What is Cyclical in Credit Cycles?

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Credit Cycles Facts

Figure: Time Series Plot of Aggregate Credit Growth for Compustat Non-Financial Firms
Banking Sector Balance Sheet

- Examples: Brunnermeier-Sannikov, He-Krishnamurthy, Kiyotaki-Gertler
Figure: Time Series Plot of Credit Quality and Credit Growth for Compustat Non-Financial Firms
Banking Sector Balance Sheet

Intermediaries

Less Risky Assets

Net Worth N

More Risky Assets

Short-Term Debt
Mechanism

(0) Current banking sector balance sheet determines effective discount rate
Both asset and liability sides matters
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Both asset and liability sides matters

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(1) Bankers evaluates potential projects by computing their risk-adjusted present values
Prices of different types of capital evolves differently over time
Mechanism

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(2) Capital producers respond to fluctuating capital prices through optimal production decisions
Financial sector health shifts the production frontier of the aggregate economy
Mechanism

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(2) Capital producers respond to fluctuating capital prices through optimal production decisions
Financial sector health shifts the production frontier of the aggregate economy

↓

(3) Once financed, these projects stay and accumulate on banks balance sheets
Fully solved general equilibrium model to extract dynamic implications
Results

- Interaction between production heterogeneity and financial frictions generates fundamental economic forces that leads to endogenous boom-bust cycles
  - A risks buildup process
  - A slow recovery process
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- Interaction between production heterogeneity and financial frictions generates fundamental economic forces that lead to endogenous boom-bust cycles
  - A risks buildup process
  - A slow recovery process

- Financial frictions amplifies both positive and negative shocks
  - New perspective on “asymmetry” in impulse responses
... For businessmen, seeing the interest rate fall as a change of market signals: Investments, particularly in lengthy and time-consuming projects, which previously looked unprofitable now seem profitable, because of the fall of the interest charge. They expand their investment in durable equipment, in capital goods, in industrial raw material, in construction as compared to their direct production of consumer goods.

Rothbard (1969) recount of Hayek’s *Prices and Production* (1931)
Connection to Banking Literature

- “Liquidity cycles” is the next step (Moreira and Savov (2013), Sannikov (2013) )
Set up

- Three types of agents: households, bankers and capital producers.

- Risk neutral households can consume and make deposits with bankers, they maximize

  \[ E \left[ \int_{0}^{\infty} \exp(-\rho t) dC_t^H \right] \]

- Bankers hold all risky capital. I impose that bankers consume \( \lambda N dt \) (\( N \) is bankers networth). They maximize

  \[ E \left[ \int_{0}^{\infty} \exp(-\lambda t) \log(\lambda N_t) dt \right] \]

- This is a continuous time adoption of Kiyotaki-Gertler model, but with fixed risk free rate \( \rho \) and simplified effective bankers’ pricing kernel \( \theta_B^t = \exp(-\lambda t) \frac{\lambda}{N_t} \).
Two types of capital producers producing $K_j \in \{A, B\}$, both capital produces cash flow at rate $AK_j dt$, they depreciate at rate $\delta$.

But they have differential exposure to the systematic shock, in aggregate

$$\frac{dK_j}{K_j} = (\Phi_j (i_j) - \delta) \, dt + \bar{\sigma}_j dZ_t$$

“Quality” is captured by $\bar{\sigma}_A < \bar{\sigma}_B$. Cash flow from type B projects are more sensitive to macroeconomic shocks than type A projects.
Risk Adjustment

- Capital producers are owned by household, but can only sell their capital to bankers. The production function of type $j$ capital is

$$\Phi_j (i_j) = \sqrt{\frac{2i_j}{\kappa_j}}$$

- Key assumption: $\kappa_A > \kappa_B$. Supply of high quality projects are limited.

- Key endogeneous variable is the risk adjusted present value of the cash flow (net of investment) produced by type $j$ capital

$$q_j = PV_j = E \left[ \int_0^\infty A \frac{K^j_t}{K^j_0} \theta^*_B dt \right]$$

where

$$\frac{dK^j_t}{K^j_0} = -\delta dt + \bar{\sigma}_j dZ_t$$
Model Schematic

Less Risky Capital
$K_A$ Producers

More Risky Capital
$K_B$ Producers

Rebate Profit

Own

Households

Less Risky Capital

More Risky Capital
Given $q_A, q_B$, capital producers solve a static problem

$$\max_i j \Phi_j (i_j) K_j q_j - i_j K_j$$

Optimal investment follows

$$\Phi_j^* (i_j^*) = \frac{q_j}{\kappa_j}$$
Given their preference, bankers solves a portfolio problem that resembles standard mean-variance efficient investors

\[
\max_{\alpha_A, \alpha_B} E \left[ \int_0^\infty \exp(-\lambda t) \log(\lambda N_t) \, dt \right]
\]

st. \[ \frac{dN_t}{N_t} = -\lambda \, dt + (\alpha_A \pi_A + \alpha_B \pi_B + (1 - \alpha_A - \alpha_B) r_f) \, dt + (\alpha_A \sigma_A + \alpha_B \sigma_B) \, dZ_t \]

where \(\alpha_A, \alpha_B\) are portfolio shares, \(\pi_A, \pi_B\) are excess returns by investing in \(K_A, K_B\); \(\sigma_A, \sigma_B\) are return volatilities for \(K_A, K_B\)
Equilibrium Investment (1)

- $K_A$ and $K_B$ offers same amount of expected cash flow $\rightarrow$ PV difference comes from differential risk adjustments applied to the cash flow.
Equilibrium Investment (2)
An **equilibrium** of this economy consists of prices processes \((q_A, q_B, r_f)\), and decisions, \((c_H, \alpha_A, \alpha_B, i_A, i_B)\), such that

1. Given prices, households, bankers and capital producers solve their optimization problems.

2. Given decisions, markets for risky capital \((K_A, K_B)\) and risk-free bond clears. This pins down bankers’ portfolio choices \(\alpha_A, \alpha_B\).

3. Market for goods clear

\[
A (K_A + K_B) = i_A K_A + i_B K_B + C_H
\]
Solving the Model

1. Conjecture the model has two scaled state variables: “size” and “quality” of intermediaries balance sheet

\[
\eta = \frac{N}{q_A K_A + q_B K_B} \\
\sigma_s = \frac{K_B}{K_A + K_B}
\]

2. Conjecture

\[
\begin{align*}
d\eta &= \mu_\eta dt + \sigma_\eta dZ_t \\
ds &= \mu_s dt + \sigma_s dZ_t
\end{align*}
\]

3. Key endogeneous variables are \(q_A (\eta, s)\) and \(q_B (\eta, s)\), express \(\mu_s\), \(\sigma_s\) as

\[
\begin{align*}
\mu_s &= s(1-s)(\Phi_B (i_B) - \Phi_A (i_A) + \bar{\sigma}_A^2 (1-s) - \bar{\sigma}_B^2 s + \bar{\sigma}_A \bar{\sigma}_B (2s-1)) \\
\sigma_s &= s(1-s)(\bar{\sigma}_B - \bar{\sigma}_A)
\end{align*}
\]
Solving the Model

1. Our goal from now on is to express $\mu_\eta$, $\sigma_\eta$, $\pi_A$, $\pi_B$, $\sigma_A$, $\sigma_B$ as functions of $q_A$, $q_B$ and their first and second order partial derivatives.

2. By definition, we have

$$\pi_j = \left(-\delta + \frac{A + \mu^i q + \bar{\sigma}_j \sigma^j}{q_j}\right) - r_f$$

$$\sigma_j = \bar{\sigma}_j + \frac{\sigma^i \eta^i}{q_j} + \frac{\sigma^i \eta^i}{q_j}$$

3. Finally, use Ito’s lemma on $\eta = \frac{N}{q_A K_A + q_B K_B}$ we get two more equations in $\mu_\eta$ and $\sigma_\eta$

4. Solve this 6 equations in 6 unknowns – so far everything is mechanical
Solving the Model

1. Optimality conditions are summarized in basic asset pricing equations

\[
\frac{\pi_A}{\sigma_A} = \frac{\pi_B}{\sigma_B} = \alpha_A \sigma_A + \alpha_B \sigma_B
\]

2. Above equations solved on \([\eta, s] \in [\epsilon, 1 - \epsilon] \times [0, 1]\). Boundary conditions

2.1 \(s = 0, 1 \rightarrow\) Single technology economy, solved in ODE
2.2 \(\eta = \epsilon\), impose \(q_{\eta} = 0 \rightarrow\) justified by some type of entry
2.3 \(\eta = 1 - \epsilon\), reduce to a system of lower order equations

3. Numerically, I use projection method (5-7th order Chebshev polynomials) to minimize PDE error over a grid.
## Parameters

<table>
<thead>
<tr>
<th>Interpretation</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ Household Time Discount Rate</td>
<td>0.01</td>
<td>0.01</td>
<td>Risk Free Rate</td>
</tr>
<tr>
<td>$\lambda$ Bankers' Time Discount Rate</td>
<td>0.15</td>
<td>0.19</td>
<td>Unconditional Moments</td>
</tr>
<tr>
<td>$\sigma_A$ Cash Flow Volatility of $K_A$</td>
<td>0.02</td>
<td>0.046</td>
<td>Output Volatility</td>
</tr>
<tr>
<td>$\sigma_B$ Cash Flow Volatility of $K_B$</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_A$ Adjustment Cost of $K_A$</td>
<td>10.00</td>
<td>9.10</td>
<td>Investment Volatility</td>
</tr>
<tr>
<td>$\kappa_B$ Adjustment Cost of $K_B$</td>
<td>7.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A$ Productivity</td>
<td>0.16</td>
<td>0.16</td>
<td>Investment-Capital Ratio</td>
</tr>
<tr>
<td>$\delta$ Depreciation</td>
<td>0.10</td>
<td>0.10</td>
<td>Literature</td>
</tr>
</tbody>
</table>

Model 1: Heterogeneous Production. Model 2: Homogeneous Production.
Figure: Solid blue line corresponds to the solution for median $s$. Shaded area plots the solution corresponding to 25% – 75% distribution of $s$. Median output volatility = 0.046, top to bottom quartile of the distribution of output volatility is [0.025, 0.071].
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## Unconditional Moments

<table>
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<tr>
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<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>Median Output Volatility(%)</td>
<td>4.60</td>
<td>4.60</td>
<td>2.0 $\sim$ 5.0</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Return Volatility of $K_A$ (%)</td>
<td>5.11</td>
<td>8.54</td>
<td>19.00</td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>Return Volatility of $K_B$ (%)</td>
<td>15.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SR$</td>
<td>Sharpe Ratio</td>
<td>0.33</td>
<td>0.35</td>
<td>0.50</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>Consumption Growth(%)</td>
<td>1.65</td>
<td>1.77</td>
<td>2.00</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Consumption Growth Volatility(%)</td>
<td>2.45</td>
<td>2.31</td>
<td>1.90</td>
</tr>
<tr>
<td>$\sigma_{\Phi(i_A)}$</td>
<td>Investment Volatility of $K_A$ (%)</td>
<td>3.70</td>
<td>6.53</td>
<td>8.13</td>
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<tr>
<td>$\sigma_{\Phi(i_B)}$</td>
<td>Investment Volatility of $K_B$ (%)</td>
<td>10.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_A$</td>
<td>Investment / Capital Ratio for $K_A$ (%)</td>
<td>10.9</td>
<td></td>
<td>11.20</td>
</tr>
<tr>
<td>$i_B$</td>
<td>Investment / Capital Ratio for $K_B$ (%)</td>
<td>12.2</td>
<td>11.40</td>
<td></td>
</tr>
</tbody>
</table>

Model 1: Heterogeneous production. Model 2: Homogeneous production.
Conditional Implications

My model delivers a precise formulation of the following “concepts”

- Risks “Buildup”
  - Without production heterogeneity, positive shocks always push the economy away from crisis state
  - Therefore, well capitalized banks (higher $\eta$) are associated with lower risks of entering a crisis
  - In my framework, well capitalized banks have strong incentive to take on additional risks — this will show up in the term structure of crisis probability
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- **“Slow” Recovery**
  - In the model, bank equity grows by earning this risk premium associated with its asset
  - Risk premium is higher in crisis state, so return on equity is high $\rightarrow$ recovery is fast
  - When risk taking is endogenous, banks substitute risky, high-yield projects with safe, low-yield ones $\rightarrow$ return on equity $\downarrow$ in crisis
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- **“Volatility Paradox” — details coming later**
Figure: Left Panel: Investment Ratio as a function of $\eta$ and $s$. Right Panel: Drifts of the state variable when starting from $\eta = 0.6$ and median $s$. 

\[ \frac{\Phi(i\eta)}{\Phi(i\eta_a)} \]
Risks Buildup

Figure: Left Panel: Investment Ratio as a function of $\eta$ and $s$. Right Panel: Drifts of the state variable when starting from $\eta = 0.6$ and median $s$. 
Figure: Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of hitting the top 25% of the Sharpe Ratio when starting from $\eta = 0.6$. 
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**Recovery Dynamics**

**Figure:** Left Panel: Investment Ratio as a function of $\eta$ and $s$. Right Panel: Drifts of the state variable when starting from $\eta = 0.2$ and median $s$. 
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Recovery Dynamics

![Graph showing the conditional probability of staying in the top 25% of the Sharpe Ratio when starting from $\eta = 0.2$.]

**Figure**: Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of staying in the top 25% of the Sharpe Ratio when starting from $\eta = 0.2$. 
Volatility Paradox

- No consensus has emerged to define “volatility paradox” – generally refers to the observation that prolonged period of low volatility tends to precede a crisis
  - Brunnermeier Sannikov (2013): compare a series of models differing in their fundamental volatility, banks in low-volatility economies take on more leverage
  - Adrian Boyarchenko (2013): banks run by VaR role, lower financial volatility corresponds to higher leverage → Shorter distance to restructuring boundary
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- My model endogenize both fundamental and financial volatilities
  - Low financial volatility symptomatic of lower risk prices
  - Riskier projects come into the money and get financed
  - Negative correlation between financial volatility and growth in fundamental volatility
  - Accumulation of riskier project tend to coincide with a period of low financial volatility and pushes economy closer to a crisis
Figure: Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. Simulated 200 years.
Conclusion

- **Main Economic Message:** Through financial sector’s optimal financing decision, the real economy accumulates different types of productive capital over the credit cycles.

- **Main Empirical Message:** In the data, credit quality of the marginal borrowers is counter-cyclical.

- **Main Theoretical Message:** Need to keep track of both asset and liability side of the financial sector. Requires sophisticated numerical / technical methods to solve the model.

- **Main Quantitative Message:** I extract model’s conditional implications from the term structure of distress probabilities.