What is Cyclical in Credit Cycles?

Rui Cui

November 5, 2013

Credit Cycles Facts

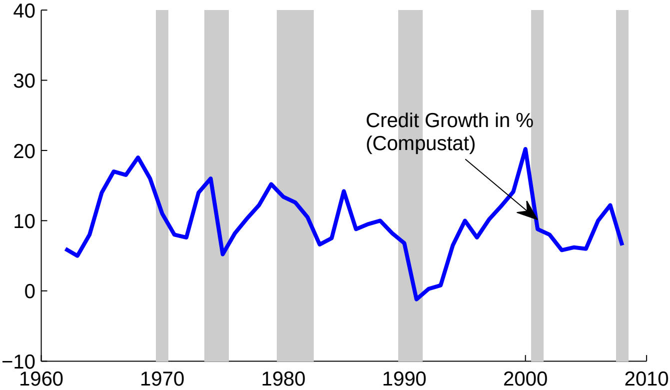


Figure : Time Series Plot of Aggregate Credit Growth for Compustat Non-Financial Firms

Banking Sector Balance Sheet

- Examples: Brunnermeier-Sannikov, He-Krishnamurthy, Kiyotaki-Gertler

Credit Cycles Facts

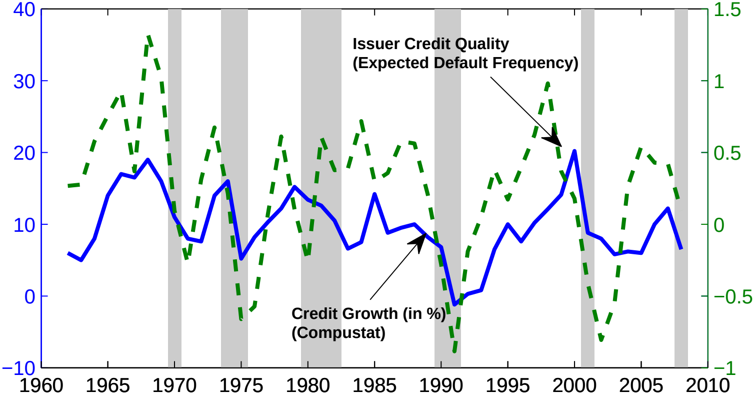


Figure : Time Series Plot of Credit Quality and Credit Growth for Compustat Non-Financial Firms

Banking Sector Balance Sheet

Mechanism

(0) Current banking sector balance sheet determines effective discount rate

Both asset and liability sides matters

Mechanism

(0) Current banking sector balance sheet determines effective discount rate
Both asset and liability sides matters

$\Downarrow$

(1) Bankers evaluates potential projects by computing their risk-adjusted present values
Prices of different types of capital evolves differently over time

Mechanism

(0) Current banking sector balance sheet determines effective discount rate
Both asset and liability sides matters

$\Downarrow$

(1) Bankers evaluates potential projects by computing their risk-adjusted present values
Prices of different types of capital evolves differently over time

$\Downarrow$

(2) Capital producers respond to fluctuating capital prices through optimal production decisions
Financial sector health shifts the production frontier of the aggregate economy

Mechanism

(0) Current banking sector balance sheet determines effective discount rate
Both asset and liability sides matters

$\Downarrow$

(1) Bankers evaluates potential projects by computing their risk-adjusted present values
Prices of different types of capital evolves differently over time

$\Downarrow$

(2) Capital producers respond to fluctuating capital prices through optimal production decisions
Financial sector health shifts the production frontier of the aggregate economy

$\Downarrow$

(3) Once financed, these projects stay and accumulate on banks balance sheets
Fully solved general equilibrium model to extract dynamic implications

Results

- Interaction between production heterogeneity and financial frictions generates fundamental economic forces that leads to endogeneous boom-bust cycles
  - A risks buildup process
  - A slow recovery process

Results

- Interaction between production heterogeneity and financial frictions generates fundamental economic forces that leads to endogeneous boom-bust cycles
  - A risks buildup process
  - A slow recovery process
- Financial frictions amplifies both positive and negative shocks
  - New perspective on “asymmetry” in impulse responses

Connection to Austrian Business Cycle Theory

... For businessmen, seeing the interest rate fall as a change of market signals: Investments, particularly in lenghty and time-consuming projects, which previously looked unprofitable now seem profitable, because of the fall of the interest charge. They expand their investment in durable equipment, in capital goods, in industrial raw material, in construction as compared to their direct production of consumer goods.

Rothbard (1969) recount of Hayek's *Prices and Production* (1931)

Connection to Banking Literature

- Diamond and Dybvig (1983), Holmstrom and Tirole (1997,1998)
- "Liquidity cycles" is the next step (Moreira and Savov (2013), Sannikov (2013))

Set up

- Three types of agents: households, bankers and capital producers.
- Risk neutral households can consume and make deposits with bankers, they maximize

$$E\left[\int_0^\infty exp\left(-\rho t\right) dC_t^H\right]$$

- Bankers hold all risky capital. I impose that bankers consume $\lambda N dt$ (N is bankers networth). They maximize

$$E\left[\int_0^\infty exp\left(-\lambda t\right) log\left(\lambda N_t\right) dt\right]$$

- This is a continuous time adoption of Kiyotaki-Gertler model, but with fixed risk free rate ρ and simplified effective bankers' pricing kernel $\theta_B^t = exp\left(-\lambda t\right) \frac{\lambda}{N_t}$.

Capital Producers

- Two types of capital producers producing $K^{j\in\{A,B\}}$, both captial produces cash flow at rate $AK^j dt$, they depreciate at rate δ.
- But they have differential exposure to the systematic shock, in aggregate

$$\frac{dK^j}{K^j} = \underbrace{(\Phi_j(i_j) - \delta)}_{\text{net investment}} dt + \bar{\sigma}_j dZ_t$$

- "Quality" is captured by $\bar{\sigma}_A < \bar{\sigma}_B$. Cash flow from type B projects are more sensitive to macroeconomic shocks than type A projects.

Risk Adjustment

- Capital producers are owned by household, but can only sell their capital to bankers. The production function of type j capital is

$$\Phi_j(i_j) = \sqrt{\frac{2i_j}{\kappa_j}}$$

- Key assumption: $\kappa_A > \kappa_B$. Supply of high quality projects are limited.

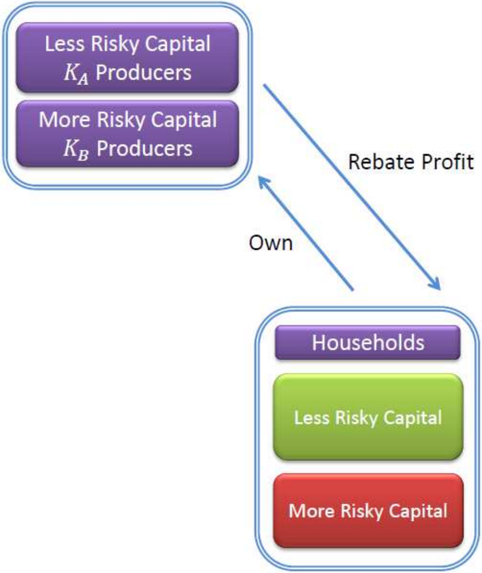
- Key endogeneous variable is the risk adjusted present value of the cash flow (net of investment) produced by type j capital

$$q_j = PV_j = E\left[\int_0^\infty A\frac{K_t^j}{K_0^j}\theta_B^t dt\right]$$

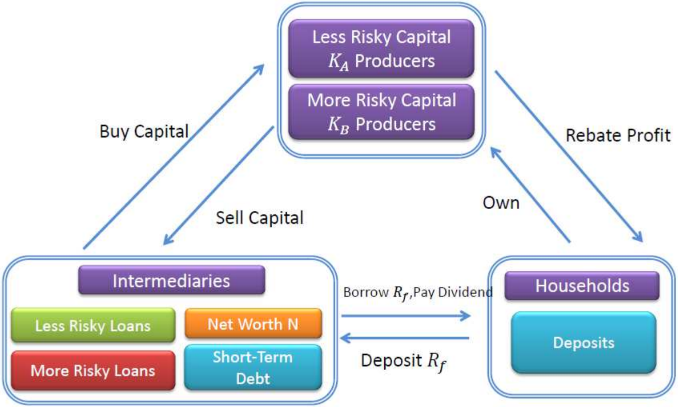
where

$$\frac{dK_t^j}{K_0^j} = -\delta dt + \bar{\sigma}_j dZ_t$$

Model Schematic



Model Schematic



Capital Producers' Problem

- Given q_A, q_B, capital producers solve a static problem

$$max_{i_j} \Phi_j \left(i_j\right) K_j q_j - i_j K_j$$

- Optimal investment follows

$$\Phi_j^* \left(i_j^*\right) = \frac{q_j}{\kappa_j}$$

Bankers Problem

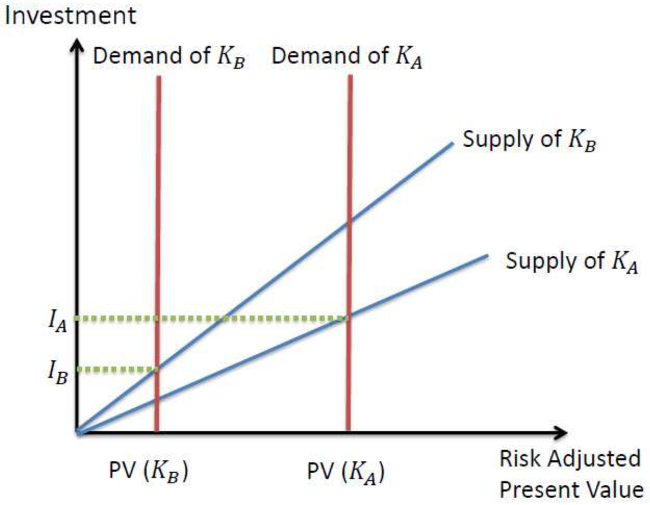
- Given their preference, bankers solves a portfolio problem that resembles standard mean-variance efficient investors

$$max_{\alpha_A,\alpha_B} E\left[\int_0^\infty exp\left(-\lambda t\right) log\left(\lambda N_t\right) dt\right]$$

$$st. \quad \frac{dN_t}{N_t} = -\lambda dt + \left(\alpha_A\pi_A + \alpha_B\pi_B + \left(1 - \alpha_A - \alpha_B\right) r_f\right) dt + \left(\alpha_A\sigma_A + \alpha_B\sigma_B\right) dZ_t$$

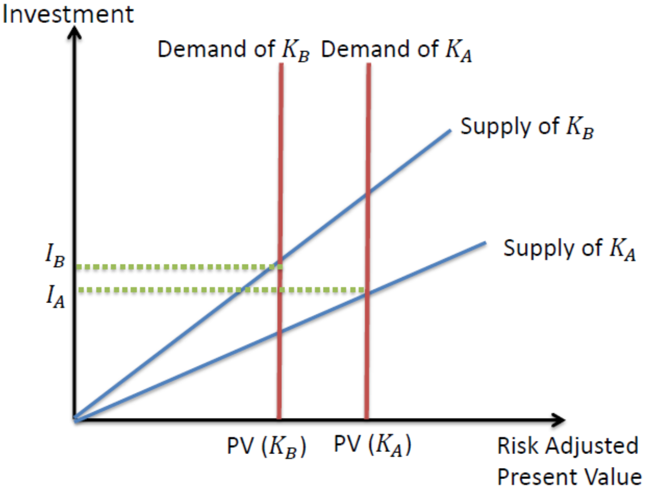
where α_A, α_B are portfolio shares, π_A, π_B are excess returns by investing in K_A, K_B; σ_A, σ_B are return volatilities for K_A, K_B

Equilibrium Investment (1)



- K_A and K_B offers same amount of expected cash flow $\rightarrow$PV difference comes from differential risk adjustments applied to the cash flow.

Equilibrium Investment (2)



Equilibrium Definition

An **equilibrium** of this economy consists of prices processes (q_A, q_B, r_f), and decisions, $(c_H, \alpha_A, \alpha_B, i_A, i_B)$, such that

1. Given prices, households, bankers and capital producers solve their optimization problems.

2. Given decisions, markets for risky capital (K_A, K_B) and risk-free bond clears. This pins down bankers' portfolio choices α_A, α_B

3. Market for goods clear

$$A(K_A + K_B) = i_A K_A + i_B K_B + C_H$$

Solving the Model

1. Conjecture the model has two scaled state variables: "size" and "quality" of intermediaries balance sheet

$$\eta = \frac{N}{q_A K_A + q_B K_B}$$
$$s = \frac{K_B}{K_A + K_B}$$

2. Conjecture

$$d\eta = \mu_\eta dt + \sigma_\eta dZ_t$$
$$ds = \mu_s dt + \sigma_s dZ_t$$

3. Key endogeneous variables are $q_A(\eta, s)$ and $q_B(\eta, s)$, express μ_s, σ_s as

$$\mu_s = s(1-s)\left(\Phi_B(i_B) - \Phi_A(i_A) + \bar{\sigma}_A^2(1-s) - \bar{\sigma}_B^2 s + \bar{\sigma}_A\bar{\sigma}_B(2s-1)\right.$$
$$\sigma_s = s(1-s)(\bar{\sigma}_B - \bar{\sigma}_A)$$

Solving the Model

1. Our goal from now on is to express $\mu_\eta, \sigma_\eta, \pi_A, \pi_B, \sigma_A, \sigma_B$ as functions of q_A, q_B and their first and second order partial derivatives.

2. By definition, we have

$$\begin{aligned}\pi_j &= \left(-\delta + \frac{A + \mu_q^j + \bar{\sigma}_j \sigma_q^j}{q_j}\right) - r_f \\ \sigma_j &= \bar{\sigma}_j + \frac{\sigma_\eta q_\eta^j}{q_j} + \frac{\sigma_s q_s^j}{q_j}\end{aligned}$$

3. Finally, use Ito's lemma on $\eta = \frac{N}{q_A K_A + q_B K_B}$ we get two more equations in μ_η and σ_η

4. Solve this 6 equations in 6 unknowns – so far everything is mechanical

Solving the Model

1. Optimality conditions are summarized in basic asset pricing equations

$$\frac{\pi_A}{\sigma_A} = \frac{\pi_B}{\sigma_B} = \alpha_A \sigma_A + \alpha_B \sigma_B$$

2. Above equations solved on $[\eta, s] \in [\epsilon, 1-\epsilon] \times [0,1]$. Boundary conditions
   2.1 $s = 0, 1 \longrightarrow$Single technology economy, solved in *ODE*
   2.2 $\eta = \epsilon$, impose $q^j_\eta = 0 \longrightarrow$ justified by some type of entry
   2.3 $\eta = 1-\epsilon$, reduce to a system of lower order equations

3. Numerically, I use projection method (5-7th order Chebshev polynomials) to minimize PDE error over a grid.

Parameters

		Model Parameters		
	Interpretation	Model 1	Model 2	Justification
ρ	Household Time Discount Rate	0.01	0.01	Risk Free Rate
λ	Bankers' Time Discount Rate	0.15	0.19	Unconditional Moments
$\bar{\sigma}_A$	Cash Flow Volatility of K_A	0.02	0.046	Output Volatility
$\bar{\sigma}_B$	Cash Flow Volatility of K_B	0.10		
κ_A	Adjustment Cost of K_A	10.00	9.10	Investment Volatility
κ_B	Adjustment Cost of K_B	7.50		
A	Productivity	0.16	0.16	Investment-Capital Ratio
δ	Depreciation	0.10	0.10	Literature

Model 1: Heterogeneous Production. Model 2: Homogeneous P roduction.

Model Solution

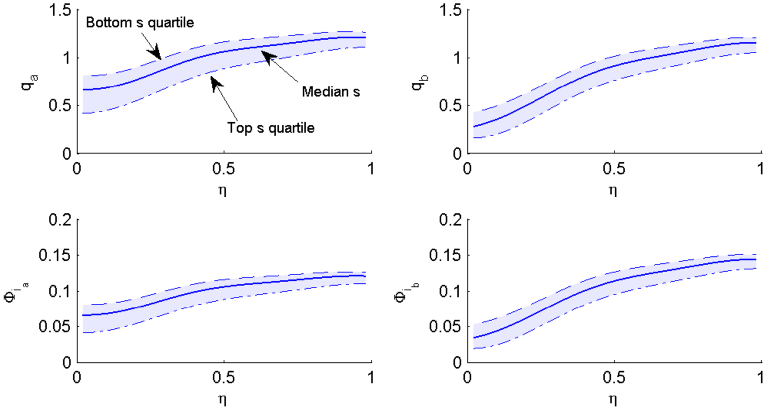


Figure : Solid blue line corresponds to the solution for median s. Shaded area plots the solution corresponding to $25\% - 75\%$ distribution of s. Median output volatility$= 0.046$, top to bottom quartile of the distribution of output volatility is $[0.025, 0.071]$.

Model Solution

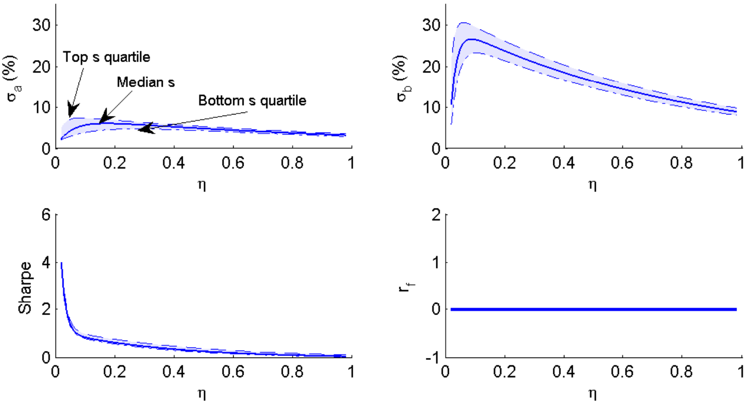


Figure : Solid blue line corresponds to the solution for median s. Shaded area plots the solution corresponding to $25\% - 75\%$ distribution of s. Median output volatility$= 0.046$, top to bottom quartile of the distribution of output volatility is $[0.025, 0.071]$.

Unconditional Moments

Moment	Interpretation	Model 1	Model 2	Data
σ_Y	Median Output Volatility(%)	4.60	4.60	2.0 ~5.0
σ_A	Return Volatility of K_A(%)	5.11	8.54	19.00
σ_B	Return Volatility of K_B(%)	15.23		
SR	Sharpe Ratio	0.33	0.35	0.50
μ_c	Consumption Growth(%)	1.65	1.77	2.00
σ_c	Consumption Growth Volatility(%)	2.45	2.31	1.90
$\sigma_{\Phi(i_A)}$	Investment Volatility of K_A(%)	3.70	6.53	8.13
$\sigma_{\Phi(i_B)}$	Investment Volatility of K_B(%)	10.12		
i_A	Investment / Captial Ratio for K_A(%)	10.9	11.20	11.40
i_B	Investment / Capital Ratio for K_B(%)	12.2		

Model 1: Heterogeneous production. Model 2: Homogeneous production.

Conditional Implications

My model delivers a precise formulation of the following "concepts"

- Risks "Buildup"
  - Without production heterogeneity, positive shocks always push the economy away from crisis state
  - Therefore, well captialized banks (higher η) are associated with lower risks of entering a crisis
  - In my framework, well capitalized banks have strong incentive to take on additional risks — this will show up in the term structure of crisis probability

Conditional Implications

My model delivers a precise formulation of the following "concepts"

- Risks "Buildup"
  - Without production heterogeneity, positive shocks always push the economy away from crisis state
  - Therefore, well captialized banks (higher η) are associated with lower risks of entering a crisis
  - In my framework, well capitalized banks have strong incentive to take on additional risks — this will show up in the term structure of crisis probability
- "Slow" Recovery
  - In the model, bank equity grows by earning this risk premium associated with its asset
  - Risk premium is higher in crisis state, so return on equity is high $\longrightarrow$ recovery is fast
  - When risk taking is endogeous, banks substitute risky, high-yield projects with safe, low-yield ones $\longrightarrow$ return on equity $\Downarrow$ in crisis

Conditional Implications

My model delivers a precise formulation of the following "concepts"

- Risks "Buildup"
  - Without production heterogeneity, positive shocks always push the economy away from crisis state
  - Therefore, well captialized banks (higher η) are associated with lower risks of entering a crisis
  - In my framework, well capitalized banks have strong incentive to take on additional risks — this will show up in the term structure of crisis probability
- "Slow" Recovery
  - In the model, bank equity grows by earning this risk premium associated with its asset
  - Risk premium is higher in crisis state, so return on equity is high $\longrightarrow$ recovery is fast
  - When risk taking is endogeous, banks substitute risky, high-yield projects with safe, low-yield ones $\longrightarrow$ return on equity $\Downarrow$ in crisis
- "Volatility Paradox" — details coming later

Risks Buildup

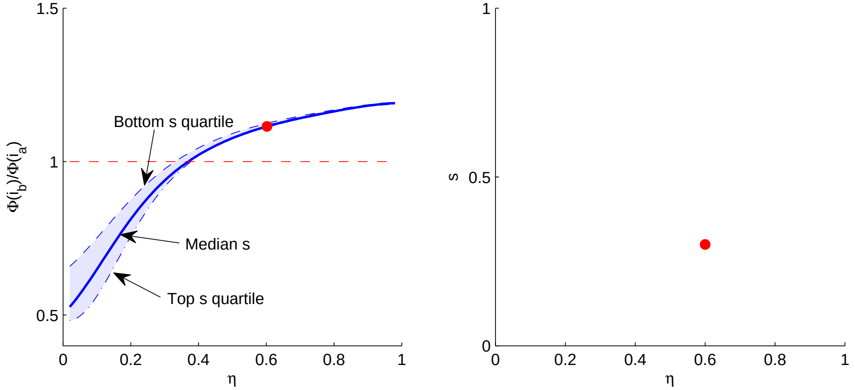


Figure : Left Panel: Investment Ratio as a function of η and s. Right Panel: Drifts of the state variable when starting from $\eta = 0.6$ and median s.

Risks Buildup

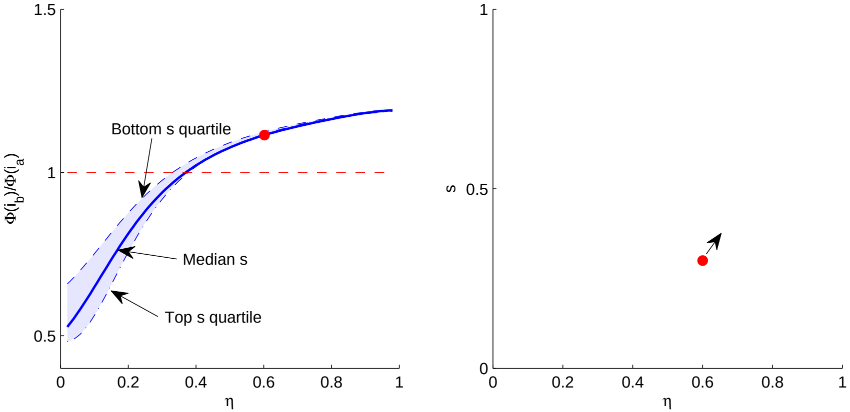


Figure : Left Panel: Investment Ratio as a function of η and s. Right Panel: Drifts of the state variable when starting from $\eta = 0.6$ and median s.

Risks Buildup

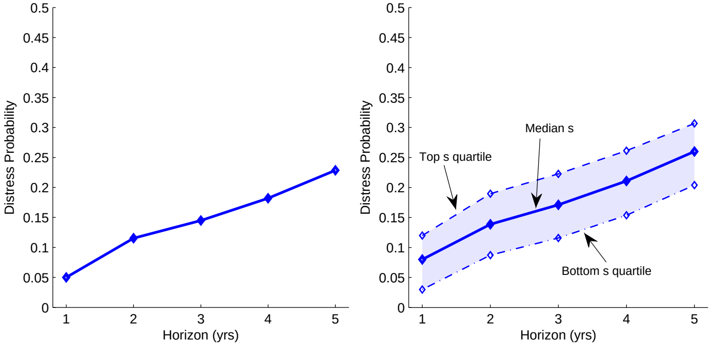


Figure : Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of hitting the top 25% of the Sharpe Ratio when starting from $\eta = 0.6$.

Risks Buildup

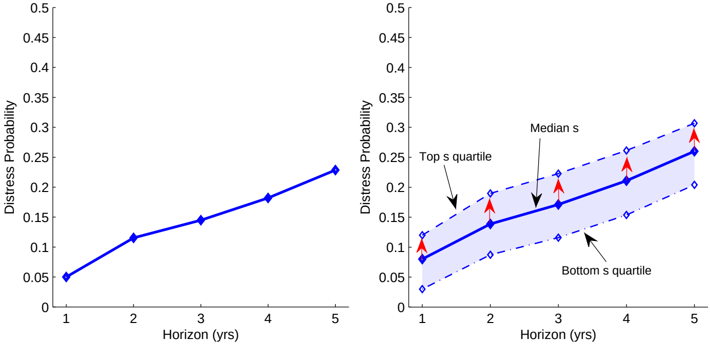


Figure : Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of hitting the top 25% of the Sharpe Ratio when starting from $\eta = 0.6$.

Recovery Dynamics

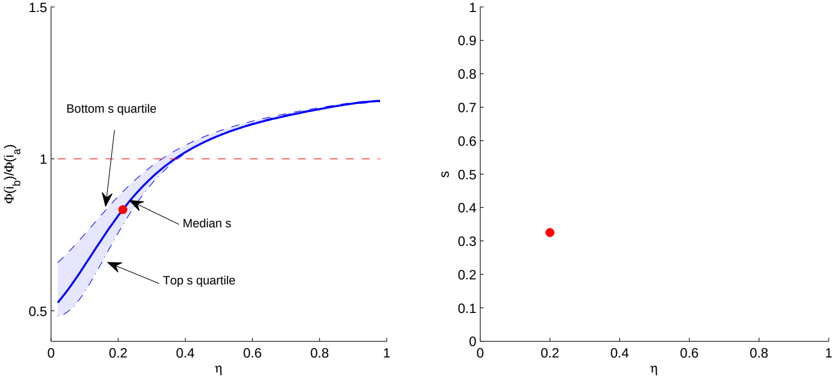


Figure : Left Panel: Investment Ratio as a function of η and s. Right Panel: Drifts of the state variable when starting from $\eta = 0.2$ and median s.

Recovery Dynamics

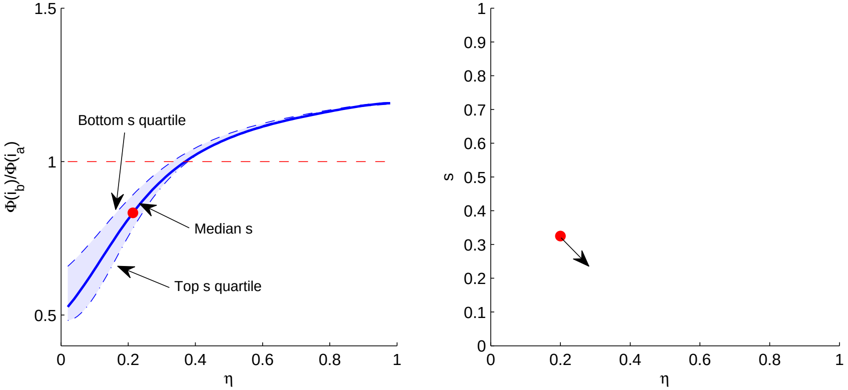


Figure : Left Panel: Investment Ratio as a function of η and s. Right Panel: Drifts of the state variable when starting from $\eta = 0.2$ and median s.

Recovery Dynamics

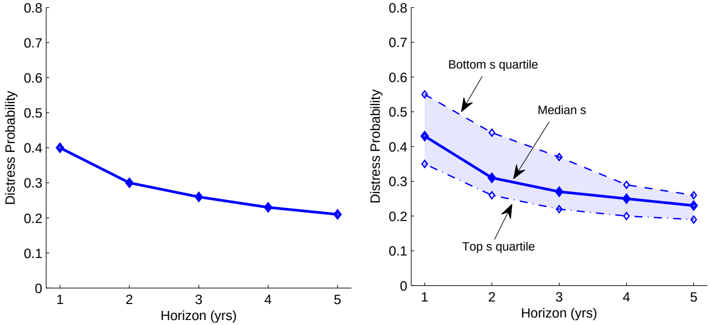


Figure : Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of staying in the top 25% of the Sharpe Ratio when starting from $\eta = 0.2$.

Recovery Dynamics

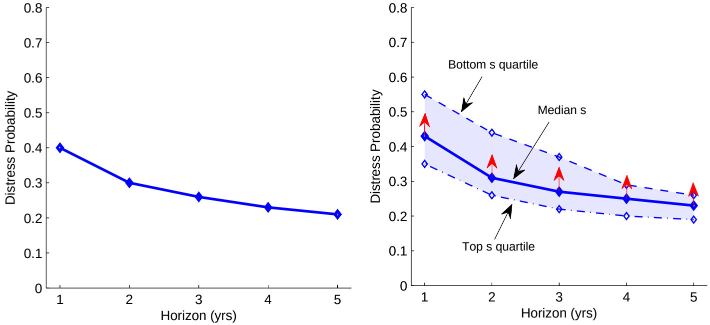


Figure : Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of staying in the top 25% of the Sharpe Ratio when starting from $\eta = 0.2$.

Volatility Paradox

- No concensus has emerged to define "volatility paradox" – generally refers to the observation that prolonged period of low volatility tends to precede a crisis
  - Brunnermeier Sannikov (2013) : compare a series of models differing in their fundamental volatility, banks in low-volatility economies take on more leverage
  - Adrian Boyarchenko (2013): banks run by VaR role, lower financial volatility corresponds to higher leverage $\longrightarrow$ Shorter distance to restructuring boundary

Volatility Paradox

- No concensus has emerged to define "volatility paradox" – generally refers to the observation that prolonged period of low volatility tends to precede a crisis
  - Brunnermeier Sannikov (2013) : compare a series of models differing in their fundamental volatility, banks in low-volatility economies take on more leverage
  - Adrian Boyarchenko (2013): banks run by VaR role, lower financial volatility corresponds to higher leverage $\longrightarrow$ Shorter distance to restructuring boundary

- My model endogenize both fundamenal and financial volatilities
  - Low financial volatility symptomatic of lower risk prices
  - Riskier projects come into the money and get financed
  - Negative correlation between financial volatility and growth in fundamental volatility
  - Accumulation of riskier project tend to coincide with a period of low financial volatility and pushes economy closer to a crisis

Volatility Paradox

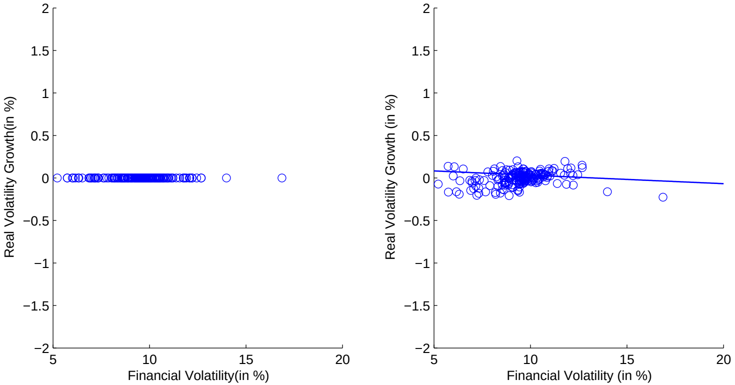


Figure : Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. Simulated 200 years.

Conclusion

- **Main Economic Message:** Through financial sector's optimal financing decision, the real economy accumulates different types of productive capital over the credit cyles.
- **Main Empirical Message:** In the data, credit quality of the marginal borrowers is counter-cyclical.
- **Main Theoretical Message:** Need to keep track of both asset and liability side of the financial sector. Requires sophisticated numerical / technical methods to solve the model.
- **Main Quantitative Message:** I extract model's conditional implications from the term structure of distress probabilities.