence of the slope of the Phillips curve and microeconomic evidence on the frequency of price changes jointly, the quantitative differences between the linear and nonlinear solutions are noticeably smaller, albeit still quantitatively important in long-lived liquidity traps. In this variant of the model, it is the
linearization of the pricing equation which accounts for the bulk of the differences between the nonlinear and linearized solutions.

Our paper is closely related to the papers by Benova, Braun and Eggertsson (2016) and Eggertsson and Singh (2016). When it comes to the latter, our results in the stylized model without real rigidites compare well with Eggertsson and Singh (2016), who argue that the difference in multipliers derived from the linearized and nonlinear solutions is small. But our analysis make clear that the findings in Eggertsson and Singh (2016) is contingent on considering a variant of the New Keynesian model with firm-specific labor markets, which implies that movements in the price distortion term is irrelevant for equilibrium dynamics. Relative to Boneva, Braun and Waki (2016), our results with the stylized model are in line with their paper in the sense that they suggest that the multiplier schedule is muted when the model is solved nonlinearly. Even so, an important difference is that we, even in the stylized model with real rigidities, find that the multiplier can be elevated in a long-lived liquidity trap (about twice as high as in normal times).

Also the results in the workhorse model augmented with the financial accelerator mechanism suggest that the multiplier may be twice as high in a long-lived liquidity trap (lasting 3 years or longer). However, in contrast to the stylized model, in which a duplication of the multiplier merely means an increase from 0.3 to 0.6 (which is not a gamechanger from an substantive viewpoint), a duplication in the workhorse model means an increase the first-year average multiplier from 0.9 in normal times to 1.8 in a long-lived liquidity trap. This difference is significant from a substantive viewpoint, and implies that the workhorse model maintains the possibility that a suitably sized transient fiscal stimulus (austerity) may come at a low cost or even be “a fiscal free lunch” (or be self-defeating).

The paper is organized as follows. Section 2 presents the small scale New Keynesian model and Section 3 the results. Section 4 examines the implications of the medium-sized New Keynesian model. Section 5 concludes.

2. A Stylized New Keynesian Model

The simple model we study is very similar to the one developed Erceg and Linde (2014), which in turn builds on the baseline Eggertsson and Woodford (2003) model with the exception that it allows for real effects of price distortions by dropping the assumption that labor cannot be reallocated between different firms (or industries). We deviate from Erceg and Linde (2014) in two ways; first by allowing for a Kimball (1995) aggregator (with the standard Dixit and Stiglitz (1977) specification
as a special case), and second, by including a discount factor, or more generally savings, shock. Below, we outline the model and its key nonlinear equations. In the Appendix A, we describe the linearized version.

2.1. Model

2.1.1. Households

The utility functional for the representative household is

$$
\max_{\{C_t, N_t, B_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left\{ \log (C_t - C\nu_t) - \frac{N_t^{1+\chi}}{1 + \chi} \right\}
$$

(1)

where the discount factor $\beta$ satisfies $0 < \beta < 1$ and is subject to an exogenous shock $\zeta_t$. As in Erceg and Lindé (2014), the utility function depends on the household’s current consumption $C_t$ as deviation from a “reference level” $C\nu_{t+j}$. The exogenous consumption taste shock $\nu_t$ raises the reference level and marginal utility of consumption. The utility function also depends negatively on hours worked $N_t$.

The household’s budget constraint in period $t$ states that its expenditure on goods and net purchases of (zero-coupon) government bonds $B_{G,t}$ must equal its disposable income:

$$
P_t C_t + B_t = (1 - \tau_N) W_t N_t + (1 + i_{t-1}) B_{t-1} - T_t + \Gamma_t
$$

(2)

Thus, the household purchases the final consumption good at price $P_t$. The household is subject to a constant distortionary labor income tax $\tau_N$ and earns after-tax labor income $(1 - \tau_N) W_t N_t$. The household pays lump-sum taxes net of transfers $T_t$ and receives a proportional share of the profits $\Gamma_t$ of all intermediate firms.

Utility maximization yields the standard consumption Euler equation

$$
1 = \beta E_t \left\{ \delta_{t+1} \frac{(1 + i_t)}{1 + \pi_{t+1}} \frac{C_t - C\nu_t}{C_{t+1} - C\nu_{t+1}} \right\},
$$

(3)

where we have defined

$$
\delta_{t+1} = \frac{s_{t+1}}{s_t}
$$

(4)

and introduced the notation $1 + \pi_{t+1} = P_{t+1}/P_t$. We also have the following labor supply schedule:

$$
N_t^\chi = \frac{1 - \tau}{C_t - C\nu_t} \frac{W_t}{P_t}
$$

(5)

Equations (3) and (5) are the key equations for the household side of the model.
2.1.2. Firms and Price Setting

**Final Goods Production** The single final output good $Y_t$ is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G \left( \frac{Y_t(f)}{Y_t} \right) df = 1. \quad (6)$$

Following Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007), we assume that $G(\cdot)$ is given by the following strictly concave and increasing function:

$$G \left( \frac{Y_t(f)}{Y_t} \right) = \frac{\omega}{1 + \psi} \left[ (1 + \psi) \frac{Y_t(f)}{Y_t} - \psi \right]^{-\frac{1}{\varphi}} - \left[ \frac{\omega}{1 + \psi} - 1 \right], \quad (7)$$

where $\psi = \frac{(1-\varphi_p)p}{\varphi_p}, \omega = \frac{\varphi_p-\varphi_p(1-\varphi_p)}{1-(\varphi_p-1)\varphi_p}$. Here $\phi_p \geq 1$ denotes the gross markup of the intermediate firms. The parameter $\epsilon_p$ governs the degree of curvature of the intermediate firm’s demand curve, and in Figure 1 we show how relative demand is affected by the relative price under alternative assumptions about $\epsilon_p$ (and thus $\psi$) for given $\phi_p$.\(^3\) When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as under the standard Dixit-Stiglitz aggregator, implying a linear relationship between relative demand and relative prices ($\psi = 0$ in Figure 1). When $\epsilon_p$ is positive – as in Smets and Wouters (2007) – the firm’s instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost, especially when $\epsilon_p$ is high ($\psi = 0$ in Figure 1).

Finally, we notice that $G(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index $Y_t$, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price $P_t$, and hence solve the following problem:

$$\max \{ Y_t, Y_t(f) \} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df \quad (8)$$

\(^3\) The figure is taken from Levin, Lopez-Salido and Yun (2007), and the mapping between $\psi$ and $\epsilon_p$ is given by $\epsilon_p = -\frac{\phi_p\psi}{(\phi_p-1)}$. A value of $\psi = -8$ thus implies that $\epsilon_p$ equals 88 when the gross markup $\phi_p$ equals 1.1. [We should replace figure with lines in the $\epsilon_p$ space.]
subject to the constraint (6). The first order conditions can be written as

\[
\frac{Y_t(f)}{Y_t} = \frac{1}{1+\psi} \left( \left[ \frac{P_t(f)}{P_t} \right]^{\frac{1}{\phi_p - (\phi_p - 1)\epsilon_p}} + \psi \right),
\]

(9)

\[
P_t \Lambda_t^p = \left[ \int P_t(f) \left( \frac{1}{\phi_p - (\phi_p - 1)\epsilon_p} \right) df \right]^{-\phi_p - 1},
\]

\[
\Lambda_t^p = 1 + \psi - \psi \int \frac{P_t(f)}{P_t} df,
\]

where \(\Lambda_t^p\) denotes the Lagrange multiplier on the aggregator constraint (7). Note that for \(\epsilon_p = 0\), this problem leads to the usual Dixit and Stiglitz (1977) expressions

\[
\frac{Y_t(f)}{Y_t} = \left[ \frac{P_t(f)}{P_t} \right]^{\frac{1}{\phi_p - 1}}, \quad P_t = \left[ \int P_t(f) \frac{1}{\phi_p} df \right]^{1-\phi_p}
\]

Intermediate Goods Production A continuum of intermediate goods \(Y_t(f)\) for \(f \in [0, 1]\) is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand schedule from the final goods firms through the solution to the problem in (8) that varies inversely with its output price \(P_t(f)\) and directly with aggregate demand \(Y_t\).

Aggregate capital \((K)\) is assumed to be fixed, so that aggregate production of the intermediate good firm is given by

\[
Y_t(f) = K(f)^{\alpha} N_t(f)^{1-\alpha}.
\]

(10)

Despite the fixed aggregate stock \(K = \int K(f) df\), shares of it can be freely allocated across the \(f\) firms, implying that real marginal cost, \(MC_t(f)/P_t\) is identical across firms and equal to

\[
\frac{MC_t}{P_t} = \frac{W_t/P_t}{MPL_t} = \frac{W_t/P_t}{(1 - \alpha)K^\alpha N_t^{-\alpha}},
\]

(11)

where \(N_t = \int N_t(f) df\).

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm \(f\) faces a constant probability, \(1 - \xi_p\), of being able to reoptimize its price \(P_t(f)\). The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price according to the following formula

\[
\tilde{P}_t = (1 + \pi) P_{t-1},
\]

(12)

where \(\pi\) is the steady-state (net) inflation rate and \(\tilde{P}_t\) is the updated price.
Given Calvo-style pricing frictions, firm \( f \) that is allowed to reoptimize its price \( P_t^{opt}(f) \) solves the following problem

\[
\max_{P_t^{opt}(f)} \mathbb{E}_t \sum_{j=0}^{\infty} \left( \beta \xi_p \right)^j \varsigma_{t+j} \Lambda_{t,t+j} \left[ (1 + \pi)^j P_t^{opt}(f) - MC_{t+j} \right] Y_{t+j}(f)
\]

where \( \Lambda_{t,t+j} \) is the stochastic discount factor (the conditional value of future profits in utility units, recalling that the household is the owner of the firms), and demand \( Y_{t+j}(f) \) from the final goods firms is given by the equations in (9).

### 2.1.3. Monetary and Fiscal Policies

The evolution of nominal government debt is determined by the following equation

\[
B_{G,t} = (1 + i_{t-1}) B_{G,t-1} + P_t G_t - \tau N W_t N_t - T_t
\]

where \( G_t \) denotes real government expenditures on the final good \( Y_t \). Following the convention in the literature on fiscal multipliers, we assume that lump-sum taxes stabilize government debt as share of nominal trend GDP, \( b_{G,t} = \frac{B_{G,t}}{P_t Y_t} \). Specifically, we follow Erceg and Linde (2014) and assume that net lump-sum taxes as share of nominal trend GDP, \( \tau_t = \frac{T_t}{P_t Y_t} \), follow the simple rule:

\[
\tau_t - \tau = \varphi_b (b_{G,t-1} - b_G),
\]

where variables without time subscript denote steady state. Finally, government spending, \( g_{G,t} = \frac{G_t}{Y_t} \) is exogenous.

Turning to the central bank, it is assumed to adhere to a Taylor-type policy rule that is subject to the zero lower bound:

\[
1 + i_t = \max \left( 1, (1 + i) \left[ \frac{1 + \pi_t}{1 + \pi} \right]^{\gamma_x} \left[ \frac{Y_t}{Y_t^{pot}} \right]^{\gamma_x} \right)
\]

where \( Y_t^{pot} \) denotes the level of output that would prevail if prices were flexible, and \( i \) the steady-state (net) nominal interest rate, which is given by \( r + \pi \) where \( r \equiv 1/\beta - 1 \). In the linearized model, (15) is written

\[
i_t = \max \left( 0, i + \gamma_\pi (\pi_t - \pi) + \gamma_x x_t \right)
\]

where \( x_t = \ln \left( Y_t / Y_t^{pot} \right) \) denotes the output gap.
2.1.4. Aggregate Resource Constraint

We now turn to discuss the derivation of the aggregate resource constraint. Let $Y_{t}^{\text{sum}}$ denote the unweighted average (sum) of output for each firm $f$, i.e.

$$Y_{t}^{\text{sum}} = \int_{0}^{1} Y_{t}(f) df.$$  

which from (10) and the observation that all firms have the same capital-labor ratio can be rewritten as

$$Y_{t}^{\text{sum}} = \int \left( \frac{K(f)}{N_{t}(f)} \right)^{\alpha} N_{t}(f) df = \left( \frac{K}{N_{t}} \right)^{\alpha} \int N_{t}(f) df = K^{\alpha} N_{t}^{1-\alpha} \quad (17)$$

Recalling that $Y_{t+j}(f)$ is given from (??), it follows that

$$Y_{t}^{\text{sum}} = Y_{t} \int_{0}^{1} \frac{1}{1+\psi} \left( \left[ \frac{P_{t}(f)}{P_{t}} \right] \frac{1}{N_{t}} \right) - \frac{\phi_{p}-(\phi_{p}-1)\rho_{p}}{\phi_{p}-1} + \psi \right) df,$$

or equivalently, using (17):

$$Y_{t} = (p_{t}^{*})^{-1} K^{\alpha} N_{t}^{1-\alpha}, \quad (18)$$

where

$$p_{t}^{*} = \int_{0}^{1} \frac{\phi_{p}}{\phi_{p}-(\phi_{p}-1)\rho_{p}} \left( \left[ \frac{P_{t}(f)}{P_{t}} \right] \frac{1}{N_{t}} \right) - \frac{\phi_{p}-(\phi_{p}-1)\rho_{p}}{\phi_{p}-1} + \psi \right) df.$$  

In a technical appendix, we show how to develop a recursive formulation of the sticky price distortion term $p_{t}^{*}$.

Now, because actual output $Y_{t}$ is what is available for private consumption and government spending purposes, it follows that:

$$C_{t} + G_{t} \leq \overbrace{(p_{t}^{*})^{-1} K^{\alpha} N_{t}^{1-\alpha}}^{\equiv Y_{t}^{\text{sum}}} \equiv Y_{t}^{\text{sum}}. \quad (19)$$

The sticky price distortion introduces a wedge between input use and the output available for consumption (including by the government).\footnote{As the economy is assumed to be endowed with a fixed aggregate capital stock $K$ which does not depreciate, no resources is devoted to investment. An alternative formulation would have embodied a constant capital depreciation rate in which case output would have been used for $C_{t}$, $I$ and $G_{t}$.} Even so, this term vanishes in the log-linearized version of the model.
2.2. Parameterization

Our benchmark calibration — essentially adopted from Erceg and Linde (2014) — is fairly standard at a quarterly frequency. We set the discount factor $\beta = 0.995$, and the steady state net inflation rate $\pi = .005$; this implies a steady state interest rate of $i = .01$ (i.e., four percent at an annualized rate). We set the intertemporal substitution elasticity $\sigma = 1$ (log utility), the capital share parameter $\alpha = 0.3$, the Frisch elasticity of labor supply $\frac{1}{\lambda} = 0.4$, and the steady state value for the consumption taste shock $\nu = 0.01$.\(^5\) As a compromise between the low estimate of $\phi_p$ in Altig et al. (2011) and the higher estimated value by Smets and Wouters (2007), we set $\phi_p = 1.1$. This leaves us with two additional deep parameters to pin down; the price contract duration parameter $\xi_p$, and the Kimball elasticity demand parameter $\epsilon_p$. To pin down these parameters, our starting point is the loglinearized New Keynesian Phillips Curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_{mc} \hat{mct},$$  \hspace{1cm} (20)

which obtains in our model where $\hat{mct}$ denotes marginal cost as log-deviation from its steady state value. The parameter $\kappa_{mc}$, i.e. the slope of the Phillips curve, is given by

$$\kappa_{mc} = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p} \frac{1}{1+(\phi_p-1)\epsilon_p}.$$  \hspace{1cm} (21)

A large body of microeconomic evidence, see e.g. Klenow and Malin (2010) and Nakamura and Steinsson (2012) and the references therein, suggest that firms change their prices rather frequently, on average somewhat more often than once a year. Based on this micro evidence, we set $\xi_p = 0.667$, implying an average price contract duration of 3 quarters ($\frac{1}{1-0.667}$). On the other hand, the macroeconomic evidence suggest that the sensitivity of aggregate inflation to variations in marginal cost is very low, see e.g. Altig et al. (2011). To capture this, we adopt a value for $\epsilon_p$ so that the slope of the Phillips curve ($\kappa_{mc}$) — given our adopted values for $\beta$, $\xi_p$ and $\phi_p$ — equals 0.012.\(^6\) This calibration allows us to match both the micro- and macroevidence on price setting behavior and is aimed at capturing the resilience of core inflation, and measures of expected inflation, during the recent global recession.

We assume a government debt to annualized output ratio of 0.6 (consistent with U.S. pre-crisis federal debt level). We set government consumption as a share of output $g_y = 0.2$. Further, we

\(^5\) By setting the steady value of the consumption taste shock to a small value, we ensure that the dynamics for alternative shocks are roughly invariant to the presence of $-C\nu_t$ in the period consumption utility function.

\(^6\) The median estimates of the Phillips Curve slope in recent empirical studies by e.g. Adolfson et al (2005), Altig et al. (2011), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007) are in the range of 0.009 — 0.014.
set net lump-sum taxes $\tau = 0$ in steady state. The above assumptions imply a steady state labor income tax $\tau_N = 0.33$. The parameter $\varphi$ in the tax rule (14) is set equal to 0.01, which implies that the contribution of lump-sum taxes to the response of government debt is negligible in the first couple of years following a shock (so that almost all variation in tax revenues reflect fluctuations in labor tax revenues). For monetary policy, we use the standard Taylor (1993) rule parameters $\gamma_{\pi} = 1.5$ and $\gamma_x = .125$.

In order to facilitate comparison between the nonlinear and linear model, we specify processes for the exogenous shocks such that there is no loss in precision due to an approximation. In particular, the preference, discount and government spending shocks are assumed to follow AR(1) processes:

\begin{align}
g_{y,t} - g_y &= \rho_g (g_{y,t-1} - g_y) + \varepsilon_{g,t}, \\
\nu_t - \nu &= \rho_\nu (\nu_{t-1} - \nu) + \sigma_{\nu,t}, \\
\delta_t - \delta &= \rho_\delta (\delta_{t-1} - \delta) + \sigma_\delta \varepsilon_{\delta,t},
\end{align}

where $\delta = 1$. Our baseline parameterization of these processes adopts a persistence coefficient of 0.95, so that $\rho_\nu = \rho_g = \rho_\delta = 0.95$ in (22). But following some prominent papers in the literature on fiscal multipliers, we also investigate the sensitivity of our results when the processes are assumed to be moving average (MA) processes. Those results are reported in Appendix A.

### 2.3. Solving the Model

We compute the linearized and nonlinear solutions using the Fair and Taylor (1983) method. This method imposes certainty equivalence on the nonlinear model, just as the linearized solution does by definition. In other words, the Fair and Taylor solution algorithm traces out the implications of not linearizing the equilibrium equations for the resulting multiplier. An alternative approach would have been to compute solutions where uncertainty about future shock realizations matters for the dynamics of the economy following for instance Adam and Billi (2006, 2007) within a linearized framework and Fernández-Villaverde et al. (2015) and Gust, Herbst, López-Salido and Smith (2016) within a nonlinear framework. These authors have shown that allowing for future shock uncertainty can potentially have important implications for equilibrium dynamics, especially when inflation expectations are less well anchored because the conduct of monetary policy is far from non-optimal and prices are quick to adjust. We nevertheless confine ourself to study perfect foresight simulations for the following three reasons. First, because much of the existing literature
have used a perfect foresight approach, retaining this approach allows us to parse out the effects of going from a linearized to a nonlinear framework. Second, the high degree of real rigidities we introduce in order to fit the micro- and macroeconomic evidence implies that prices adjust very slowly, which in turn means that the impact of future shock uncertainty is small.\textsuperscript{7} Third, the perfect foresight assumption allows us to readily study the robustness in a larger scale model with many state variables. So far, the solution algorithms used to solve models with shock uncertainty have typically not been applied to models with more than 4-5 state variables.\textsuperscript{8}

To solve the model, we feed the relevant equations in the nonlinear and log-linearized versions of the model to Dynare. Dynare is a pre-processor and a collection of MATLAB routines which can solve nonlinear models with forward looking variables, and the details about the implementation of the algorithm used can be found in Juillard (1996). We use the perfect foresight simulation algorithm implemented in Dynare using the ‘simul’ command. The algorithm can easily handle the ZLB constraint: one just writes the Taylor rule including the max operator in the actual model equations. The solution algorithm reliably calculates the model solution is fractions of a second.

For the linearized model, we used the algorithm outlined in Hebden, Linde and Svensson (2012) to check for uniqueness of the local equilibrium associated with a positive steady state inflation rate. As noted earlier, we rule the well-known problems associated with steady state multiplicity emphasized by Benhabib, Schmitt-Grohe and Uribe (2001) by restricting our attention to the steady state with a positive inflation rate. However, for the nonlinear version of the model we cannot rule out the possibility that there exists other solutions in addition to the one found by Dynare, but note that the work by Christiano, Eichenbaum and Johannsen (2016) suggest that alternative solutions may not be relevant (i.e. not stable under learning).

3. Results for the Stylized Model

In this section, we report our main results in the linearized and non-linear solution of the model outlined in the Section above. As mentioned earlier, our aim is to compare spending multipliers in linearized and nonlinear versions of the model economy. Specifically, we seek to characterize how the difference between the multiplier in the linear and nonlinear frameworks varies with the

\textsuperscript{7} We have verified this in the model with the Dixit-Stiglitz aggregator by comparing the decision rules under perfect foresight with the decision rules obtained when allowing for shock uncertainty (when calibrating the variance of the shocks in the model so that the ZLB binds in 4 percent of the stochastic simulations with the model. Preliminary results (which needs to be refined) for the variant of our model with Kimball aggregator are in line with the Dixit-Stiglitz results.

\textsuperscript{8} A recent paper by Judd, Maliar and Maliar (2011) provides a promising avenue to compute the stochastic solution of larger scale models efficiently.
expected duration of the liquidity trap. We start out by reporting how we construct the baseline scenarios and then report the marginal fiscal multipliers.

3.1. Construction of Baseline Scenarios

To construct a baseline where the interest rate is bounded at zero for \( ZLB_{DUR} = 1, 2, 3, \ldots, T \) periods, we follow the previous fiscal multiplier literature (e.g. Christiano, Eichenbaum and Rebelo, 2011) and assume that the economy is hit by a large adverse shock that triggers a deep recession and drives interest rates to zero. The longer the expected liquidity trap duration (i.e. the larger value of \( ZLB_{DUR} \)) we want to have, the larger the adverse shock has to be. The particular shock we consider is a negative consumption taste shock \( \nu_t \) (see the equations 1 and 22) following Erceg and Linde (2014).

9 To provide clarity on how we pick the shock sizes, Figure 2 reports the linear and nonlinear solutions for the same negative taste shock (depicted in the bottom right panel). The economy is in the deterministic steady state in period 0, and then the shock hits the economy in period 1. As is evident from Figure 2, the same-sized shock has a rather different impact on the economy depending on whether the model is linearized or solved in its original nonlinear form. For instance, we see from panel 3 that while the nominal interest rate is bounded by zero from periods 1 to 8 in the linearized model, the same-sized consumption demand shock (panel 9) only generates a two quarter trap in the nonlinear model. Hence, we need to subject the nonlinear model to a more negative consumption demand shock — as shown in panel 9 in Figure 3 — to generate \( ZLB_{DUR} = 8 \) for the interest rate (panel 3).

10 Important insights about the differences between the linearized and nonlinear solutions can be gained from Figures 2 and 3. Starting with Figure 2, we see from the fifth panel that the drop in the potential real rate is about the same in both models. Still, the linearized model generates a much longer liquidity trap because inflation and expected inflation falls much more (panel 2), which in turn causes the actual real interest rate (panel 4) to rise much more initially. The larger initial rise in the actual real interest rate, and thus in gap between the actual and potential real rates, triggers a larger fall in the output gap (panel 1) and consequently real GDP falls more in the

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9 In Appendix A, we present results when the recession is instead assumed to be triggered by the discount factor shock \( \delta \) that was used in the seminal papers by Eggertsson and Woodford (2003), and Christiano, Eichenbaum and Rebelo (2011). For the linearized solution, we show that the results are invariant w.r.t. the choice of the baseline shock (see Erceg and Linde, 2014, for proof), and in the nonlinear solution, we show that the multiplier schedules are nearly identical (Figure A.1).

10 Figure 2 also depicts a third line (“Nonlinear model with linear NKPC and Res. Con.”), which we will discuss further in Section 3.2.
linearized model as well (because the impact on potential GDP is about the same, as implied by
the similarity of the potential real interest rate response).

Turning to Figure 3, we first note from the third panel that the paths for the policy rate are
bounded at zero for 8 quarters and display a very similar path upon exit from the liquidity trap.
Moreover, panel 9 shows that it takes a much larger adverse consumption demand shock in the
nonlinear model to trigger a liquidity trap of the same expected duration as in the linearized model.
This implies that the drop in the potential real rate and real GDP (panels 5 and 7) is much more
severe in the nonlinear model. Even so, and perhaps most important, we see that inflation – panel
2 – falls substantially less in the nonlinear model. This suggests that the difference between the
linearized and nonlinear solutions too a large extent is driven by the pricing block of the model.

3.2. Marginal Fiscal Multipliers

As previously noted, we are seeking to compare fiscal multipliers in liquidity traps of same ex-
pected duration in the linearized and nonlinear frameworks. Accordingly, we allow for differently
sized shocks in the linearized and nonlinear models so that each model variant generates a liq-
uidity trap with the same expected duration $ZLB_{DUR} = 1, 2, 3, ..., T$. Let $B_t^{linear}(\sigma_{\nu,t}^{linear})$ and
$B_t^{nonlin}(\sigma_{\nu,t}^{nonlin})$ denote vectors with simulated variables in the linear and nonlinear models,
respectively. This notation reflects that the innovations, $\sigma_{\nu}$, to the consumption demand shock $\nu_t$, in eq. (22) are set so that

$$\sigma_{\nu,t}^{linear} \Rightarrow ZLB_{DUR} = i,$$

and

$$\sigma_{\nu,t}^{nonlin} \Rightarrow ZLB_{DUR} = i,$$

where we consider $i = 1, 2, ..., T$. In the specific case of $i = 8$, panel 9 in Figure 3 shows that
$\sigma_{\nu,linear,8} = -.18$ and $\sigma_{\nu,nonlin,8} = -.42$.

To these different baseline paths, we add the fiscal response in the first period the ZLB binds,
which happens in the same period as the adverse shock hits ($t = 1$). By letting $S_t^{linear}(\sigma_{\nu,t}^{linear}, \sigma_G)$
and $S_t^{nonlin}(\sigma_{\nu,t}^{nonlin}, \sigma_G)$ denote vectors with simulated variables in the linear and nonlinear solutions
when both the negative baseline shock $\sigma_{\nu}$ and the positive government spending shock $\sigma_G$
hits the economy, we can compute the partial impact of the fiscal spending shock as

$$P_t^f(ZLB_{DUR}) = S_t^f(\sigma_{\nu,t}^{f}, \sigma_G) - B_t^f(\sigma_{\nu,t}^{f})$$
for $j = \{\text{linear, nonlin}\}$ and where we write $I_j^T(ZLB_{DUR})$ to highlight its dependence on the liquidity trap duration. Notice that the fiscal spending shock is the same for all $i$ and is scaled so that $ZLB_{DUR}$ remains the same as under the baseline shock only. By setting the fiscal impulse so that the liquidity trap duration remains unaffected, we retrieve “marginal” spending multipliers in the sense that they show the impact of a “tiny” change in the fiscal instrument.\footnote{Had we considered a larger fiscal intervention that altered the duration of the liquidity trap, there would have been an important distinction between the average (i.e. the total response) and marginal (i.e. the impact of a small change in $g_t$ which leaves $ZLB_{DUR}$ unchanged) multiplier as discussed in further detail in Erceg and Linde (2014).}

In Figure 4 we report the results of our exercise. The upper panels report results for the benchmark calibration with the Kimball aggregator. The lower panels report results under the Dixit-Stiglitz aggregator, in which case $\epsilon_p = 0$. This parametrization implies a substantially higher slope of the linearized Phillips curve (see eq. 21) and thus a much stronger sensitivity of expected inflation to current and expected future marginal costs (and output gaps). We will first discuss the results under the Kimball parameterization, and then turn to the results under Dixit-Stiglitz.

The left panels report the output-spending multiplier on impact, i.e. simply

$$m_i = \Delta Y_{t,i} / \Delta G_{t,i} = \Delta Y_{t,i} / \Delta g_y t$$

where the $\Delta$-operator represents the difference between the scenario with the spending change and the baseline without the spending change and $Y$ denotes the steady state level of output. We compute $m_i$ for $ZLB_{DUR} = 1, ..., 12$, but also include results for the case when the economy is at the steady state, so that $ZLB_{DUR} = 0$.

As the linear approximation is more accurate the closer the economy is to the steady state, it is not surprising that the difference between the “linear” and “nonlinear” multiplier increases with the duration of the liquidity trap. In a three-year liquidity trap, the multiplier is about twice as high (0.65) compared to normal times (when it is about 0.30), whereas it is almost 8 times higher in the linearized solution. So for a three year liquidity trap, the multiplier in the linearized solution is about three times as large as in the nonlinear solution. For shorter-lived liquidity traps, the differences are notably more modest, and in the special case when the economy is in the steady state ($ZLB_{DUR} = 0$ in the figure) we note that the multipliers are identical in both economies. The difference in government debt (as share of actual annualized GDP) response after 1 year, shown in the upper right panel, largely follows the pattern for $m_i$ and increases with $ZLB_{DUR}$\footnote{For ease of interpretability, we have normalized the response of debt and inflation so that they correspond to an initial change in government spending (as share of steady state output) by one percent.}.

The substantial differences in the output and debt responses between the linearized and nonlinear solutions begs the question of which factors account for them. The middle upper panel, which
shows the response of the one-period ahead expected annualized inflation rate (i.e., \(4E_t \pi_{t+1}\)), sheds some light on this. As can be seen from the panel, expected inflation responds much more in a long-lived trap in the linearized model than in the nonlinear model. The sharp increase in expected inflation triggers a larger reduction in the actual real rate relative to the potential real rate (not shown) in the linearized model, and thereby induces a more favorable response of private consumption which helps to boost output relative to the nonlinear model.

Turning to the Dixit-Stiglitz case shown in the lower panels, we see that the differences between the linear and nonlinear solutions are even more pronounced in this case, with the multiplier in an 8-quarter trap being over 100 \((27.5/0.25)\) times larger than in normal times in the linearized solution, but only roughly 10 times higher in the nonlinear solution.\(^{13}\) The larger discrepancy in the Dixit-Stiglitz case is to a large extent driven by the fact that we are in effect allowing a substantially higher slope (i.e. \(\kappa_{nc}\)) of the New Keynesian Phillips curve in eq. (20). Taken together, the results in Figure 4 suggest that the findings of the papers in the previous literature which relied on linearized models were more distorted to the extent that they relied on a calibration with a higher slope of the Phillips curve and thus a larger sensitivity of expected inflation.

The key question is then why expected inflation responds so much more in the linearized economy, and particularly so in the Dixit-Stiglitz case? To shed light on this, we simulate two additional variants of the nonlinear model. In the first, we linearize the pricing equations of the model, e.g. replace all pricing equations in the nonlinear model with the standard linearized Phillips curve. In the second, we linearize all the pricing equations and remove the price distortion term from the aggregate resource constraint (19). Following the approach with the linear and fully nonlinear models, we construct baseline scenarios for the two additional variants of the model as described in Section 3.1 for \(ZLB_{DUR} = 1, \ldots, 12\). The blue dash-dotted line in Figure 3 depicts the eight quarter liquidity trap baseline in the variant with linearized pricing equations and resource constraint (second additional variant described above). Clearly, the simulated paths of the variables in this variant of the model are very similar to those in the fully linearized solution. Therefore, given that the consumption demand \(\nu_t\) generating the baseline and the added government spending shock both work through the demand side of the economy, is it not surprising that the results in Figure 5 for this model (blue dashed-dotted line, referred to as “Linearized Resource Constraint and NKPC”) also display a striking similarity with the linearized model. Hence, we can draw the conclusion that it is the linearization of the resource constraint and the Phillips curve (20), and

\(^{13}\) We only show results up to 8 quarters with the Dixit-Stiglitz aggregator to be able to show the differences more clearly in the graph.
not the aggregate demand part of the model, which account for the bulk of the differences between the effects of fiscal spending in a long-lived liquidity trap in the linear and nonlinear models under Kimball. In fact, as shown by the green dash-dotted line in the upper panels of Figure 5, it is almost sufficient to only linearize the NKPC to account for most of the discrepancy between the linearized and nonlinear solution under the Kimball aggregator.

Even so, we see from the lower panels in Figure 5 that with the Dixit-Stiglitz aggregator, log-linearization of the New Keynesian Phillips curve only is not sufficient to explain the large discrepancies between the linear and nonlinear solutions. Accounting for the price distortion in the aggregate resource constraint is necessary, i.e. log-linearizing the resource constraint so that movements in the price distortion term $p_t^*$ in eq. (18) becomes irrelevant for equilibrium dynamics. The reason for this difference between the Kimball and the Dixit-Stiglitz specifications is that the price distortion variable moves much more for the latter specification, as re-optimizing firms will adjust their prices much more under Dixit-Stiglitz compared to Kimball for given $\xi_p$ (recalling the insights from Figure 1). So in a Dixit-Stiglitz world where firms adjust prices a lot when they re-optimize, the bulk of the difference between the linearized and nonlinear solutions is driven by movements in the price distortion, whereas in the Kimball world where firms adjust prices by less, the bulk of the differences is driven by the pricing equations directly.

Boneva, Braun and Waki (2016) argue that the key is to account for the price distortion, and our results suggest their claim is valid given that they are considering a model framework in line with the Dixit-Stiglitz aggregator. In terms of multiplier sizes, it is important to note that we report lower multipliers that Boneva, Braun and Waki in Figure 4 (for comparable degree of price adjustment) because our spending process is assumed to be a fairly persistent AR(1) process. If we assume that spending follows an uniform MA process and is only increased as long as policy rates are constrained by the ZLB, we obtain a marginal multiplier of unity in both the linear and nonlinear solutions already in a one-quarter liquidity trap (as we should, see Woodford, 2011, for proof). Even so, there is an important difference between the linearized and nonlinear solution for longer-lived liquidity traps, where multiplier rises to 5 in a three-year trap in the former case but only to 1.03 in the nonlinear solution. This is in line with Boneva, Braun and Waki, who reports a maximum multiplier of 1.05 in their model.

Our results can also be used to understand the results in Eggertsson and Singh (2016). Eggertsson and Singh consider a model with firm-specific labor in which the price distortion does not

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14 These results are discussed and presented in Appendix A, see Figure A.2.
affect equilibrium allocations. This implies that they are effectively working with nonlinear variant without the price distortion, i.e. the blue dash-dotted line in Figure 5. And since the results in the fully linearized model are very similar to those in this variant of the nonlinear model, our results confirm their conclusion that nonlinearities are unimportant for this variation of the NK model.\footnote{Strictly speaking, the Eggertsson and Singh (2016) model only omits the price distortion but retains the nonlinear pricing equations. Our blue dash-dotted line linearizes the pricing block in addition to removing the price distortion. However, the green dashed line shows that non-linearities in the pricing block matters very little in this specification.}

To further tease out the difference between the Kimball vs. Dixit-Stiglitz aggregator, Panel A in Figure 6 compares outcomes when the sticky price parameter $\xi_p$ is adjusted in the Dixit-Stiglitz version so that the slope of the linearized Phillips curve (20) is the same as in our benchmark Kimball calibration. Both the Kimball and Dixit-Stiglitz versions hence now feature a linearized Phillips curve with an identical slope coefficient ($\kappa_{mc} = 0.012$, see 21), but the Dixit-Stiglitz version of the model achieves this with a substantially higher value of $\xi_p$ (0.90). However, since only the value of $\kappa_{mc}$ matters in the linearized solution, the multiplier schedules are invariant w.r.t. the mix of $\xi_p$ and $\epsilon_p$ that achieves a given $\kappa_{mc}$ for the linearized models. The linearized solution for the Dixit-Stiglitz specification is thus identical to the Kimball solution, and given by the solid black line in the upper panel in Figure 4. But for the nonlinear solutions, shown in Panel A in Figure 6, the results differ. In particular, we see that the Dixit-Stiglitz aggregator implies that expected inflation and output multiplier responds more when the duration of the liquidity trap increases. Thus, when the Kimball parameter $\epsilon_p$ is reduced, the more will expected inflation and output multiplier respond when $\text{ZLB}_{\text{DUR}}$ increases; conversely, increasing $\epsilon_p$ and lowering $\xi_p$ flattens the output multiplier schedule even more. The explanation behind this finding is that a higher value of $\epsilon_p$ induces the elasticity of demand to vary more with the relative price differential among the intermediate good firms as shown in Figure 1, and this price differential increases when the economy is far from the steady state. Thus, intermediate firms which only infrequently are able to re-optimize their price will optimally choose to respond less to a given fiscal impetus far from the steady state when price differentials are larger as they perceive that they have a much larger impact on their demand for a given change in their relative price. As a result, aggregate current and expected inflation are less affected far from the steady state in the Kimball case relative to the Dixit-Stiglitz case for which the elasticity of demand is independent of relative price differentials. This demonstrates that the modeling of price frictions matters importantly within a nonlinear framework, especially so when nominal wages are flexible.

So far, we have followed the convention in the literature and assumed that non-optimizing firms
index their prices w.r.t. the steady state inflation rate, see eq. (12). This is a convenient benchmark modelling assumption as it simplifies the analysis by removing steady state price distortions. However, this assumption have been critized for being inconsistent with the microevidence on price setting (many firms’ prices remain unchanged for several subsequent quarters, whereas whey always change under our benchmark indexation scheme). Thus – there is an important issue to what extent it matters for the dynamics, especially in the nonlinear model. To examine this, we respecify the model assuming no indexation among the non-optimizing firms following e.g. Ascari and Ropele (2007), i.e.

\[
\tilde{P}_t = P_{t-1}.
\]  

(23)

In Panel B in Figure 6, we report the results when comparing the nonlinear baseline model (black solid line, which features indexation) with the nonlinear variant without indexation for the non-optimizing firms (red dotted line). From the panels, we see that abandoning the conventional assumption of full indexation results in a somewhat steeper multiplier schedule, mostly explained by the somewhat higher sensitivity of expected inflation in the “no-indexation” model. But by and large, the message in Panel B is that the results hold up well for this perturbation of the model.

4. Analysis with a Workhorse New Keynesian Model

The benchmark model studied so far is useful for highlighting many of the key factors likely to shape how a change in fiscal stance would affect the economy. However, to the extent that the stylized benchmark model featured a very low multiplier in normal conditions when the economy is not far off the steady state (about 0.30, as can be seen from Figure 4), it may well understate the aggregate effects of fiscal policy due to the exclusion of Keynesian accelerator effects on household and business spending. A consequence is that the aggregate multiplier remains relatively modest even in a long-lived liquidity trap unless inflation rises significantly.

Hence, in this section we move on to a substantive analysis with the aim of examining the multiplier schedule in a more quantitatively realistic model environment. Specifically, we consider a workhorse New Keynesian model with endogenous investment that closely follows the seminal model of Christiano, Eichenbaum and Evans (2005). This model has been successfully estimated by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007). In the following, we sketch the model and then turn to discussing the results. Appendix B provides a detailed exposition of the model and its parameterization.
From now on, we focus exclusively on the nonlinear model, as the previous analysis indicated substantial differences between the linear and nonlinear model for longer-lived liquidity traps.

4.1. Model

Following CEE, the model includes both sticky nominal wages and prices, allowing for some intrinsic persistence in both components; habit persistence in consumption; and embeds a $Q$–theory investment specification modified so that changing the level of investment (rather than the capital stock) is costly. However, our model departs from this earlier literature in two substantive ways. First, we introduce consumption (VAT), labor income and capital income taxes to get a realistic calibration of fiscal flows in the model. This enables us to study the impact changes in fiscal spending has on the evolution of government debt. Second, we incorporate a financial accelerator using Christiano, Motto and Rostagno’s (2008) variant of the Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism.\footnote{CMR assumes the entrepreneurs borrow nominal debt, whereas BGG assumed all debt were real.}

These features boost the natural real interest rate following a spending shock and hence tend to amplify the spending multiplier in normal times even if inflation doesn’t respond very much. And consistent with the findings of many influential papers in the empirical literature on government spending multipliers, e.g. Blanchard and Perotti (2002), we work with a parameterization of the model that implies a multiplier of about unity in the short term.\footnote{To generate a unit multiplier in the model in the short-term, we work with a relatively high degree of habit formation in consumer preferences and large investment adjustment costs relative to the empirical evidence. An alternative avenue to generate a higher multiplier would be to assume that a fraction of the households are “Keynesian”, and simply consume their current after-tax income. Gali, López-Salido and Vallés (2007) show that the inclusion of non-Ricardian households helps account for structural VAR evidence indicating that private consumption rises in response to higher government spending.} This number is reasonable, given that the VAR literature which has examined the effects of government spending shocks on the post-WWII pre-financial crisis period when monetary policy had latitude to adjust interest rates suggest a multiplier in the range of 0.5 to somewhat above unity (1.5), see e.g. Hall (2009), Ramey (2011), Blanchard, Erceg and Lindé (2016) and the references therein.

4.2. Results

Analogously with how we proceeded in the stylized model, the analysis with the workhorse model starts by feeding it with a set of baseline shocks which generate a baseline scenario where the interest rate is bounded at zero for $ZLB_{DUR} = 1, 2, 3, ..., T$ periods. In addition to using a negative consumption demand shock ($v_t$), we also feed the large scale model with a negative net worth
(entrepreneur survival) and a positive bond risk-premium shock. The consumption demand shock exhibits a correlation of 0.85, whereas the financial market shocks are assumed to be white noise. Together, these shocks exert a significant adverse impact on the economy, in which economy activity dampens, interest rate spreads rise, and inflation falls. As a result, the policy rate is driven towards successively longer zero lower bound spells when we increase the magnitude of these baseline shocks.

Figure 7 depicts the specific baseline for an 8-quarter liquidity trap in the nonlinear specification of the model. We think about an 8-quarter trap as roughly corresponding to the situation United States were facing in the first half of 2009. As can be seen from the figure, output and consumption fall by roughly 8 percent, and investment by twice as much (relative to trend). Consistent with the U.S. episode, interest rate spreads rise roughly 3 percentage points before receding towards their steady state value. Government debt as share of GDP rises about 10 percent. Core inflation falls to almost 0 percent, but consistent with the U.S. evidence the model avoids deflation for a protracted period. Even so, amid the negative output gap and low inflation, the policy interest rate is driven to nil for eight quarters starting in period 8 (in the figure, the economy starts out in the steady state in period 0).

Against this background, we consider a marginal increase in fiscal spending announced and undertaken the first period the ZLB binds (in the 8-quarter liquidity trap case described in Figure 7, the change in fiscal spending occurs in period 8) for all $\text{ZLB}_{\text{DUR}} = 1, 2, 3, \ldots, T$. As before, we can then construct impulse responses by deducting from the scenario with the spending hike the baseline with no change in fiscal spending.

In Figure 8 we show the corresponding marginal multiplier as function of liquidity trap duration. As before we show results for $\text{ZLB}_{\text{DUR}} = 1, \ldots, 12$ but also include the impact at steady state ($\text{ZLB}_{\text{DUR}} = 0$). The output multiplier is the 1-year cumulated multiplier, calculated as the sum of output responses during the first year divided by the sum of spending hikes during the same period. In the middle panel, we show expected inflation during the first year, calculated as $\ln P_{t+4}/ \ln P_t$. Finally, the government debt multiplier in the right panels is computed as the level of debt after 3 years divided by the absolute average change in spending during this period, so that a negative value implies that government debt falls following a hike in spending whereas a positive value implies that debt rises at this horizon.

As shown by the black solid line in the figure (which report results for the benchmark specification with Kimball aggregator in the price setting block of the model), the results are qualitatively
similar to the results with the stylized model in Figure 4. However, an important difference is that the inflation response is now an increasing function of the liquidity trap duration, as opposed to the results in the simple model in which case expected inflation responded less in a liquidity trap. As a result, the output multiplier schedule is somewhat more convex in the workhorse model compared to the stylized model, even though sticky nominal wages slows the speed of wage adjustment. Evidently, Keynesian accelerator mechanisms dominate the slower wage adjustment. As a result of the elevated effects on output and inflation, tax revenues rise sufficiently and debt service costs drop sufficiently that government debt multiplier turns negative in a sufficiently long-lived trap.

The red-dotted line shows results for a calibration of the model with the Dixit-Stiglitz aggregator variant of the model when the slope of the Phillips curve is kept unchanged at 0.012, corresponding to the results in Panel A of Figure 6 for the stylized model. As can be seen from the figure, the output multiplier schedule under the Dixit-Stiglitz aggregator essentially equals that obtained with the Kimball aggregator, due to the fact that the inflation schedule is only moderately elevated under the Dixit-Stiglitz calibration. This finding differs from the analysis in the stylized model, in which case the output multiplier was about twice as high with Dixit-Stiglitz aggregator in a 12-quarter liquidity trap (see Figure 6). But again, the results in the stylized model were contingent on flexible wages, and the differences between Dixit-Stiglitz and Kimball aggregators would be elevated if we jointly changed from Kimball to Dixit-Stiglitz in wage setting.

A key mechanism in the workhorse model that elevates the multiplier schedule is the financial accelerator mechanism embedded into the model. To see this, the bottom panels in Figure 8 compare the multiplier schedules with (solid black lines) and without (blue dash-dotted lines) financial accelerator mechanism in the model. As can be seen by comparing the variant without the financial accelerator to the benchmark specification, the output multiplier and inflation responses are shifted down and not amplified to the same extent as the duration of the liquidity trap is extended (multiplier only increases from 0.8 in normal times to 1.2). This implies that the government debt multiplier does not change sign even in a long-lived trap in the no-accelerator model, implying that debt increases after 3 years in the no-financial accelerator variant of the model for a persistent hike in spending (recalling that the results in Figure 8 are conditioned on an AR(1) process for government spending with persistence 0.95).

From a substantive perspective, the results in the workhorse model paint a quite different

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18 In this variant of the model, we replace the negative net worth shock with a negative investment-specific technology shock to generate a qualitatively similar baseline in the no financial accelerator model as plotted for the benchmark model in Figure 7.
picture than the stylized model. Importantly, the larger scale model implies that the multiplier is sufficiently elevated in a longer-lived liquidity trap so that fiscal stimulus intended to stimulate economic activity might actually be close to “a free lunch” (self-financed), as in Erceg and Lindé (2014). This result did not obtain in the nonlinear variant of the stylized model, for which the government debt multiplier always remained positive. In fact, an aggressive spending hike that is perceived to be transient is in fact associated with a fiscal free lunch at all horizons in a protracted liquidity trap, as shown by the results in Figure 9. In the figure, the solid blue lines show results for the benchmark model in a 12-quarter liquidity trap of a uniform hike in spending for 12-quarters (i.e. as long as the ZLB is expected to last). As can be seen from the figure, this transient spending hike elevates the marginal multiplier even further (to about 2 in the benchmark model), and causes debt to converge back to nil from below (as in the simple model, lump-sum taxes move to stabilize debt). An important corollary is that fiscal austerity, implemented through aggressive spending cuts that are not perceived to be persistent, may actually be self-defeating, at least for some time, in a long-lived liquidity trap.

Finally, Figure 9 also report results for the no-accelerator variant of the model (red dashed lines). Comparing the solid and dashed lines we see that financial accelerator effects plays a key role behind the magnified response in a deep liquidity trap. The favorable impact is substantially mitigated in the variant without the financial accelerator as investment (and consumption) is not crowded-in. As a result, government debt only falls initially before rising and there is no “fiscal free lunch” for the treasury as in the benchmark model.

5. Conclusions

[Remains to be written.]
References


Figure 1: Quasi-Kinked Demand

- Blue line: $\psi = 0$
- Green dashed line: $\psi = -2$
- Red dashed line: $\psi = -8$

The graph plots relative price (%) against relative demand (Index=100). The curves show the relationship between price and demand for different values of $\psi$. The demand is normalized, and the price changes are represented along the vertical axis.
Figure 3: Baselines for 8–Quarter Liquidity Trap

1. Output Gap
2. Yearly Inflation ($\ln(P_t/P_{t-4})$)
3. Nominal Interest Rate (APR)
4. Real Interest Rate (APR)
5. Potential Real Interest Rate (APR)
6. Price Dispersion
7. Real GDP
8. Government Debt to GDP
9. Consumption Demand Shock

Legend:
- **Linear Model**
- **Nonlinear Model**
- **Nonlinear Model with Linear NKPC and Res. Con.**
Figure 4: Marginal Multipliers

Impact Spending Multiplier

Expected inflation \(4E_t\pi_{t+1}\)

Govt Debt to GDP (After 1 Year)

Benchmark Calibration

Alternative Calibration: Dixit Stiglitz

Linear Model

Nonlinear Model
Figure 5: Marginal Multipliers

<table>
<thead>
<tr>
<th>Linear Model</th>
<th>Nonlinear Model</th>
<th>Linearized Resource Constraint and NKPC</th>
<th>Linearized NKPC only</th>
</tr>
</thead>
</table>

### Impact Spending Multiplier

![Impact Spending Multiplier Graph](image1)

### Expected Inflation ($4E_t \pi_{t+1}$)

![Expected Inflation Graph](image2)

### Govt Debt to GDP (After 1 Year)

![Govt Debt to GDP Graph](image3)

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**Benchmark Calibration**

- Linear Model
- Nonlinear Model
- Linearized Resource Constraint and NKPC
- Linearized NKPC only

**Alternative Calibration: Dixit Stiglitz**

- Linear Model
- Nonlinear Model
- Linearized Resource Constraint and NKPC
- Linearized NKPC only

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**Notes:**

- The graphs illustrate the marginal multipliers for different models and calibration scenarios over varying ZLB durations.
- The linear model shows a steady increase with a slope that remains constant, while the nonlinear model exhibits a more erratic, non-linear trend.
- The linearized model with resource constraint and NKPC introduces a sharp drop at certain points, indicating constraints at the zero lower bound.
- The linearized NKPC only model shows a gradual decrease with a smoother trend compared to the combined model.

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**Explanation:**

- The Impact Spending Multiplier measures how much the economy responds to an increase in government spending.
- Expected inflation is a key factor influencing real interest rates and investment decisions.
- Government debt to GDP ratios are critically important for long-term fiscal sustainability and market confidence.

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**Data Source:**

The data and models used are based on econometric analyses by leading economists, incorporating real-world macroeconomic indicators and policy scenarios.
Figure 6: Marginal Impact of Changes in Spending: Sensitivity Analysis in Nonlinear Model

Panel A: Kimball ($\xi_p=0.667$; $\varepsilon_p>0$) vs. Dixit-Stiglitz ($\xi_p=0.9$; $\varepsilon_p=0$)

Panel B: Impact of Indexation Assumption for Non-Optimizing Firms
Figure 7: Baseline for an 8-quarter Liquidity Trap in Workhorse Model
Figure 8: Marginal Multipliers in Workhorse Model

Benchmark Model: Kimball vs. Dixit-Stiglitz Aggregator in Price Setting

Assessing the Role of the Financial Accelerator Mechanism
Figure 9: Impulses to Transient Fiscal Stimulus in Workhorse Model

Benchmark Model

No Financial Accelarator Variant of Model
Appendix A. Additional Results for the Stylized Model

In this appendix, we state the log-linearized variant of the stylized model and present some additional results.

A.1. The Log-linearized Stylized Model

As shown in the technical appendix (available upon request from the authors), the equations of the log-linearized model can be written as follows:

\[ x_t = x_{t+1|t} - \dot{\sigma}(i_t - \pi_{t+1|t} - \nu_t^\text{pot}), \]
\[ \pi_t = \beta \pi_{t+1|t} + \kappa_p x_t, \]
\[ y_t^\text{pot} = \frac{1}{\phi_{mc}\sigma} [g_{y,t} + (1 - g_y)\nu_t], \]
\[ r_t^\text{pot} = -\dot{\delta} + \frac{1}{\sigma} \left( 1 - \frac{1}{\phi_{mc}\sigma} \right) \left[ (g_{y,t} - g_{y,t+1|t}) + (1 - g_y)\nu_t(\nu_t - \nu_{t+1|t}) \right], \]
\[ b_{G,t} = (1 + r)b_{G,t-1} + b_G(i_{t-1} - \pi_t) + g_{y,t} - \tau_N s_N (y_t + \phi_{mc} x_t) - \tau_t, \]
\[ y_t = x_t + y_t^\text{pot} \]

where \( \dot{\sigma}, \kappa_p, \phi_{mc} \) and \( s_N \) are composite parameters defined as:

\[ \dot{\sigma} = \sigma (1 - g_y)(1 - \nu_c), \]
\[ \kappa_p = \frac{(1 - \xi_p)(1 - \beta\xi_p)}{\xi_p} \frac{1}{1 + (\phi_p - 1) \epsilon_p} \phi_{mc}, \]
\[ \phi_{mc} = \frac{\chi}{1 - \alpha} + \frac{1}{\dot{\sigma}} + \frac{\alpha}{1 - \alpha}, \]
\[ s_N = \frac{1 - \alpha}{\phi_p}. \]

In slight abuse of previous notation, all variables above are measured as percent or percentage point deviations from their steady state level.A.1

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\[ b_{G,t} = (1 + r)b_{G,t-1} + b_G(i_{t-1} - \pi_t) + g_{y,t} - \tau_N s_N (y_t + \phi_{mc} x_t) - \tau_t, \]  
\[ y_t = x_t + y_t^\text{pot} \]  

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\[ \dot{\sigma} = \sigma (1 - g_y)(1 - \nu_c), \]  
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\[ s_N = \frac{1 - \alpha}{\phi_p}. \]  

In slight abuse of previous notation, all variables above are measured as percent or percentage point deviations from their steady state level.A.1

\[ A.1 \] We use the notation \( y_{t+j|t} \) to denote the conditional expectation of a variable \( y \) at period \( t+j \) based on information available at \( t \), i.e., \( y_{t+j|t} = E_t y_{t+j} \). The superscript ‘pot’ denotes the level of a variable that would prevail under completely flexible prices, e.g., \( y_t^\text{pot} \) is potential output.
Equation (A.1) expresses the “New Keynesian” IS curve in terms of the output and real interest rate gaps. Thus, the output gap \( x_t \) depends inversely on the deviation of the real interest rate \( (i_t - \pi_{t+1|t}) \) from its potential rate \( r_{pot}^t \), as well as on the expected output gap in the following period. The parameter \( \hat{\sigma} \) determines the sensitivity of the output gap to the real interest rate; as indicated by (A.7), it depends on the household’s intertemporal elasticity of substitution in consumption \( \sigma \), the steady state government spending share of output \( g_y \), and a (small) adjustment factor \( \nu_c \) which scales the consumption taste shock \( \nu_t \). The price-setting equation (A.2) specifies current inflation \( \pi_t \) to depend on expected inflation and the output gap, where the sensitivity to the latter is determined by the composite parameter \( \kappa_p \). Given the Calvo-Yun contract structure, equation (A.8) implies that \( \kappa_p \) varies directly with the sensitivity of marginal cost to the output gap \( \phi_{mc} \), and inversely with the mean contract duration \( (\frac{1}{1+\phi_{mc}}) \). The marginal cost sensitivity equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in (A.9), \( \phi_{mc} \) varies inversely with the Frisch elasticity of labor supply \( \frac{1}{\chi} \), the interest-sensitivity of aggregate demand \( \hat{\sigma} \), and the labor share in production \( (1 - \alpha) \). The equations (A.3) and (A.4) determinate potential output and the potential (or natural) real rate. The evolution of government debt is determined by equation (A.5), and depends on variations in the service cost of debt, government spending as well as labor income and lump-sum tax revenues. Equation (A.6) is a simple definitional equation for actual output \( y_t \) (in logs). Finally, the policy rate \( i_t \) follows a Taylor rule subject to the zero lower bound (equation 16 in the main text) and the exogenous shocks follows the processes in eqs. (22).

A.2. Sensitivity Analysis in the Simple Model

Figure A.1 report results for an alternative shock driving the baseline in Figures 2. The upper panels in the figure confirm the results in by Erceg and Lindé (2014) by showing that the fiscal spending multiplier at the margin in the linearized version is independent of the shock driving the baseline, as long as it generates an equally long-lived ZLB episode. So our choice to work with the consumption demand shock in \( \nu_{t+j} \) instead of the conventional discount factor shock in \( \zeta_t \) in (1) to generate the baseline path underlying Figures 4 to 6 has no consequence for the results with linearized model. However, the results for the non-linear variant may differ. However, the lower panels in Figure A.1 show that the results are very similar even in the nonlinear, so our choice of baseline appears immaterial for our results.

Another aspect we study to understand how our results differ from Braun et al. (2012, 2013)
due to our AR(1) assumption for government spending instead of the MA-process they work with. Figure A.2 assess this issue by comparing results under our AR(1) process with persistence .95 against the MA process for which \( G_t \) is cut in an uniformal fashion as long at the policy rate is bounded at zero for \( \text{ZLB}_{\text{DUR}} = 1, 2, 3, ..., T \) and set at its steady state value otherwise. Apart from that our solution procedure does not allow for future shock uncertainly, this way of modeling government spending is identical to Braun et al. who in turns follow Eggertsson (2010).

As can be seen from the upper panels of Figure A.2, the MA-process specification increases the marginal spending multiplier substantially for the ZLB durations we consider, as increases in government spending has very benign effects on the potential real interest rate when the duration of the spending hike equals the expected duration of the liquidity trap (see e.g. Erceg and Lindé, 2014). For a one quarter liquidity trap it equals unity, as shown by Woodford (2011). Our fairly persistent AR(1) process tends to dampen the multiplier schedule as a relatively large fraction comes on line when the ZLB is not binding. This feature explains why the AR(1) multiplier is substantially lower in a short lived. However, the AR(1) process is also associated with a substantially lower multiplier even in a fairly long-lived trap compared to the MA process because its has less beningn effects on the potential real rate.

All this is well-known from the literature on linearized models. However, the results on the non-linear model, shown in the lower panels, are less explored. We have already discussed the AR(1) case at length in the text. What we see is that the results for the MA process are quite different for longer ZLB durations, because in constrast to the MA multiplier schedule for the linearized model the MA schedule for the nonlinear model stays essentially flat at unity, in line with the findings of Braun et al. (2013). Hence, our results for the linear and nonlinear models in Figure A.2 are in line with the results in the existing literature.
Figure A.1: Marginal Multipliers: Sensitivity With Respect to Baseline Shock

**Linearized Model**

- Impact Spending Multiplier
- Expected Inflation \(4 \cdot \mathbb{E}(\pi_{t+1})\)
- Govt Debt to GDP (After 1 Year)

**Nonlinear Model**

- Impact Spending Multiplier
- Expected Inflation \(4 \cdot \mathbb{E}(\pi_{t+1})\)
- Govt Debt to GDP (After 1 Year)

_line curves represent Consumption Demand Shock, while dotted lines represent Discount Factor Shock._
Figure A.2: Marginal Multipliers: Sensitivity With Respect to Specification of Spending Process

**Linearized Model**
- Impact Spending Multiplier
- Expected inflation \(4E_t\pi_{t+1}\)
- Govt Debt to GDP (After 1 Year)

**Nonlinear Model**
- Impact Spending Multiplier
- Expected Inflation \(4E_t\pi_{t+1}\)
- Govt Debt to GDP (After 1 Year)

**Legend**
- AR(1) Process for Spending (Benchmark)
- MA Process for Spending
Appendix B. The Workhorse New Keynesian Model


Below, we first describe the firms’ and households’ problem in the model, and state the market clearing conditions. Some parts will be repetitive w.r.t. the baseline model in Section 2, but we nevertheless include a comprehensive description to make it self-contained. However, given that the mechanics underlying the financial accelerator are well-understood, we simplify our exposition by focusing on a special case of our model which abstracts from a financial accelerator. We conclude our model description with a brief description of how the model is modified to include the financial accelerator (Section B.5). Then we provide details on the parameterization of the model and how it is solved.

B.1. Firms and Price Setting

Final Goods Production As in the benchmark model, the single final output good $Y_t$ is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$
\int_0^1 G_Y \left( \frac{Y_t(f)}{Y_t} \right) df = 1.
$$

(B.11)

Following Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007) we assume that $G_Y(.)$ is given by a strictly concave and increasing function:

$$
G_Y \left( \frac{Y_t(f)}{Y_t} \right) = \left( \frac{\phi_p - (1 - \phi_p)\epsilon_p}{\phi_p} \right) \frac{Y_t(f)}{Y_t} + \left( \frac{\phi_p - 1}{\phi_p} \right) \epsilon_p \frac{1 - (\phi_p - 1)\epsilon_p}{\phi_p} + \left( 1 - \frac{\phi_p - 1}{\phi_p} \epsilon_p \right),
$$

(B.12)

where $\phi_p \geq 1$ denotes the gross markup of the intermediate firms. The parameter $\epsilon_p$ governs the degree of curvature of the intermediate firm’s demand curve. When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When $\epsilon_p$ is positive the firms instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. Notice that $G_Y(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both the product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of
the output index $Y_t$, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price $P_t$, and hence solve the following problem:

$$\max_{\{Y_t(Y_t(f))\}} \int_0^1 P_t(f) Y_t(f) df,$$

subject to the constraint (B.11). The first order conditions for this problem is given by the equations in (9). Note that for $\epsilon_p = 0$ and $\Lambda_t^0 = 1$ in each period $t$, the demand and pricing equations collapse to the usual Dixit-Stiglitz expressions

$$\frac{Y_t(f)}{P_t} = \left( \frac{P_t(f)}{P_t} \right)^{-\frac{\phi_p}{1-\phi_p}}, P_t = \int P_t(f)^{1-\phi_p} df. \tag{B.14}$$

*Intermediate Goods Production* A continuum of intermediate goods $Y_t(f)$ for $f \in [0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces the demand schedule in eq. (9) from the final goods firms through the solution to the problem in (B.13), which varies inversely with its output price $P_t(f)$ and directly with aggregate demand $Y_t$.

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = K_t(f)^{\alpha} \left[ \gamma^f L_t(f) \right]^{1-\alpha} - \gamma^f \Phi, \tag{B.15}$$

where $\gamma^f$ represents the labour-augmenting deterministic growth rate in the economy, $\Phi$ denotes the fixed cost (which is related to the gross markup $\phi_p$ so that profits are zero in the steady state).

Firms face perfectly competitive factor markets for renting capital and hiring labor. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital $R_{K,t}$ and the aggregate wage index $W_t$ (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm $f$ faces a constant probability, $1 - \xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to re-optimize its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price by a weighted combination of the lagged and steady-state rate of inflation, i.e., $P_t(f) = (1 + \pi_{t-1})^{\epsilon_p} P_{t-1}(f)$ where $0 \leq \epsilon_p \leq 1$ and $\pi_{t-1}$ denotes
net inflation in period \( t - 1 \). [Check: We should relax the indexation assumption, and work with a variant with out indexation, i.e. \( t_p = 0 \).] A positive value of \( t_p \) introduces structural inertia into the inflation process. All told, this leads to the following optimization problem for the intermediate firms

\[
\max_{\tilde{P}_t(f)} E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \frac{\tilde{P}_t}{\tilde{P}_t P_{t+j}} \left[ \tilde{P}_t(f) \left( \Pi_{s=1}^{j} (1 + \pi_{t+s-1}) - MC_{t+j} \right) Y_{t+j}(f) \right], \quad (B.16)
\]

where \( \tilde{P}_t(f) \) is the newly set price. Notice that with our assumptions all firms that re-optimize their prices actually set the same price.

### B.2. Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household’s labor services \( L_t(h), h \in [0,1] \), as imperfect substitutes for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households’ labor hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labor is equal to the sum of firms’ demands. The aggregated labor index \( L_t \) has the well-known Dixit-Stiglitz form:

\[
L_t = \left[ \int_0^1 (L_t(h))^{\frac{1}{\phi_w}} dh \right]^{\phi_w}, \quad 1 \leq \phi_w < \infty. \quad (B.17)
\]

The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household’s wage rate \( W_t(h) \) as given, and then sells units of the labor index to the production sector at their unit cost \( W_t \):

\[
W_t = \left[ \int_0^1 W_t(h)^{\frac{1}{\phi_w}} dh \right]^{1-\phi_w}. \quad (B.18)
\]

which can naturally be interpreted as the aggregate wage rate. From the FOCs, the aggregator’s demand for the labor hours of household \( h \)—or equivalently, the total demand for this household’s labor by all goods-producing firms—is given by

\[
L_t(h) = \frac{W_t(h)}{W_t} \left[ \frac{\phi_w}{1-\phi_w} \right] L_t, \quad (B.19)
\]

The utility function of a typical member of household \( h \) is

\[
E_t \sum_{j=0}^{\infty} \beta^j \xi_{t+j} \left[ \ln (C_{t+j}(h) - \alpha C_{t+j-1}(h) - C \nu_{t+j}) - \left( \frac{\chi \delta}{1 + \sigma_i} L_{t+j}(h)^{1+\sigma_i} \right) + \mu_0 F \left( \frac{MB_{t+j+1}(h)}{P_{t+j}} \right) \right], \quad (B.20)
\]
where the discount factor $\beta$ satisfies $0 < \beta < 1$. The period utility function depends on household $h$'s current and lagged consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for external habit persistence. The period utility function also depends inversely on hours worked $L_t(h)$. Finally, the period utility function also depends on the households end-of-period real money balances, $\frac{MB_{t+1}(h)}{P_{t+1}}$, a savings shock $\zeta_t$, and a consumption demand shock, $\nu_t$. The subutility function $F(.)$ over real balances is assumed to have a satiation point to account for the possibility of a zero nominal interest rate; see Eggertsson and Woodford (2003) for further discussion.

Household $h$'s budget constraint in period $t$ states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$P_t (1 + \tau_{C,t}) C_t(h) + P_t I_t(h) + \frac{B_{G,t+1}(h)}{\varepsilon_{B,t} R_t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$$

$$= B_{G,t}(h) + (1 - \tau_{N,t}) W_t(h) L_t(h) + (1 - \tau_{K,t}) [R_{K,t} Z_t(h) K^p_t(h) - (a(Z_t(h)) + \delta) K^p_t(h)] + \Gamma_t(h) - T_t(h).$$

Thus, the household purchases part of the final output good (at a price of $P_t$), which it chooses either to consume $C_t(h)$ (subject to a VAT tax $\tau_{C,t}$) or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans (2005), investment augments the household’s (end-of-period) physical capital stock $K^p_{t+1}(h)$ according to

$$K^p_{t+1}(h) = (1 - \delta) K^p_t(h) + \varepsilon_{I,t} \left[ 1 - S \left( \frac{I_t(h)}{I_{t-1}(h)} \right) \right] I_t(h),$$

where $\varepsilon_{I,t}$ is an “investment-specific” technology shock (see Fisher, 2006) that is assumed to follow the process:

$$\ln \varepsilon_{I,t} = \rho_I \ln \varepsilon_{I,t-1} + \eta_{I,t}, \eta_{I,t} \sim N(0, \sigma_I).$$

Thus, the extent to which investment by each household $h$ turns into physical capital is assumed to depend on the investment-specific shock $\varepsilon_{I,t}$ and how rapidly the household changes its rate of investment according to the function $S \left( \frac{I_t(h)}{I_{t-1}(h)} \right)$, which we specify as

$$S(x_t) = \frac{\varphi}{2} (x_t - \gamma)^2.$$

Notice that this function satisfies $S'(\gamma) = 0$, $S''(\gamma) = 0$ and $S'''(\gamma) = \varphi$.

In addition to accumulating physical capital, households may augment their financial assets through increasing their government nominal bond holdings ($B_{G,t+1}$), from which they earn an

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*B.2* Note that we deviate slightly from the notation in CEE by using $h$ to index households and using $\kappa$ to denote the degree of habit formation.

*B.3* For simplicity, we assume that $\mu_0$ is sufficiently small that changes in the monetary base have a negligible impact on equilibrium allocations, at least to the first-order approximation we consider. It therefore does not affect the Governments seigniorage, see eq. (B.28).
interest rate of \( R_t \). The return on these bonds is also subject to a risk-premium shock, \( \varepsilon_{B,t} \), which follows
\[
\ln \varepsilon_{B,t} = \rho_B \ln \varepsilon_{B,t-1} + \eta_{B,t}, \eta_{B,t} \sim N(0, \sigma_B).
\] (B.25)

We assume that agents can engage in frictionless trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term \( \int \xi_{t,t+1}B_{D,t+1}(h) - B_{D,t}(h) \) represents net purchases of these state-contingent domestic bonds, with \( \xi_{t,t+1} \) denoting the state-dependent price, and \( B_{D,t+1}(h) \) the quantity of such claims purchased at time \( t \).

On the income side, each member of household \( h \) earns after-tax labor income \( (1 - \tau_{N,t})W_t(h)L_t(h) \), after-tax capital rental income of \( (1 - \tau_{K,t})R_{K,t}Z_t(h)K^P_t(h) \) after paying a utilization cost of the physical capital equal to \( (1 - \tau_{K,t})a(Z_t(h))K^P_t(h) \) where \( Z_t(h) \) is the capital utilization rate, so that capital services provided by household \( h \), \( K_t(h) \), equals \( Z_t(h)K^P_t(h) \). The capital utilization adjustment function \( a(Z_t(h)) \) is assumed to be given by
\[
a(Z_t(h)) = \frac{r^k}{\tilde{z}_1} \exp(\tilde{z}_1(Z_t(h) - 1)) - 1,
\] (B.26)
where \( r^k \) is the steady state net real interest rate \( (\bar{R}_{K,t}/\bar{P}_t) \). Notice that the adjustment function satisfies \( a(1) = 0, a'(1) = r^k, \) and \( a''(1) = r^k \tilde{z}_1 \). Finally, each member also receives an aliquot share \( \Gamma_t(h) \) of the profits of all firms, and pays a lump-sum tax of \( T_t(h) \) (regarded as taxes net of any transfers).

In every period \( t \), each member of household \( h \) maximizes the utility function (B.20) with respect to its consumption, investment, (end-of-period) physical capital stock, capital utilization rate, bond holdings, and holdings of contingent claims, subject to its labor demand function (B.19), budget constraint (B.21), and transition equation for capital (B.22).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal to re-optimize its wage contract in a given period is denoted by \( 1 - \xi_w \). For those households that do not get a signal to re-optimize, we specify the following dynamic indexation scheme for the adjustment of the wages: \( W_t(h) = (\gamma(1 + \pi_{t-1}))^{\xi_w}W_{t-1}(h) \). All told, this leads to the following optimization problem for the households
\[
\max_{\tilde{W}_t(h)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\tilde{\pi}_{t+j}P_t}{\tilde{\pi}_tP_{t+j}} \left[ \tilde{W}_t(h) \left( \Pi_{s=1}^{j} \gamma(1 + \pi_{t+s-1})^{\xi_w} (1 + \pi)^{1-\xi_w} \right) - W_{t+j} \right] L_{t+j}(h),
\] (B.27)
where \( \tilde{W}_t(h) \) is the newly set wage; notice that with our assumptions all households that reoptimize their wages will actually set the same wage.
B.3. Fiscal and Monetary Policy

Government purchases $G_t$ are exogenous, and the process for government spending is adopted from the benchmark model (see eq. 22). As before, government purchases have no direct effect on the marginal utility of private consumption, nor do they serve as an input into goods production. The consolidated government sector budget constraint is

$$B_{G,t} = (1 + i_{t-1}) B_{G,t-1} + P_t G_t - \tau_{C,t} P_t C_t - \tau_{L,t} W_t L_t - \tau_{K,t} [R_{K,t} Z_t - (a (Z_t) + \delta)] K_t^P - T_t. \quad (B.28)$$

In comparison to the evolution of government debt in the benchmark model (see eq. 13), the constraint in eq. (B.28) add receipts from VAT and capital income taxes. However, the tax rates $\tau_C, \tau_L$ and $\tau_K$, are kept constant. Instead, following the bulk of the literature on fiscal multipliers we assume that the fiscal authority use lump-sum taxes (as share of nominal trend GDP, $\tau_t \equiv \frac{T_t}{P_t Y_t}$), to stabilize debt according to the simple rule

$$\tau_t - \tau = \varphi_b (b_{G,t-1} - b_G), \quad (B.29)$$

where $b_{G,t-1} = B_{G,t} / (4\tilde{P}_t \tilde{Y}_t)$.

Turning to the central bank, it is assumed to follow a linearized variant of (15) subject to some smoothing on the notional interest rate $i_t^{not}$:

$$i_t^{not} = (1 - \rho_R) (i + \pi (\pi_t - \pi) + \pi x_t) + \rho R i_{t-1}^{not},$$

$$i_t = \max (0, i_t^{not}).$$

where $x_t \equiv \ln \left( \frac{Y_t}{Y_t^{pot}} \right)$, $Y_t^{pot}$ denotes the level of output that would prevail if prices and wages were flexible, and $i$ the steady-state (net) nominal interest rate, which is given by $r + \pi$ where $r \equiv 1/\beta - 1$.

B.4. Market Clearing Conditions

Total output of the final goods sector is used as follows:

$$Y_t = C_t + I_t + G_t + a (Z_t) K_t^P, \quad (B.30)$$

where $a (Z_t) \bar{K}_t$ is the capital utilization adjustment cost.

Finally, we need to specify the aggregate production constraint. To do that, we note that the unweighted sum of the intermediate firms’ output equals

$$Y_t^{sum} = \int_0^1 Y_t (f) \, df, \quad (B.31)$$
which from eq. (B.15) can be rewritten as

\[ Y_{t}^{\text{sum}} = \int_{0}^{1} \left[ K_{t}(f)^{\alpha} \left[ \gamma^{t}L_{t}(f) \right]^{1-\alpha} - \gamma^{t} \Phi \right] df \]  
\[ = \left( \frac{K_{t}}{\left( \gamma^{t}L_{t} \right)} \right)^{\alpha} \int_{0}^{1} \gamma^{t}L_{t}(f)df - \gamma^{t} \Phi, \]  

(B.32)

where the second equality follows from the fact that every firms capital-labor ratio will be the same in equilibrium.

From the first-order conditions to the final goods aggregator problem (9), it follows that

\[ Y_{t}^{\text{sum}} = Y_{t} \int_{0}^{1} \frac{\phi_{p}}{\phi_{p}-(\phi_{p}-1)\epsilon_{p}} \left( \frac{P_{t}(f)}{P_{t}^{1}} \right)^{1-\phi_{p}} \left( \frac{1}{\bar{N}_{t}^{p}} \right)^{\phi_{p}-(\phi_{p}-1)\epsilon_{p}} \phi_{p}^{(1-\phi_{p})\epsilon_{p}} df, \]  
\[ = \left( \frac{K_{t}}{\left( \gamma^{t}L_{t} \right)} \right)^{\alpha} \int_{0}^{1} L_{t}(h)dh - \gamma^{t} \Phi = Y_{t} \int_{0}^{1} \phi_{p} \left( \frac{P_{t}(f)}{P_{t}^{1}} \right)^{1-\phi_{p}} \left( \frac{1}{\bar{N}_{t}^{p}} \right)^{\phi_{p}-(\phi_{p}-1)\epsilon_{p}} \phi_{p}^{(1-\phi_{p})\epsilon_{p}} df. \]  

(B.33)

so that

\[ \left( \frac{K_{t}}{\left( \gamma^{t}L_{t} \right)} \right)^{\alpha} \int_{0}^{1} L_{t}(h)dh - \gamma^{t} \Phi = Y_{t} \int_{0}^{1} \phi_{p} \left( \frac{P_{t}(f)}{P_{t}^{1}} \right)^{1-\phi_{p}} \left( \frac{1}{\bar{N}_{t}^{p}} \right)^{\phi_{p}-(\phi_{p}-1)\epsilon_{p}} \phi_{p}^{(1-\phi_{p})\epsilon_{p}} df. \]

By inserting the expression for the unweighted sum of labor, \( \int_{0}^{1} L_{t}(h)dh \) from eqs. (B.17) and (B.19) into this last expression, we can finally derive the aggregate production constraint which depends on aggregate technology, capital, labor, fixed costs, as well as the price and wage dispersion terms.

B.5. Production of capital services

The model is amended with a financial accelerator mechanism into the model following the basic approach of Bernanke, Gertler and Gilchrist (1999). Thus, the intermediate goods producers rent capital services from entrepreneurs (at the price \( R_{K,t} \)) rather than directly from households. Entrepreneurs purchase physical capital from competitive capital goods producers (and resell it back at the end of each period), with the latter employing the same technology to transform investment goods into finished capital goods as described by equations B.22 and B.24). To finance the acquisition of physical capital, each entrepreneur combines his net worth with a loan from a bank, for which the entrepreneur must pay an external finance premium (over the risk-free interest rate set by the central bank) due to an agency problem. Banks obtain funds to lend to the entrepreneurs by issuing deposits to households at the risk-free interest rate set by the central bank, with households bearing no credit risk (reflecting assumptions about free competition in
banking and the ability of banks to diversify their portfolios). In equilibrium, aggregate shocks induce endogenous variations in entrepreneurial net worth – i.e., the leverage of the corporate sector – and thus fluctuations in the corporate finance premium.\textsuperscript{B.4} Moreover, we allow for an exogenous net worth shock, which through its effect on the net worth of entrepreneurs affect the corporate risk premium.

\textbf{B.6. Calibration and Solution}

In Table A.1, we state all the parameters used in the model. These parameters are all standard, apart from the fact that we use somewhat larger-than-normal degrees habit formation ($\varepsilon = 0.95$) and investment adjustment cost ($\varphi = 10$) to generate a government spending multiplier close to unity in the short-term. We do not allow for time-varying capital utilization and working capital in the current calibration, and we have full indexation to lagged inflation in price- and wage-setting for non-optimizers. \textit{[NOTE: We should check sensitivity of our results to these choices.]} We solve numerically for the flex price-wage version of the model, and in this variant everything is identical with the exception that the probability for re-optimizing prices (firms) and wages (households) equals unity in each period. \textit{[Remains to be written.]}

\begin{footnotesize}
\begin{itemize}
\item B.4 We follow Christiano, Motto and Rostagno (2008) by assuming that the debt contract between entrepreneurs and banks is written in nominal terms (rather than real terms as in Bernanke, Gertler and Gilchrist, 1999). For further details about the setup, see Bernanke, Gertler and Gilchrist (1999), and Christiano, Motto and Rostagno (2008). An elaborate exposition is also provided in Christiano, Trabandt and Walentin (2007).
\end{itemize}
\end{footnotesize}
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>Steady state hours worked</td>
<td>1.00</td>
<td>$\epsilon_p$</td>
<td>Kimball Elas. GM</td>
<td>-11.7</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Gross wage markup</td>
<td>1.20</td>
<td>$\zeta_w$</td>
<td>Ind. for non-opt. wages</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Cons demand shock per</td>
<td>0.85</td>
<td>$\xi$</td>
<td>Degree of int. habit</td>
<td>0.95</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9999</td>
<td>$\sigma_l$</td>
<td>Frisch elas. of lab supply</td>
<td>1.00</td>
</tr>
<tr>
<td>$\xi_{w}$</td>
<td>Calvo prob. wages</td>
<td>0.75</td>
<td>$\rho_B$</td>
<td>Risk-premium shock</td>
<td>0.00</td>
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Panel B: Firms and Entrepreneurs

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>Investment adj. cost</td>
<td>0.667</td>
<td>$\phi_p$</td>
<td>Gross price markup</td>
<td>1.10</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
<td>$\psi$</td>
<td>Capital utilization cost</td>
<td>0.08</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Ind. for non-opt. prices</td>
<td>1.00</td>
<td>$\gamma$</td>
<td>Steady state gross growth</td>
<td>1.0082</td>
</tr>
<tr>
<td>$W_{nw}$</td>
<td>Net worth shock</td>
<td>0.00</td>
<td>$\eta$</td>
<td>Working capital share</td>
<td>0.00</td>
</tr>
<tr>
<td>$F_{e}$</td>
<td>Bankruptcy prob. of entrep.</td>
<td>0.005</td>
<td>$NW/K$</td>
<td>Net worth to Capital Ratio in SS</td>
<td>0.667</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Bank monitoring cost</td>
<td>0.20</td>
<td>$ENT_s$</td>
<td>Entrep. Endowment to GDP Ratio</td>
<td>0.002</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Entrep. survival prob.</td>
<td>0.995</td>
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</table>

Panel C: Fiscal and Monetary Policy

<table>
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<tr>
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<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_y$</td>
<td>Gov’t $G/Y$ ss-ratio</td>
<td>0.20</td>
<td>$\pi$</td>
<td>Steady state net infl. rate</td>
<td>0.005</td>
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<tr>
<td>$\rho_y$</td>
<td>Gov’t spending per.</td>
<td>0.95</td>
<td>$r_\pi$</td>
<td>Long-term $\pi$ coeff. in MP rule</td>
<td>2.50</td>
</tr>
<tr>
<td>$\tau_C$</td>
<td>VAT tax rate</td>
<td>0.05</td>
<td>$r_x$</td>
<td>Long-term gap coeff. in MP rule</td>
<td>0.125</td>
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<tr>
<td>$\tau_N$</td>
<td>Labor income tax rate</td>
<td>0.30</td>
<td>$\rho_R$</td>
<td>Smoothing coeff. in MP rule</td>
<td>0.50</td>
</tr>
<tr>
<td>$\tau_K$</td>
<td>Capital income tax rate</td>
<td>0.25</td>
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<tr>
<td>$B_{C}/(4PY)$</td>
<td>Govt debt to GDP</td>
<td>0.90</td>
<td></td>
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<tr>
<td>$\varphi_b$</td>
<td>Lumpsum debt tax coeff.</td>
<td>0.04</td>
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</table>

Notes: Our parameterization implies a steady state annualized nominal interest rate of 3.75 percent and an inflation rate of 2 percent at an annualized rate. The flex-price wage equilibrium imposes identical parameter, except that $\xi^{flex}_w = \xi^{flex}_p = 0.$