Savers and Spenders: Fiscal Policy as a Potential Resolution to the Treasury Bond Premium Puzzle

Alex C. Hsu*
Georgia Institute of Technology

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Abstract

This paper provides an explanation for the bond premium puzzle: Treasury bond risk premia that match historical averages are obtained in a general equilibrium production economy populated by savers and spenders with constant relative risk aversion around 3. With Epstein-Zin-Weil recursive utilities, the ability to optimize intertemporally raises the stochastic discount factor of the representative saver as the agent’s future wealth becomes riskier due to the prospect of switching to the spender type down the road. Government spending and tax rate shocks account for almost all of the model-implied unconditional mean risk premium on 10-year nominal bonds.

*alex.hsu@scheller.gatech.edu. This work is based on the first chapter of my dissertation. This paper has benefited from discussions with Robert Barsky, Max Croce, Robert Dittmar and Francisco Palomino. All the usual disclaimers apply.
1 Introduction

The Treasury bond term premium summarizes the compensation for risk of holding a long-term bond. Empirically, it has been shown that the unconditional mean of the 10-year Treasury bond risk premium is larger than one percentage point per annum in the post-war data. Theoretically, however, it is still unclear what the economic forces are behind the large observed average term premium. Analogous to the Mehra and Prescott (1985) equity premium puzzle, Backus, Gregory, and Zin (1989) and Donaldson, Johnsen, and Mehra (1990) show that macroeconomic models are incapable of generating long-term bond risk premia of more than a few basis points with reasonable levels of risk aversion. Despite advances in macroeconomic modeling over the past two decades, the puzzle persists in dynamic stochastic general equilibrium (DSGE) models such as Rudebusch, Sack, and Swanson (2007), who demonstrate that the term premium is negligible in a typical New-Keynesian model. Furthermore, Rudebusch and Swanson (2008) calculate the term premium implied by the Christiano, Eichenbaum, and Evans (2005) model, which is widely viewed as a sophisticated tool for modern monetary policy analysis, and find the term premium to be about one basis point.

Resolving the bond premium puzzle is important because policy-makers and practitioners need to understand the fundamental macroeconomic forces making long-term Treasuries risky. Indeed, bond risk premia not only add to the cost of borrowing for the government, but they also impact the financing decisions of households and firms. Wachter (2006) constructs an endowment economy using Campbell and Cochrane (1999) habit-formation in the representative agent’s utility specification. With a curvature parameter on the utility of consumption equals to 2, she is able to replicate observed risk premia for Treasury bonds. However, the partial equilibrium setup is less than ideal to study the underlying mechanism driving bond risk premia because it is silent on the reason behind consumption and bond price comovements. The DSGE framework provides more discipline in this regard as consumption, investment and production processes are internally consistent with household preferences. Furthermore, to the extent that monetary and fiscal policies
have first order effects on bond prices, DSGE models are more robust to structural policy changes that can impact bond risk premia.

In this paper, I develop an DSGE model with heterogeneous agents in which endogenously determined economic output and government borrowing dictate the likelihood of the representative household’s ability to optimize intertemporally such that saving and investing are possible. Bond supply is the crucial state variable for the benchmark model to generate large risk premia on long-term bonds, and shocks to the government spending-to-output ratio and shocks to tax rates account for almost all of the average 10-year nominal term premium. This focus on bond supply is novel, since the theoretical literature does not provide clear guidance on how fiscal policy and bond supply affect the riskiness of long-term bonds.

In the model herein, Ricardian Equivalence breaks down due to the presence of heterogeneous agents and distortionary taxes. Government borrowing becomes relevant to the representative pricer’s marginal utility and bond pricing. Extending the savers-spenders framework outlined in Mankiw (2000), I show that the switching of household types coupled with fiscal policy shocks allow the model to reproduce high risk premia for nominal Treasury bonds. Compared to the existing papers in the literature on general equilibrium term structure, such as Piazzesi and Schneider (2006), Rudebusch and Swanson (2012), Andreasen (2012), van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) and Hsu, Li, and Palomino (2013) to name a few, the current model is successful in simultaneously matching the second moments of macroeconomic quantities as well as the level of the term structure and bond risk premia in the data using a much lower value of risk aversion. The constant relative risk aversion (CRRA) coefficient used for calibration here is 3.4 compared with CRRA of 75 in the baseline model of Rudebusch and Swanson (2012), for example.

In the typical rational expectations framework, the representative agent can smooth consumption perfectly without any regard to the possibility of ever being

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1 One exception is the recent revival of the preferred habitat theory by Vayanos and Vila (2009) and Greenwood and Vayanos (2012), where the yield curve is determined in equilibrium due to the presence of risk-averse arbitrageurs who buy and sell Treasury bonds across maturities.
financially constrained. This leads to relatively stable marginal utilities of the representative agent, and the risk premia on long-term bonds are minuscule in this setting. The savers-spenders model augments the makeup of the population by introducing a new class of agents, the spenders, who are unable to smooth their consumption through the financial markets but rather follow the “rule-of-thumb” of consuming their entire current income. Mankiw (2000) argues that this setup is much more realistic to study fiscal policy because it fits better with stylized facts regarding consumption and wealth distribution that we observe from data. In particular, Mankiw (2000) points out the advantages of the savers-spenders model over the more traditional fiscal policy models on three fronts: consumption smoothing is far from perfect, many people have net worth near zero, and bequests are an important factor with wealth accumulation. However, the savers-spenders dichotomy by itself is not enough to generate any interesting asset pricing implications because the savers are still the marginal price setters in the economy, and they can smooth their consumption perfectly. The possibility of switching from a saver (optimizing household) to a spender (rule-of-thumb household) takes on the leading role in delivering the quantitative results of the model in this paper.

Following Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2012), I include four fiscal policy variables in the economy: the share of government spending over output, the tax rate on consumption expenditure, the tax rate on labor income, and the tax rate on the return of capital. Fiscal policy plays a pivotal role in the model’s ability to generate high risk premia on long-term nominal bonds. Shocks to government spending and distortionary taxes jointly determine the path of government borrowing, and the resulting debt level dictates the switching probabilities of the savers and spenders. These fiscal policy shocks exert great influence on the magnitude of the nominal term premium, above and beyond that resulting from the transitory productivity shock or the monetary policy shock, and accounts for roughly all of the model-implied average compensation for long-term bond risk.

When the probability of the representative saver to switch is high following a negative spending-to-output shock (or a positive tax rate shock), it means the agent is more likely to become a financially constrained spender and loses the capability
to smooth consumption over time. In that case, the agent’s intertemporal optimization problem is terminated following the switch, and he/she will only obtain the within-period utility optimized over consumption and labor. On the other hand, if the probability to switch is low, the agent is more likely to obtain his/her continuation utility under Epstein and Zin (1989), Weil (1990) recursive preferences. This scenario raises the present values of expected consumption and labor income, thus increasing the return on wealth of the representative saver. Under standard calibration of the preference parameters, the representative saver prefers early resolution of uncertainty as opposed to late, and high return on wealth is risky because future wealth is extremely uncertain due to the prospect of been switched to the spender type down the road. In the benchmark model, the preference for early resolution of uncertainty combined with the highly uncertain future wealth make the representative saver’s marginal utility respond strongly to shocks that cause the likelihood of remaining as the saver type to be high. As a result of this increased variability of the stochastic discount factor (SDF), or the pricing kernel, the conditional covariance between the SDF and the price of long-dated nominal bonds becomes more negative, thus generating large term premia.

The switching probability is specified in reduced form such that it is driven by macroeconomic conditions. Patterned after the fiscal policy rules originally taken from Fernández-Villaverde et al., the switching probability is a function of two endogenous macroeconomics variables: output and real short-term debt outstanding. In good times, characterized by high output and high real debt issuance (equivalent to high household savings), the probability of switching from a saver to a spender is low while the probability of switching from a spender to a saver is high. In bad times, characterized by low output and low real debt issuance, the probability of switching from a saver to a spender is high while the probability of switching form a spender to a saver is low. The intuition behind the negative relationship between the probability of switching from a saver to a spender and government borrowing is straight forward: fiscal theory tells us that the government’s budget constraint

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2 This is part of the mechanism driving the results in the long-run risk (LRR) literature popularized by Bansal and Yaron (2004). However, there is no LRR component in the model here strictly speaking, and the elasticity of intertemporal substitution (EIS) is calibrated to be much less than 1.
implies, at any moment in time, total debt obligations has to be fully endorsed by the present value of discounted future surpluses. This means, ceteris paribus, the more the government is able to borrow in real terms, the better the economic outlook is. As a result, it is less likely that an unconstrained household (saver) will become financially constrained (spender) when bond supply, or savings, is high. The switching probabilities are modeled using logistic functions.

The rest of this paper is organized as follows. Section 2 discusses related literature. Section 3 presents the benchmark model where the probability of switching types is a function of output and real debt. Section 4 summarizes the results with analyses of different model specifications. Impulse responses following various exogenous shocks are examined. Finally, section 5 concludes.

2 Related Literature

The results presented here also contribute to our understanding of the term structure and bond risk premia in equilibrium. [Campbell (1986)] specifies an endowment economy in which utility maximizing agents trade bonds of different maturities. When the exogenous consumption growth process is negatively autocorrelated, term premia on long-term bonds are positive, generating upward sloping yield curves because they are bad hedges against consumption risk compared to short-term bonds. The intuition is straightforward; high current consumption growth means low expected future consumption growth and low prices for long-term bonds. On the other hand, high current consumption growth means marginal utility of consumption is high now. Therefore, long-term bonds always have low payoffs in the states of the world where investors want to consume more. More recently, [Piazzesi and Schneider (2006)], using [Epstein and Zin (1989)] preferences, show that inflation is the driver that generates a positive term premium on nominal long-term bonds. Negative covariance between consumption growth and inflation translates into high inflation when consumption growth is low and marginal utility to consume is high. [Wachter (2006)] generates upward sloping nominal and real
yield curves employing habit formation. In her model, bonds are bad hedges for consumption as agents wish to preserve previous level of consumption as current consumption declines. Bansal and Shaliastovich (2013) attempt to understand the return predictability of Treasury bonds using long-run risk. Given exogenously defined consumption growth and inflation processes, their estimated partial equilibrium model can resolve the bond predictability puzzle with the inclusion of stochastic volatilities. In their setting, the estimated coefficient of risk aversion is 20.9.

The models in all of the above papers are constructed with endowment economies. Rudebusch and Swanson (2008) and Rudebusch and Swanson (2012) examine bond risk premia in general equilibrium where utility-maximizing agents supply labor to profit-maximizing firms, who are producers of the single consumption good. Rudebusch and Swanson (2008) find that an economy with habit-forming agents does a poor job explaining the dynamics of bond yields. They calculate model-implied term premia in a number of state-of-the-art DSGE models with various frictions, namely real and nominal rigidities, and conclude that the bond premium puzzle is unresolved in this setting. The best-fit model in Rudebusch and Swanson (2012), on the other hand, is successful in matching the basic empirical properties of the term structure, but this comes at a cost of employing a high parameter of relative risk aversion at 75.

Andreasen (2012) and van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) estimate two medium-scale DSGE models with recursive preferences to fit the term structure of interest rates in the U.K and U.S., respectively. Andreasen (2012) simultaneously fits the U.K. real and nominal term structures and is able to explain the fall in inflation risk premia from 1992 to 2008. However, the estimated relative risk aversion in the model is 80 after adjusting for variable labor supply. Similar to Rudebusch and Swanson (2012), the author speculates that the estimated RRA is so high stemming from the model compensating for the low consumption variability. In the current paper, I am able to calibrate the model to match the volatility of consumption in the data while constructing a volatile SDF employing such low constant relative risk aversion because the representative saver’s return on wealth is highly variable due to the prospect of getting switched to the spender
type in the future. van Binsbergen et al. find their estimated model to have high level of risk aversion, between 40 and 85, and elasticity of intertemporal substitution greater than 1 as well as large adjustment costs. While they contribute greatly on the methodological front in our ability to estimate these DSGE models using both macroeconomic and financial data, their model fails to jointly account for the slope of the nominal yield curve and its volatilities. Citing Rudebusch and Swanson (2012), van Binsbergen et al. suggest fiscal policy as one viable extension to enhance the fit of their model to the financial time series. In a way, this is precisely what this paper is attempting to accomplish, albeit without the full blown estimation.

This paper is closely related to the theoretical literature on fiscal policy and asset pricing. Two recent publications stand out: Croce, Kung, Nguyen, and Schmid (2012) and Gomes, Michaelides, and Polkovnichenko (2013). Croce et al. focus on the effects of corporate taxation on firm investment and productivity in a production economy. They find that distortionary corporate tax generates large risk premia on the firm’s equity by lowering investment and productivity growth while increasing the debt level. Similarly, the model presented here confirms the importance of the distortionary tax levied on the return of capital for asset returns, although the underlying mechanism is different as household investment in the firms is a major source of risk for the representative agent’s future wealth in the current paper. Gomes et al. use an overlapping generation model with incomplete markets to generate high equity risk premium employing a plausible value of risk aversion. Fiscal policy has significant impact on asset returns because equity and government debt are not perfect substitutes in their model, and the crowding out effect of government debt plays the central role in households’ portfolio and investment decisions. The savers-spenders model also has elements of the heterogenous agents model, but the difference is that markets are complete because the spenders are kept out of the financial markets. Furthermore, there are no uninsurable idiosyncratic risk for the savers. The effect of fiscal policy on investment is also more direct in the savers-spenders model as government debt determines the switching probability of the savers and in turn drives the riskiness of capital accumulation.
3 The Benchmark Model

The benchmark model presented in this section is built upon the traditional dynamic stochastic general equilibrium framework with production and capital accumulation. I include three additional features to the standard DSGE model. The first additional feature is the saver-spender dichotomy introduced in Mankiw (2000). As discussed above, the endogenous switching between the optimizing households and the rule-of-thumb households is crucial to the model’s ability to produce large Treasury bond risk premia. The second additional feature of the full model is the introduction of monopolistic producers and price rigidities in the setting of a New-Keynesian economy. The New-Keynesian framework is the workhorse of modern monetary economics. The microfoundations underlying the New-Keynesian framework generates frictions in the firm’s first order condition relating its real marginal cost to its marginal product of labor. The resulting output gap between actual output and output under flexible prices summarizes aggregate economic activity. Furthermore, the output gap is forward-looking and is driven by the expectation of future output gap as well as the real short-term interest rate. The monetary authority implements the Taylor rule and sets the nominal short rate as a function of inflation and output gap. The nominal short rate in turn affects the real short rate thus making monetary policy non-neutral with respect to output gap and the real economy. Finally, I introduce a fiscal authority in the model to determine fiscal policy in terms of government spending as a share of output and tax rates on consumption expenditure, labor income, and capital income. For brevity, I outlined the households and firms in this section and leave the details of the monetary and fiscal policies in the online technical appendix.

\footnote{For a detailed exposition on the New-Keynesian framework, see Clarida, Gali, and Gertler (1999).}
3.1 The Saver’s Problem

There are two types of households in the economy. Following Mankiw (2000), I will call them savers and spenders according to their budget constraints. The savers have the ability to save current income in order to smooth future consumption by purchasing government bonds. In contrast, the spenders required to consume their entire after tax income. With Epstein and Zin (1989), the representative agent of the savers maximizes lifetime utility by solving the following:

$$\begin{align*}
\max & \quad V(C_t^o, N_t^o) = \left[ \frac{C_t^o^{1-\psi}}{1-\psi} - \frac{N_t^o^{1+\omega}}{1+\omega} \right] + \beta E_t \left[ \{\lambda_t^o V_{t+1} + (1-\lambda_t^o)U_{t+1}^o\}^{\frac{1-\gamma}{1-\psi}} \right], \\
\text{s.t.} & \quad (1+\tau_c^c)P_t C_t^o + P_t I_t^o + \sum_{j=1}^{\infty} Q_t^{(j)} [B_t(t+j) - B_{t-1}(t+j)] + P_t \text{Lump}_t \\
& = (1-\tau_l^l)P_t W_t N_t^o + (1-\tau_k^k)P_t R_t^k K_t^o + \tau_k^k \delta P_t K_t^{o, book} + B_{t-1}(t) + P_t \Psi_t,
\end{align*}$$

where $U_{t+1}^o = \frac{C_{t+1}^{1-\psi}}{1-\psi} - \frac{N_{t+1}^{1+\omega}}{1+\omega}$, and the $o$ and $r$ superscripts denote the optimizing household and the rule-of-thumb household, respectively. For the ease of exposition, I will drop the $o$ superscript from the notation in the next two paragraphs. $\beta$ is the time discount factor, $\psi$ is the inverse of the elasticity of intertemporal substitution (EIS), $\gamma$ is the coefficient of relative risk aversion, and $\omega$ is the inverse of the Frisch elasticity of labor supply.

$C_t$ and $N_t$ are real consumption and labor, respectively. $I_t$ denotes investment in real terms. $P_t$ is the price level in the economy. $B_t(t+1)$ is the amount of nominal bonds outstanding at the end of period $t$ and due in period $t+1$. $W_t$ refers to real labor income, which is the same across households in the economy. $\text{Lump}_t$ is real lump-sum tax collected by the fiscal authority to keep the real debt process from exploding, and $\Psi_t$ is dividend income coming from the firms. There are three distortionary taxes, $\tau_c^c$, $\tau_l^l$, and $\tau_k^k$, on consumption expenditure, labor income, and

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$^4$For the rest of the paper, I will interchangeably refer to savers and spenders as optimizing households and rule-of-thumb households, respectively.
return on capital, respectively. \( K_t \) is capital, \( R^K_t \) is the return on capital, \( \delta \) is the depreciation on capital. Notice the term \( \tau^K_t \delta P_t K_{t-1}^{o,book} \) denotes the depreciation tax shield of capital, which is taken on the book value of capital only.

\( V_t \) is the value function of the dynamic programming problem for the representative saver, and \( V_{t+1} \) is the “continuation utility” of the value function. \( \lambda_{t+1} \) is the probability of the saver to not switch types and remain a saver at time \( t+1 \). What the expression \( \lambda_{t+1} V_{t+1} + (1 - \lambda_{t+1}) U^R_{t+1} \) says is that, with probability \( \lambda_{t+1} \), the optimizing household will obtain his/her continuation utility next period and remain unconstrained to purchase financial assets in order to smooth consumption. With probability \( 1 - \lambda_{t+1} \), the optimizing household will switch to a rule-of-thumb household and be locked out of the financial market starting next period. In that case, the agent will only able to obtain the within-period utility, \( U^R_{t+1} \), and lose the ability to smooth consumption intertemporally after time \( t+1 \). Once switched to a spender type, the agent will remain a spender and consume according to the rule-of-thumb until a switch back to being an optimizer in period \( t+s \) for some \( s > 1 \).

The budget constraint states that the agent has periodic after-tax income from labor, capital, and dividends as well as bonds maturing at time \( t \) and long-term bonds repurchased by the government at time \( t \) before they are due. The agent then decides how much to consume after taxes, how much to invest, and how much to pay for newly issued bonds at time \( t \) at price \( Q_t^{(j)} \).

**Proposition I** The nominal pricing kernel written in terms of return on consumption and return on labor income of the savers-spenders economy with distortionary taxes is

\[
M_{t+1}^S = \beta \left( \frac{C_{t+1}^o}{C^o_t} \right)^{-\psi} \left( \frac{1 + \tau^c_t}{1 + \tau^c_{t+1}} \right)^{1-\gamma} (1 - share_t) R^c_{t+1} + share_t R^l_{t+1} \right)^{\psi-\gamma}.
\]
where

\[ R_{t+1}^c = \frac{(1 + \tau_{t+1}^c)C_{t+1}^o + \lambda_{t+1}^o PV_{t+1}^c}{PV_{t}^c} \quad \text{and} \quad R_{t+1}^l = \frac{(1 - \tau_{t+1}^l)LI_{t+1} + \lambda_{t+1}^o PV_{t+1}^l}{PV_{t}^l}. \]

PV’s are present values, and LI means labor income.

**Proof:** Please see the online technical summary.

### 3.2 The Saver’s Investment Decision

In this economy, the savers are also the owners of capital. They rent out capital to the firms in exchange for earning the return on capital, \( R_t^k \). This return is subject to the capital taxation, \( \tau_t^k \). The capital accumulation equation is standard with convex quadratic adjustment cost:

\[ K_{t}^o = (1 - \delta)K_{t-1}^o + \Phi \left( \frac{I_{t-1}^o}{K_{t-1}^o} \right) K_{t-1}^o. \]

In the event of a switch to the spender type from a saver type, I assume the representative household no longer obtains the return on investment made in the previous period. Furthermore, all capital accumulated by the household prior to the switch is transferred to the incoming savers.

**Proposition II** The saver’s optimal investment strategy has to satisfy the following equation in the presence of distortionary taxes on the return to capital.

\[
Q_t^{inv} = \mathbb{E}_{t} \left[ M_{t,t+1}^{noswitch} \lambda_{t+1}^o \left( (1 - \tau_{t+1}^k)R_{t+1}^k + \frac{I_{t+1}^o}{K_{t+1}^o} \right) + \Phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] - \Phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right),
\]

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where

\[
M_{t,t+1}^{\text{noswitch}} = \beta \left( \frac{C_t^o}{C_t^{o'}} \right)^{-\psi} \left[ \frac{(1 + \tau_t^c)}{(1 + \tau_{t+1}^c)} \right] \left[ \frac{(V_{t+1})^{1-\psi}}{E_t \left[ \frac{1-\gamma}{V_{t+1}^{1-\psi}} \right]} \right]^{\psi-\gamma}. 
\]

\(Q_t^{\text{inv}}\) is the shadow price of investment, and \(V_{t+1}\) is the continuation utility in the value function of the saver.

**Proof:** Please see the online technical summary.

Similar to the standard investment first order condition from q-theory, I derive here the intertemporal relationship of investment’s Q as a function of the return on capital, the rate of depreciation, and the marginal rate of investment adjustment cost. Unlike the traditional first order condition, however, the expectation in this case is not discounted by the pricing kernel of the representative saver but rather the term \(M_{t,t+1}^{\text{noswitch}}\lambda_t^{o'},\) which is the standard Epstein-Zin-Weil pricing kernel augmented by the probability of staying as a saver.

### 3.3 The Spender’s Problem

Without the ability to save through purchasing government bonds or investing in capital, the representative agent of the spenders cannot smooth consumption intertemporally and maximizes the following within-period utility over his/her lifetime:

\[
\max E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{C_{t+j}^{r} - \frac{N_{t+j}^{r} \omega}{1 + \omega}}{1 - \gamma} \right) \right],
\]

while following a budget constraint such that

\[
(1 + \tau_t^c)^3 P_t C_t^r + P_t Lumped = (1 - \tau_t^l)^3 P_t W_t N_t^r,
\]

where the r superscript now refers to the rule-of-thumb household. After dividing through both sides of the equality by the price level \(P_t\), this budget constraint says
that, every period, the saver is forced to consume his/her entire after-tax labor income sans the lump-sum transfer. Notice that the parameters $\beta$, $\gamma$, and $\omega$ are the same across both types of households. The first order condition relating wage and labor supply is:

$$W_t = C_t^{\gamma}N_t^{\omega}. \quad (1)$$

This “intra-temporal” substitution between real consumption and labor supply is the same across the two households. Therefore, there exists an unique real market wage in the economy.

### 3.4 The Probability of Switching Types

The savers and spenders in the economy can switch types endogenously driven by the macroeconomic conditions. To capture the idea that during a boom the probability of switching from a saver to a spender is low while the probability of switching from a spender to a saver is high and vice versa during a recession, I construct the probabilities of NOT switching types for both households in reduced-form using logistic functions that have feedbacks from the macroeconomic conditions.

$$\lambda_t^o = \frac{1}{1 + 1/exp \left(\hat{\lambda}_t^o\right)},$$

$$\hat{\lambda}_t^o = \theta_\lambda + \phi_\lambda \hat{\lambda}_{t-1}^o + \phi_{\lambda,y} [Y_{t-1} - Y] + \phi_{\lambda,b} \left[\frac{D_{t-1}(t)P_{t-1}^{real}}{Y_{t-1}} - \frac{D}{Y}\right],$$

$$\lambda_t^r = \frac{1}{1 + 1/exp \left(\hat{\lambda}_t^r\right)},$$

$$\hat{\lambda}_t^r = \theta_\lambda + \phi_\lambda \hat{\lambda}_{t-1}^r - \phi_{\lambda,y} [Y_{t-1} - Y] - \phi_{\lambda,b} \left[\frac{D_{t-1}(t)P_{t-1}^{real}}{Y_{t-1}} - \frac{D}{Y}\right].$$

Under this specification, $1 - \lambda_t^o$ is the probability of switching for the representative saver to become a spender, and $1 - \lambda_t^r$ is the probability of switching for the representative spender to become a saver. In general, $\lambda_t^o$ and $\lambda_t^r$ can also be thought of as the fraction of savers and spenders, respectively, who stay as the same type.
the following period. For an agent of the saver type, a higher $\lambda^o_t$ means a greater chance he/she will stay unconstrained financially and obtain the continuation utility under recursive preferences.

I use two macroeconomic variables to proxy for the general economic conditions: lagged output ($Y_{t-1}$) and real maturing debt ($D_{t-1}(t)$). Real debt is defined as the nominal debt deflated by the price level, or $\frac{R_{t-1}(t)}{P_t}$. The use of output is straightforward since we define booms and recessions according to GDP growth. The inclusion of real debt is somewhat unconventional. However, I argue that real debt is a forward-looking variable that summarizes economic outlook that is not captured by the current output. This is because from the government’s budget constraint, we can write real debt as the present value sum of future real surpluses discounted by the stochastic discount factor. In other words, all outstanding real debt issued in the economy has to be endorsed by the future revenue stream of the government. The higher the debt level is today, the better the future economic outlook is, and the more likely a saver household will remain unconstrained. Therefore, an increase in $D_{t-1}(t)$ leads to an increase in $\lambda^o_t$ and a decrease in $\lambda^r_t$.

More specifically, I use the lagged output gap ($X_{t-1}$) instead of lagged output in the logistic equation, where the output gap is defined as the ratio of current output to the full output determined in the absence of nominal rigidities. Furthermore, I multiply real debt by its lagged market price ($P_{t-1}^{real}$) to get the market value of debt from the notional value, then I scale the market value by lagged output to get the lagged debt-to-GDP ratio. Deviations of the lagged debt-to-GDP ratio from the steady state debt-to-GDP ratio determine the switching probabilities.

Finally, to keep track of the percentage of savers and spenders in the population due the asymmetry in the switching probabilities, I construct a variable, $\mu_t$, as the fraction of spenders in the economy:

$$\mu_t = \theta\mu + \frac{\lambda^r_t - \lambda^o_t}{2}.$$

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5In the case of one-period real debt, the budget constraint can be written as $D_{t-1}(t) = \mathbb{E}_t \left[ \sum_{j=0}^{\infty} M_{t+j} (T_{t+j} - G_{t+j}) \right]$, where $T_{t+j} - G_{t+j}$ is the primary surplus and $M_{t+j}$ is the real pricing kernel between times $t$ and $t+j$. 
θ_μ is the unconditional mean of the percentage of spenders in the population. If \( \lambda'_r > \lambda'_o \), this means the fraction of spenders who are staying is higher than the fraction of savers who are staying, then the weight of spenders in the wealth distribution will increase. Here, I am using a simplifying assumption that at the beginning of each period, the population composition is at its unconditional mean so I do not have to keep track of the entire history of \( \mu_t \). Although not ideal, this assumption is innocuous to the asset pricing implications of the model while greatly reducing the complexity of the solution. The variable \( \mu_t \) does not appear in the pricing kernel derived in Proposition I, and it is only critical for aggregation purposes described in the next subsection.

### 3.5 Aggregation

The aggregate real consumption, labor supply and taxes in the economy can be expressed by the following weighted average of the corresponding variables of each type of households:

\[
C_t = \mu_t C_r^t + (1 - \mu_t) C_o^t
\]

\[
N_t = \mu_t N_r^t + (1 - \mu_t) N_o^t
\]

\[
Tax_t = \mu_t Tax_r^t + (1 - \mu_t) Tax_o^t,
\]

where \( \mu_t \) denotes the time-varying fraction of rule-of-thumb consumers in the economy. Furthermore, investment and capital are unique only to the optimizing households, I differentiate the aggregate quantities of investment and capital by the weighted averages below:

\[
Inv_t = (1 - \mu_t) Inv_o^t
\]

\[
K_t = (1 - \mu_t) K_o^t.
\]
3.6 The Firm’s Problem

There is a dispersion of firms, denoted by \( j \), with identical production technology in the economy. With nominal price stickiness and monopolistic competition, each firm is faced with the following optimization problem:

\[
\max_{P_t^*(j)} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \alpha^s M_{t,t+s}^s \left\{ P_t^*(j)Y_{t+s}(j) - P_{t+s} \left[ W_{t+s}N_{t+s}(j) + R_{t+s}^k K_{t+s}(j) \right] \right\} \right] \tag{7}
\]

s.t.
\[
Y_{t+s}(j) = Z_{t+s}K_{t+s-1}(j)^\kappa N_{t+s}(j)^{1-\kappa} \tag{8}
\]
\[
P_t = \left[ \int_0^1 P_t(j)^{1-\eta} \, dj \right]^{\frac{1}{1-\eta}} = \left[ (1 - \alpha)P_{t-1}^{1-\eta} + \alpha P_t^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{10}
\]

Using Calvo (1983) pricing, a firm can choose to optimally adjust price to \( P_t^*(j) \) with probability \((1 - \alpha)\) each period independent of the time elapsed between adjustments. The objective function of the firm is simply profit maximization: revenue minus labor cost and rent on capital. The within-period profits are discounted by the nominal pricing kernel and the probability that the firm has not been allowed to adjust its price optimally up to that period. Each period, with probability \( \alpha \), the firm is stuck with the price from the previous period. The cash-flow stream is discounted by the nominal stochastic discount factor between times \( t \) and \( t+s \), \( M_{t,t+s}^s \).

\( P_t^*(j)Y_{t+s}(j) \) is total sales for firm \( j \) at time \( t+s \). \( W_{t+s}N_{t+s}(j) \) and \( R_{t+s}^k K_{t+s}(j) \) are the real labor cost of and the real rental cost of capital, respectively. Notice real wage and real return on capital are determined in equilibrium with the households and are common across all firms.

There are three constraints faced by the firm in optimizing its profit. Equation \[8\] is the production function of firm \( j \), where \( Z_t \) is the transitory productivity shock and \( \kappa \) is the capital share of input in production. Equation \[9\] is the demand equation for firm \( j \)'s output as a function of the optimal price it sets at time \( t \). Lastly, equation \[10\] is the price aggregator as a weighted average of the optimal price at time \( t \) and the sticky price from time \( t-1 \).
\( P_t^*(j) \) is the optimal price the firm \( j \) charges for one unit of the consumption good set at time \( t \). \( \alpha \) is the probability in each period \( t+s \) that the firm is not allowed to adjust its price optimal so it has to keep charging \( P_t^*(j) \). If a firm is not allowed to adjust its price optimally, then it charges \( P_t^*(j) \) at time \( t+s \), as the price is not indexed. All variables indexed by \( j \) is firm-specific. For example, \( Y_{t+s}(j) \) means output of firm \( j \) at time \( t+s \) given the last time firm \( j \) was able to set its optimal price was at time \( t \). Without the index \( j \), the variable is common across all firms, such as the price level \( P_{t+s} \) and the productivity shock \( Z_{t+s} \). Finally, \( \eta \) determines the markup charged by the firm when it sets \( P_t^*(j) \) due to monopolistic competition. The parameter \( \kappa \) denotes the capital share of output in the Cobb-Douglas production function.

\( Z_t \) is the economy-wide productivity shock on output. Log productivity is an exogenous AR(1) process such that

\[
Z_t = \ln(Z_t) = \phi_z z_{t-1} + \sigma_z \epsilon^z_t,
\]

with \( \epsilon^z_t \sim \text{iid. } \mathcal{N}(0,1) \).

**Proposition III** The firm’s optimal price setting behavior has to satisfy the following equation in the presence of nominal price rigidities such that it can only adjust its price optimally each period with probability \( \alpha \).

\[
\left[ \frac{1}{1 - \alpha} \left( 1 - \alpha \left( \frac{\Pi^*}{\Pi_t} \right)^{1-\theta} \right) \right]^{-\frac{1}{\theta}} F_t = \nu \frac{\kappa (1-\kappa)^{(1-\kappa)} \kappa}{Z_t} R_k W_t^{1-\kappa} J_t,
\]

where \( \nu = \frac{\eta}{\eta-1} \) is the frictionless markup and \( \Pi^* \) is the inflation target of the central bank. \( F_t \) and \( J_t \) are recursively defined as

\[
F_t = 1 + \alpha E_t \left[ M_{t,t+1}^S \Pi_{t+1}^\eta \left( \frac{Y_{t+1}}{Y_t} \right) F_{t+1} \right]
\]

\[
J_t = 1 + \alpha E_t \left[ M_{t,t+1}^S \left( \frac{Z_t}{Z_{t+1}} \right) \left( \frac{R_{t+1}^k}{R_t^k} \right)^\kappa \left( \frac{W_{t+1}}{W_t} \right)^{1-\kappa} \Pi_{t+1}^{1+\eta} \left( \frac{Y_{t+1}}{Y_t} \right) J_{t+1} \right].
\]
Proof: Please see the online technical summary.

3.7 The Market Clearing Condition

In this economy, total output has to equal to total private consumption and private investment plus total government spending:

\[ Y_t = C_t + I_t + G_t. \]  
(12)

3.8 Model Solution and Calibration

Please refer to the online technical appendix for details and results of the calibration.

4 Analysis

4.1 The Benchmark Model

Table 1 summarizes the output of the benchmark model. SD, AR(1) and \( \bar{E} \) denote the unconditional standard deviation, first-order autocorrelation and mean, respectively, of a particular endogenous variable. Column (1) is the outcome under the benchmark specification with all six exogenous shocks turned on. I then simulate the model while shutting down all the shocks except one to tease out the contribution of each of the shocks on the macroeconomic and financial moments. The results are shown in columns (2) to (6), in order, of the shocks to government spending-to-output ratio \( (g) \), transitory productivity \( (z) \), tax rates on consumption \( (\tau^c) \), labor income \( (\tau^l) \) and return on capital \( (\tau^k) \).

Regarding the volatilities of endogenous variables, the shock to the share of government spending-to-output is the most important contributor to the average
variability of all the macroeconomic and financial variables. For example, column (2) shows that the spending shock by itself can generate 1.09% volatility in output per quarter compared with the 1.47% generated by the benchmark model. Not surprisingly, the shock to the return on capital tax rate drives the variability of investment and capital in column (6), on par with the variability generated by the spending shock in column (2). Of the three tax rate shocks, the shock to $\tau^k$ has the strongest effect on the volatilities. The shock to the labor income tax rate is next as it produces high variabilities in investment, capital and wage in column (5). The shock to the consumption tax rate is the least important, but it still is responsible for sizable shares of the macroeconomic volatilities, especially when comparing column (4) to the transitory productivity shock in column (3). All of the shocks generate high first-order autocorrelations for output, consumption and investment. The impact of the monetary policy shock is not shown in table 1 since it contributes relatively little to the volatilities in the model.

The last three rows of table 1 present the unconditional means of three proxies of long-term bond risk premium: the slope of the nominal yield curve between a 10-year bond and the short rate, the excess 1-quarter holding period return of a 10-year bond and the term premium of a 10-year bond calculated as the difference in yields between the physical measure and the risk-neutral measure. Examining $\mathbb{E}[TP^{(40)}]$, it is immediate that both the shock to the government spending-to-GDP ratio and the shock to the return on capital tax rate are major contributors to the average 10-year risk premium: roughly 35% and 40% respectively. Shocks to tax rates on consumption and labor income, in order, contribute around 5% and 20% of the average term premium. Relatively speaking, the transitory productivity shock and the monetary policy shock (not shown) generate almost none of the overall bond risk premium. The same pattern emerges for both the average slopes of the yield curve and the average excess returns of 10-year Treasuries: fiscal policy shocks, particularly government spending shocks and capital tax rate shocks, are the source of the large average term premium on long-term bonds in the benchmark model with endogenous switching.

To understand how the probability of switching types helps to generate large
average bond risk premia, I solve the benchmark model with and without switching (the no-switching model) and examine the impulse responses. Figures 1 contrasts the responses of selected endogenous variables in the benchmark model and the no-switching model following a positive one standard deviation shock to the government spending-to-output ratio. I focus on only the government spending shock for now because Table 1 shows that it alone accounts for about 35% of the average 10-year risk premium and is the most vital contributor to macroeconomic volatilities.

Under the standard savers-spenders framework without the possibility of switching, à la Mankiw (2000), the impulse responses show that higher government spending as a share of output increases output, crowds out investment and aggregate consumption. The households supply more labor to make up for the lost consumption through the wealth effect, and equilibrium real wage goes down due to smaller marginal product of labor. At the same time, the government increases tax revenue and real debt to finance the higher spending. As a result, primary surplus (tax revenue minus government spending) decreases while the debt-to-primary surplus ratio increases. However, I will explain in detail in section 4.2 the representative saver’s real and nominal marginal utilities ($m_{real,nominal}$) hardly respond to the positive spending-to-output shock in the absence of switching because the agent’s return on wealth is relatively inert to the same shock.

When the switching of household types is permitted in the model, higher government spending share also leads to greater output and lower aggregate consumption. Unlike the no-switching case, investment by the savers actually increases following the positive shock to the share of government spending, consistent with the increase in $q$ or the shadow price of investment. Aggregate labor supply goes up, but equilibrium real wage decreases. Lower wage means lower marginal cost for the firm, and equilibrium price decreases leading to lower short term inflation ($\pi$). The drastic reduction in consumption and wage result in smaller tax revenue despite the increase in capital. Figure 1 shows that real debt level goes up, which coupled with higher output increase the probability of a saver household remain a saver ($\lambda^\sigma$). The crucial insight of the switching model is the fact that higher $\lambda^\sigma$
actually makes the representative saver’s future wealth more risky, as evidenced by the substantial increase in the agents’s return on adjusted wealth comprised of returns on consumption claim ($R_{consumption\ claim}$) and labor income ($R_{labor\ income}$). This is because the representative saver is concerned with a switch to the spender type happening some time in the future while the spenders are locked out of the Treasury market and cannot save. Without the ability to smooth consumption intertemporally, the spenders ignore the impact of future wealth completely and only optimize over their with-in period utilities.

Imagine a representative saver who is faced with a known probability of getting switched to a spender type ($1 - \lambda^o$) in the following period. If the probability is high, the agent knows that if a switch happens, he/she will have to consume the entire after-tax labor income in the subsequent periods. Therefore, any concern of intertemporal consumption substitution is mute. Given the same probability of switching, in the absence of a switch in the following period, he/she will have to worry about the switch happening some time in the future. Due to the chance of a future switch to the spender type, the present value of the representative saver’s wealth becomes more uncertain and risky. As a result, the return on wealth is much higher in the model with switching than without switching. Since the agent prefers early resolution of uncertainty, high return on adjusted wealth means the marginal rate of consumption substitution is also high.

The nominal pricing kernel or the SDF of the savers-spenders switching model, defined as

$$M_{t-1,t} = \left[ \beta \left( \frac{C_t^o}{C_{t-1}^o} \right)^{-\psi} \left( \frac{1 + \tau_{t-1}^c}{1 + \tau_t^c} \right) \right]^{1-\psi} \left( \frac{1}{\Pi_t} \right) \left[ R_{wealth} \right]^{\frac{\psi - \gamma}{1-\psi}}, \quad (13)$$

is made up of consumption growth, the change in the consumption tax rate, inflation and the return on adjusted wealth. As it turns out, the response of return on wealth of the saver following the positive government spending-to-output shock in the benchmark model dominates the responses of the other three components in the SDF following the same shock. Therefore, as figure 1 confirms, $R_{consumption\ claim}$
and $R^{\text{labor income}}$ drive the responses of both the real and nominal pricing kernels. In the absence of the switching mechanism, the responses of $R^{\text{consumption claim}}$ and $R^{\text{labor income}}$ are small and negative following the positive government spending shock, and the impulse response of the SDF is relatively miniscule.

The risk premium on a long-term bond can be quantified by the covariance between the pricing kernel and the price of the long-term bond. The bottom right corner of figure 1 shows the impulse response of the 10-year Treasury yield following a positive one standard deviation shock to the share of government spending. In both the benchmark model and the no-switching model, the 10-year bond yields increase (prices decrease) by roughly the same magnitude. This is due to the fact that under both model specifications, the saver’s long-term consumption growth is expected to be high after the shock, thus driving down the rate of long-run consumption substitution and elevating the long end of the yield curve. Therefore, the large bond risk premia generated in the benchmark model above and beyond the risk premia generated in the no-switching model is best explained by the amplified variability of the nominal pricing kernel because the responses of long-term interests are similar in magnitude.

4.1.1 The Contribution of Productivity Shocks

Contrary to what Rudebusch and Swanson (2012) report as the main driver of long-term Treasury risk premium in a standard DSGE model, the productivity shock plays a minor role in the model presented here. Column (3) in table 1 reports model summary statistics when only the productivity shock is present while all other shocks are turned off. The unconditional mean of the nominal term premium on a 10-year to maturity bond is only 1.97 basis points, or less than 0.2% of the total risk premium generated by the benchmark model.

To understand the futility of the productivity shock on bond risk premium in the benchmark model, figure 2 compares the impulse responses following a one standard deviation negative transitory productivity shock to a positive government
spending-to-output shock and a negative return on capital tax rate shock. The negative productivity shock lowers output, consumption and equilibrium wage. Tax revenue falls slightly due to weaker economic activity. Similar to the case following the positive government spending-to-output shock, the negative productivity shock has a negative wealth effect on the households, and labor hours increase.

The probability of obtaining the continuation utility \( \lambda^o \) matters greatly for the representative saver’s return on consumption and labor income and the stochastic discount factor. The transitory productivity shock does very little to change the debt-to-output ratio and only has a minimal effect on output, both of which are determinants of the switching probability. As a result, return on adjusted wealth and the pricing kernel respond weakly to the productivity shock. The bottom right corner of figure 2 displays the impulse responses of the 10-year nominal yield. The long-term interest rate (bond price) has a strong positive (negative) response to low productivity that is similar in magnitude as the responses following the two fiscal policy shocks. Given the positive response of the nominal pricing kernel, the productivity shock by itself generates a small positive covariance between the SDF and the long-term bond yield, leading to positive bond risk premia. However, the size of risk premia is very small because the the productivity shock creates little variability in the nominal pricing kernel.

4.1.2 Tax Rate Rules

Fiscal policy plays an important role in the model due to the contribution of the debt-to-output ratio to the saver’s switching probability. I have already shown the impact of the shock to the share of government spending-to-output as an important driver of long-term Treasury bond risk premium in the model. I now turn the attention to examine the impact of shocks to tax rates on consumption, labor income, and return on capital. Columns (4) to (6) in table 1 are summary statistics of the benchmark model when each shock to the tax rate is activated one at a time. Most strikingly, the shock to the tax rate on the return of capital \( \tau^k \) strongly influences on the volatilities of the real variables, inflation and the nominal short
rate. The shock to the return on capital tax rate generates roughly one half of the unconditional volatilities in output, consumption, and wage in the model by itself. Furthermore, it generates around two third of the unconditional volatilities in investment and capital. Shocks to the consumption tax rate \( (\tau^c) \) and the labor income tax rate \( (\tau^l) \) each generates about a quarter of the same unconditional volatilities. The average long-term bond risk premium \( (\mathbb{E}[TP40]) \) is the largest at 40.1 basis points when the capital tax rate is turned on. The consumption tax rate shock contributes to 5.2 basis points of the overall risk premium by itself, while the labor income tax rate shock contributes to 21.3 basis points of the risk premium by itself.

To understand the dynamic impact of the tax rate shocks on the benchmark economy, I return to figures 2 to examine the impulse responses. I focus only on the return of capital tax rate shock instead of the other two tax rate shocks because it produces the highest average long-term bond risk premium. The responses following a negative one standard deviation shock to the return on capital tax rate are plotted along side those following the government spending shock and the productivity shock. The negative capital tax rate shock lowers aggregate output and consumption initially, but they turn positive after one quarter and three quarters, respectively. Due to the lower tax rate on the return of capital, the savers invest more, driving up the accumulated capital. Tax revenue falls initially due to the lower capital tax rate, but it recovers and turns positive in the long-run as economic activity picks up. Through the feedbacks in the fiscal rules, the tax rate on consumption increases following the decrease in the tax rate on the return of capital. This causes the consumption of the spenders to drop and their labor hours to increase, pushing up aggregate labor supply.

The shock to the return of capital tax rate by itself generates the same level of unconditional 10-year term premium as the shock to the share of government spending because their impact on output and government borrowing are very similar. Lower tax rate on the return of capital results in larger quantities of short-term debt, which leads to an increase in the representative saver’s probability of remaining as the saver type \( (\lambda^o) \). High \( \lambda^o \) increases returns on consumption claim and on labor income as the agent’s future wealth becomes more uncertain due to the
possibility of a switch to the spender type in the future. Moreover, since the saver prefers early resolution of uncertainty, higher return on adjusted wealth translates into higher SDF.\footnote{6 ψ−γ 1−ψ > 1.} In the non-Ricardian setting, the 10-year nominal interest rate \((\hat{r}^{(10)})\) increases following the negative tax rate shock as government bond supply expands. Combined, the shock to the return of capital tax rate induces a large and positive covariance between the nominal SDF and the long-term interest rate, and this is consistent with the large and positive \(\mathbb{E}[TP40]\) computed in table 1 column (6). Shocks to tax rates on consumption and labor income work through much the same channel as described here. For brevity, I will skip their mechanisms here.

### 4.1.3 The Effect of Monetary Policy Shocks

The benchmark model has an active monetary policy and a passive fiscal policy in the sense of \textcite{Leeper1991}, where the Taylor rule coefficient on inflation is greater than 1 and fiscal policy is such that tax revenue ensures a non-explosive debt path. It is widely accepted that monetary policy is the major determining factor of the yield curve in the literature. However, the results show that the monetary policy shock by itself generates minimal risk premia on 10-year Treasuries in the model.

To understand the role of the monetary policy shock and its failure to produce sizable term premia in the savers-spenders setting, figure 3 plots the impulse responses of the benchmark model after a one standard deviation positive shock to the share of government spending and a ten standard deviation positive monetary policy shock. I amplify the size of the monetary policy shock tenfolds to make the comparison to the government spending shock meaningful. Furthermore, I intend to emphasize the fact that the ineffectiveness of the monetary policy shock in producing bond risk premia is not a consequence of its small variance, relatively speaking, in the calibration.

In figure 3, aggregate output, consumption, tax revenue, labor hours and equilibrium wage all increase immediately after the positive shock to monetary policy is realized, but they turn negative from the steady state after roughly 1.5 quarters.
Examining the breakdown between savers and spenders, the increase in aggregate consumption in the short run is actually driven by the increase in consumption of the spenders. As it is typical in standard representative agent models, consumption of the optimizing agents (savers in this case) drops following the positive monetary policy shock, causing their labor supply to increase. The increase in equilibrium wage makes the spenders “lazier” so they work less and consume more, resulting in higher aggregate consumption and output. In the long run, however, contractionary monetary policy decreases demand, which lowers equilibrium wage as firms drive down the marginal cost of production. Lower wage causes labor supply to drop, and lower labor income coupled with depressed consumption cause tax receipt to decrease. Investment, on the other hand, decreases immediately after the positive monetary policy shock is realized but turns higher in the second quarter. Contractionary monetary policy forces firms to cut the marginal cost by lowering the return on capital. However, given the subsequent drop in the nominal interest rate, shown at the bottom of figure 3, savers choose to save by investing in capital rather than purchasing Treasury bonds. Furthermore, inflation reacts negatively from the monetary policy shock as firms lower price to account for the decrease in demand after one period.

Notice the amplified magnitude of the reactions of macro variables to the policy shock are on par with those immediately following the realization of the government spending-to-output shock, but the policy shock produces almost no variability in the nominal pricing kernel. Recall that the saver’s probability of obtaining the continuation utility ($\lambda^o$) is an increasing function in output and the debt-to-output ratio. Although output goes up after the policy shock is realized, the debt-to-output ratio goes down. The result is a very mild reaction of $\lambda^o$, and the representative saver’s returns on consumption claim and labor income exhibit much smaller perturbations following the policy shock than following the government spending shock. From equation 13, the pricing kernel is negatively driven by the saver’s consumption growth and positively driven by the return on adjusted wealth. Immediately following the positive policy shock, the representative saver’s consumption growth decreases (causing the pricing kernel to be higher) while the agent’s return on ad-
justed wealth also decreases (causing the pricing kernel to be lower). These two terms cancel each other out, and the resulting impact of the monetary policy shock on the pricing kernel is insignificant compared to the impact of the government spending shock, as reflected by the minuscule average risk premium.

Although shocks to monetary policy are not contributing to the unconditional risk premium in the benchmark case, the policy rule is still a vital element of the model because it influences the endogenous inflation dynamics. The nominal term premium encompasses compensation for inflation risk on nominal bonds, which can be summarized the covariance between the real pricing kernel and inflation. Figure 4 presents the comparative statics by adjusting the monetary policy rule parameters: the loading of the nominal short rate on the output gap ($\rho_x$), its loading on inflation ($\rho_\pi$), the constant term ($\bar{i}$) and the inflation target ($\pi^*$). From panel A, $\rho_\pi$ has an especially strong effect on the average risk premium and the unconditional switching probability. The effect is not monotonic as the response of the nominal short rate on inflation around 1.5 maximizes both the bond risk premium and the probability of switching to the spender type (one minus the fraction of savers with continuation utility). The response to the output gap, on the other hand, does very little to change $\mathbb{E}[TP40]$ and $\mathbb{E}[\lambda_o]$. Panel B in figure 4 displays the comparative statics by changing the constant and the inflation target in the policy rule. Similar to $\rho_\pi$, the average risk premium and the unconditional switching probability are non-linear functions of $\bar{i}$. Holding the $\pi^*$ constant at 0.0133, $\bar{i}$ minimizes the average bond risk premium and the mean fraction of savers with continuation utility around 0.02. Functions of $\mathbb{E}[TP40]$ and $\mathbb{E}[\lambda_o]$ in the inflation target are linear, but the direction depends on the value of the constant term. At small values of $\bar{i}$, the average risk premium and the unconditional probability to remain the saver type are increasing in $\pi^*$. As $\bar{i}$ gets large, the direction flips such that $\mathbb{E}[TP40]$ and $\mathbb{E}[\lambda_o]$ are decreasing in $\pi^*$. Taken together, monetary policy is indeed a non-trivial factor in the benchmark model where fiscal policy shocks are the main source of risk. While their impact on the representative saver’s switching probability is mostly negligible, policy rule parameters, in particular $\rho_\pi$ and $\bar{i}$, can have large effects on the average term premium.
4.2 The No-Switching Model

In this section, I examine the dynamics of the no-switching model first discussed in section 4.1 in detail. Table 2 presents the summary statistics of the no-switching model under different shock specifications. Column (1) shows the unconditional moments of the no-switching model when all of the exogenous shocks are active. Compared to the benchmark model, the volatilities of the macroeconomic moments are much lower when switching is not permitted. Aggregate output, consumption, investment, capital and labor hours \((X)\) are weighted sums of the saver’s \(X^s\) and the spenders’ \(X^r\). The shares of savers and spenders are fixed at 50% each when the switching mechanism is turned off, and aggregate macroeconomic variables become more stable. This reduction in volatility is especially apparent for aggregate investment and capital. Furthermore, the volatilities of inflation and the nominal short rate also decline substantially in the absence of switching.

The last three rows of table 2 are the unconditional means of the slope of the nominal term structure, the excess return of a 10-year to maturity nominal bond and the term premium of the same 10-year bond. From column (1), we observe that the average nominal term premium in the no-switching model is extremely low at less than one basis point. While both the benchmark model and the no-switching model report similar levels of average inflation and interest rate, \(\mathbb{E}[TP^{40}]\) in the no-switching model is minuscule relative to the 1.068% generated by the benchmark model. Column (3) shows that the overwhelming share of the average term premium derives from the transitory productivity shock. The productivity shock is also the largest contributor to the volatilities of inflation and the short rate. Fiscal policy shocks, shown to be the pivotal source of large average term premium in the benchmark model, are fairly inert in the no-switching model.

Figure 5 plots the impulse responses of the no-switching model following a positive one standard deviation government spending-to-output shock, a negative one standard deviation productivity shock and a negative one standard deviation return on capital tax rate shock. For simplicity, I leave out the other two tax rate shocks and the monetary policy shock. The impact of the government spending
shock on the no-switching model has been discussed in the previous section in contrast to its impact on the benchmark model. The main takeaway is that the return on consumption claim and the return on labor income in the no-switching model react negatively and on a much smaller scale to the positive government spending-to-output shock.

The negative productivity shock lowers output, investment, consumption, tax revenue and equilibrium wage, while labor supply goes up due to the decrease in consumption. Without switching, the return on wealth decreases following the negative productivity shock because the present value of future consumption drops. Similar to the effect of the shock to the share of government spending, the nominal pricing kernel has a positive response to the negative productivity shock due to low consumption growth. In the mean time, the long-term nominal yield reacts positively to the same shock, leading to positive term premium. Although the SDF reacts less to the negative productivity shock relative to the spending shock, the 10-year bond yield is more sensitive to the productivity shock. Thus, the average risk premium in the no-switching model is mostly generated by the shock to productivity.

5 Conclusion

The inability of the standard general equilibrium model to generate large enough average risk premium on long-term Treasuries to be matched to data is not only puzzling, but it also hinders our understanding of the macroeconomic determinants of long-term nominal risk. In this paper, I show that heterogeneity in the households in conjunction with fiscal policy uncertainty can provide a potential resolution to the Treasury premium puzzle and enhance our understanding of the risk factors underlying the issuance of long-term bonds. Extending Mankiw’s savers-spenders framework for fiscal policy analysis, I develop a model which accommodates the switching of types between the two households and document the significant effect this modification has on long-term Treasury risk premium.
A positive shock to government spending as a share of output raises the debt level in the economy as the government borrows more to finance itself. Higher debt level is a proxy for positive economic outlook, and the probability of a saver type to remain a saver increases. The increase in the probability of remaining the same type means the representative saver is more likely to obtain his/her continuation utility, and this leads to higher return on wealth. With Epstein and Zin (1989)/Weil (1990) utility parameterized to prefer early resolution of uncertainty, the representative saver’s marginal utility increases significantly, pushed up by the higher return on wealth. The result is an extremely amplified response of the stochastic discount factor following the government spending-to-output shock. Since the response of the nominal interest rate to the same shock is also positive, the covariance between the stochastic discount factor and bond yields increases greatly due to the amplification. Bond risk premia become very large in this setting because bond prices are low when the marginal benefit of consuming is extremely high. I am able to match the model-generated average risk premia on 10-year to maturity Treasury bonds to those observed from the post-war data using a constant relative risk aversion of only 3.45.

Negative shocks to tax rates also generate sizable long-term risk premia due to the same mechanism described above. This is especially true for the shock to the tax rate on the return of capital as investment is a major source of uncertain driving future wealth. Contrary to existing studies, I find that the transitory productivity shock contributes to very little of the overall risk premium. The same can be said of the monetary policy shock, which has only a small impact on long-term bond prices.
References


Hsu, Alex, Erica Li, and Francisco Palomino, 2013, What Do Nominal Rigidity and Monetary Policy Tell Us about the Real Yield Curve?, *Working Paper*.


## A Tables

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<td>0.5178</td>
<td>0.7642</td>
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Table 1: **Benchmark Model Summary Statistics for Different Shock Specifications**

The table reports quarterly model-implied statistics of selected variables. The values for inflation, the nominal rate, and risk premia are annual rates. Means and standard deviations are reported in percentage terms. Column (1) shows the summary statistics of the benchmark model. Column (2) is the model with only government spending shocks. Column (3) is the model with only transitory productivity shocks. Column (4) is the model with only shocks to the consumption tax rate rule. Columns (5) is the model with only shocks to the labor income tax rate rule. Column (6) is the model with only shocks to the capital income tax rate rule.
Table 2: No-Switching Model Summary Statistics for Different Shock Specifications

The table reports quarterly model-implied statistics of selected variables. The values for inflation, the nominal rate, and risk premia are annual rates. Means and standard deviations are reported in percentage terms. Column (2) is the model with only government spending shocks. Column (3) is the model with only transitory productivity shocks. Column (4) is the model with only shocks to the consumption tax rate rule. Columns (5) is the model with only shocks to the labor income tax rate rule. Column (6) is the model with only shocks to the capital income tax rate rule.
B Figures

Figure 1: Benchmark and No-Switching Model Comparison

Impulse responses of the endogenous variables in the benchmark model and the no-switching model. Each subplot shows the responses for 4 periods following a positive one standard deviation shock to the ratio of government spending to output. All relevant variables are on real basis unless specified otherwise.
Figure 2: Benchmark Model Shock Comparison

Impulse responses of the endogenous variables under the benchmark specifications. Each subplot shows the responses for 4 periods following a positive one standard deviation shock to the ratio of government spending to output, a negative one standard deviation shock to productivity and a negative one standard deviation shock to the tax rate on the return of capital. All relevant variables are on real basis unless specified otherwise.
Figure 3: Monetary Policy in the Benchmark Model

Impulse responses of the endogenous variables under the benchmark specifications. Each subplot shows the responses for 4 periods following a positive one standard deviation shock to the ratio of government spending to output and a positive one standard deviation shock to monetary policy. The monetary policy shock impulse responses are amplified 10 times for comparison. All relevant variables are on real basis unless specified otherwise.
Figure 4: **Comparative Statics of Monetary Policy Parameters**

Comparative statics of the model implied unconditional mean of the 10-year nominal bond risk premium (TP40) and the average fraction of savers NOT switching types each period ($\lambda^o$). Panel A is constructed by varying the nominal short rate’s response to inflation ($\rho_{\pi}$) and its response to the output gap ($\rho_x$). Panel B is constructed by varying the inflation target ($\pi^*$) and the constant in the Taylor rule ($\bar{i}$). The z-axes are reported in percentages.
Figure 5: No-Switching Model Shock Comparison

Impulse responses of the endogenous variables under the model with no switching of household types. Each subplot shows the responses for 4 periods following a positive one standard deviation shock to the ratio of government spending to output, a negative one standard deviation shock to productivity and a negative one standard deviation shock to the tax rate on the return of capital. All relevant variables are on real basis unless specified otherwise.
C Monetary Policy and Fiscal Policy in the Benchmark Model

C.1 The Monetary Authority

Disengaging monetary policy neutrality by augmenting the saver-spender model with the New-Keynesian framework, I assess the implications of fiscal policy on bond risk premia in the presence of an effective monetary authority. The Taylor rule used by the monetary authority to set the nominal short rate in the model is:

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) \left( \bar{r} + \rho_{\pi} \pi_t + \rho_{x} x_t \right) + u_t, \]

where \( \pi_t \) is log-inflation \( \log \left( \frac{P_t}{P_{t-1}} \right) \), \( x_t \) is the log-output gap, and \( u_t \) is the monetary policy shock following an autoregressive process of order one. Notice the parameter \( \rho_i \) makes the nominal short rate sticky in the model, which is more realistic than the simple case when \( \rho_i = 0 \). The parameter \( \rho_{\pi} \) has to be greater than 1 in order to satisfy the Taylor principle for a determinant equilibrium. The functional form of the monetary policy shock is

\[ u_t = \phi_u u_{t-1} + \sigma_u \varepsilon_t^u, \]

with \( \varepsilon_t^u \sim \text{iid. } \mathcal{N}(0,1) \). Impulse responses of the endogenous variables following a positive government spending-to-output shock are contrasted from those following a positive monetary policy shock to provide relative strength of the two policies.

C.2 The Government’s Budget Constraint

In the presence of long-term bonds, the government’s flow budget constraint balances resources with uses:

\[ P_t \Delta x_t + Q_t^{(1)} B_t (t+1) + \cdots + Q_t^{(m)} B_t (t+\infty) = B_{t-1} (t) + Q_t^{(1)} B_{t-1} (t+1) + \cdots + Q_t^{(m)} B_{t-1} (t+\infty) + P_t G_t, \]

where \( G_t \) is consumption by the government or government spending. \( G_t \) is not productive in the model economy. To explain the intuition on the meaning of this equation, I rearrange the terms
above to get

\[ B_{t-1}(t) - \sum_{j=1}^{\infty} Q^{(j)}(B_t(t+j) - B_{t-1}(t+j)) = P_t(Tax_t - G_t). \] (14)

I then rewrite the budget constraint in its present value form as

\[
\frac{B_{t-1}(t)}{P_t} + E_t \left[ \sum_{j=1}^{\infty} \beta^j \left( \frac{C_t^0}{C_{t+j}^0} \right)^{\gamma} B_{t-1}(t+j) \right] = E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{C_t^0}{C_{t+j}^0} \right)^{\gamma} S_{t+j} \right],
\] (15)

where \( S \) denotes the primary surplus,

\[ S_t = Tax_t - G_t. \] (16)

The present value condition tells us that, in any given period, the government’s fiscal liability has to be endorsed by the present value of expected real surpluses from now to infinity. Derivation of equation (15) can be found in the appendix.[7]

Following Cochrane (2001), I make the assumption of a geometrically declining debt structure to further simplify the government budget constraint:

\[ B_{t-1}(t+j) = \varphi B_{t+j-1}(t+j). \] (17)

Furthermore, it can be shown that the fraction of debt issued at time \( t \) maturing at time \( t+j \) is

\[
\frac{B_t(t+j) - B_{t-1}(t+j)}{B_{t+j-1}(t+j)} = \varphi^{j-1}(1 - \varphi).
\] (18)

Substituting (17) into the present value version of the government budget constraint in (15) and (18) into the intertemporal budget constraint, multiplying the latter by \( \frac{\varphi}{1-\varphi} \) and adding, I arrive at the following:

\[
\left( 1 + \frac{\varphi}{1-\varphi} \right) \frac{B_{t-1}(t)}{P_t} = E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{C_t^0}{C_{t+j}^0} \right)^{\gamma} S_{t+j} \right] + \frac{\varphi}{1-\varphi} S_t,
\] (19)

which is only expressed in terms of the amount of debt outstanding at the end of period \( t-1 \) that’s due at time \( t \). Thus, by applying the geometrically declining maturity structure, the present value budget constraint is free of long-term bonds.

---

[7]: This is a variant of the original present value equation derived by Cochrane (2001) in which consumption growth is exogenously fixed.
C.3 The Fiscal Authority

The fiscal authority levies taxes on the saver’s consumption, labor income, and return on capital while providing a rebate based on the tax shield of depreciated book value of capital. The spender only pays taxes on consumption expenditure and labor income. The tax revenues from the saver and the spender are, respectively, $Tax_s^t$ and $Tax_l^t$, where

$$Tax_s^t = \tau_s^t C_t^s + \tau_s^t W_t N_t^p + Lump_t + \frac{\delta^s_t}{\phi^s_t} K_t^u - \frac{\delta^s_t}{\phi^s_t} \Delta K_t^u$$

and

$$Tax_l^t = \tau_l^t C_t^l + \tau_l^t W_t N_t^p + Lump_t.$$  

The lump-sum tax is meant to be collected to keep the borrowing path of the government from exploding. The amount is trivial compared to the revenue from the distortionary taxes. Following standard procedure in the literature, I specify the lump-sum tax as a function of real debt.

$$Lump_t = Lump + \rho_b [D_t - D]_{t-1}.$$  

From Fernández-Villaverde et al., government spending as a share of output and the three tax rates are determined by the following fiscal rules with feedbacks from lagged output gap and lagged real debt-to-GDP ratio:

$$g_t = (1 - \phi_g)\theta_g + \phi_g g_{t-1} + \phi_{g,y} [Y_t - Y] + \phi_{g,b} \left[ \frac{D_{t-1}(t)P_{t-1}^{real}}{Y_t} - \frac{D}{Y} \right] + \exp(\sigma_g) \epsilon_t^g,$$

$$\tau_t^s = (1 - \phi_{\tau^s})\theta_{\tau^s} + \phi_{\tau^s} \tau_{t-1}^s + \phi_{\tau^s,y} [Y_t - Y] + \phi_{\tau^s,b} \left[ \frac{D_{t-1}(t)P_{t-1}^{real}}{Y_t} - \frac{D}{Y} \right] + \exp(\sigma_{\tau^s}) \epsilon_t^{\tau^s},$$

$$\tau_t^l = (1 - \phi_{\tau^l})\theta_{\tau^l} + \phi_{\tau^l} \tau_{t-1}^l + \phi_{\tau^l,y} [Y_t - Y] + \phi_{\tau^l,b} \left[ \frac{D_{t-1}(t)P_{t-1}^{real}}{Y_t} - \frac{D}{Y} \right] + \exp(\sigma_{\tau^l}) \epsilon_t^{\tau^l},$$

$$\tau_t^k = (1 - \phi_{\tau^k})\theta_{\tau^k} + \phi_{\tau^k} \tau_{t-1}^k + \phi_{\tau^k,y} [Y_t - Y] + \phi_{\tau^k,b} \left[ \frac{D_{t-1}(t)P_{t-1}^{real}}{Y_t} - \frac{D}{Y} \right] + \exp(\sigma_{\tau^k}) \epsilon_t^{\tau^k},$$

where $g_t = \frac{G_t}{Y_t}$ Denote $x$ as $[g_t, \tau_t^s, \tau_t^l, \tau_t^k]'$. The $\theta_t$’s are the unconditional means of the spending-to-output ratio and tax rates. The $\phi_t$’s are the autoregression coefficients of the fiscal policy variables. $\phi_{x,y}$’s are the reaction coefficients to the output gap, and the $\phi_{x,b}$’s are the reaction coefficients to the debt-to-GDP ratio. $X_{t-1}$ is lagged output gap, $D_{t-1}(t)$ is the notional amount of real debt outstanding at time $t - 1$ due at $t$, and $Y_{t-1}$ is lagged output. $\frac{D}{Y}$ is the steady state real debt-to-GDP ratio. Since $D_{t-1}(t)$ is expressed in terms of its face value, I scale $D_{t-1}(t)$ by the unit price of the real bond at time $t - 1$, $P_{t-1}^{real}$, to get the market value of real debt at time $t - 1$ in order to calculate the lagged debt-to-GDP ratio.
Finally, each one of the fiscal variables are driven by a rule-specific shock, \( \varepsilon_i^x \), with \( \varepsilon_i^x \sim \text{iid. } \mathcal{N}(0, 1) \). The \( \sigma_i \)'s are the log volatilities of the fiscal variable shocks. The shocks are uncorrelated, but the feedbacks, especially from debt, generate correlations among the government spending-to-output ratio and the tax rates.

## D Solving the Benchmark Model

### D.1 Households with Epstein-Zin-Weil Preference, Distortionary Taxation, and Switching Probability

The savers’ optimization problem is

\[
\max V(C^o_t, N^o_t) = \left[ \frac{C_t^{1-\psi}}{1-\psi} - \frac{N_t^{1+\omega}}{1+\omega} \right] + \beta E_t \left[ \left\{ \lambda_{t+1}^o V_{t+1} + \left( 1 - \lambda_{t+1}^o \right) U_{t+1}^r \right\}^{1-\psi} \right]^{\frac{1-\psi}{1-\gamma}},
\]

subject to

\[
(1 + \tau_c^t)P_t C_t^o + P_t I_t^o + \sum_{j=1}^{\infty} Q_{t+j} [B_t(t+j) - B_{t-1}(t+j)] + P_t \text{Lump}_t
\]

\[
= (1 - \tau_c^t)P_t W_t N_t^o + (1 - \tau_c^t)P_t R_t^o K_t^o - 1 + \tau_c^t \delta P_t K_{t-1}^{o, \text{book}} + B_{t-1}(t) + P_t \Psi_t,
\]

\[
K_t^o = (1 - \delta) K_{t-1}^o + \Phi \left( \frac{P_t}{K_t^o} \right) K_{t-1}^o,
\]

and

\[
K_t^{o, \text{book}} = (1 - \delta) K_{t-1}^{o, \text{book}} + P_t. \]

where

\[
U_{t+1}^r = \frac{C_{t+1}^{1-\psi}}{1-\psi} - \frac{N_{t+1}^{1+\omega}}{1+\omega}.
\]
The Lagrangian is

\[ \mathcal{L} \equiv V_t - \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \Lambda_{t+i} \left\{ \prod_{j=0}^{i} \lambda_{t+j} \right\} \left\{ V_{t+i} - U_{t+i} - \beta \mathbb{E}_{t+i} \left[ \left\{ \lambda_{t+i} V_{t+i+1} + (1 - \lambda_{t+i}) U_{t+i+1} \right\} \right]^\gamma \right\} \right] \]

\[ - \mathbb{E}_t \left[ \sum_{i=1}^{\infty} \Lambda_{t+i} \left\{ \prod_{j=1}^{i} \lambda_{t+j-1} \right\} (1 - \lambda_{t+i}) \{ V_{t+i} - U_{t+i} \} \right] \]

\[ - \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \Lambda_{t+i} \left\{ \prod_{j=0}^{i} \lambda_{t+j} \right\} \left\{ (1 + \tau_{t+i}^c) P_{t+i} C_{t+i}^o + P_{t+i} a_o + \sum_{k=1}^{\infty} Q_{t+i}^{(k)} [B_{t+i} (t + i + k) - B_{t+i-1} (t + i + k)] \right\} \right] \]

\[ + P_{t+i} \text{Lump}_{t+i} \left\{ (1 - \sigma_{t+i}) P_{t+i} W_{t+i} N_{t+i}^o - (1 - \delta_{t+i}) P_{t+i} R_{t+i}^c K_{t+i-1}^o - \delta_{t+i} \delta P_{t+i} \text{K}_{t+i-1}^o \right\} \]

\[ - B_{t+i-1} (t + i) - P_{t+i} \Psi_{t+i} \right\} \]

\[ - (1 - \tau_{t+i}^d) P_{t+i} W_{t+i} N_{t+i}^o - B_{t+i-1} (t + i) \right\} \]

\[ - \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \Lambda_{t+i} \left\{ \prod_{j=0}^{i} \lambda_{t+j} \right\} \left\{ (1 + \delta_{t+i}) P_{t+i} C_{t+i}^o + P_{t+i} \text{Lump}_{t+i} \right\} \right] \]

\[ - (1 - \delta_{t+i}) P_{t+i} W_{t+i} N_{t+i}^o \]

\[ - \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \Lambda_{t+i} \left\{ \prod_{j=0}^{i} \lambda_{t+j} \right\} \left\{ K_{t+i}^o - (1 - \delta) K_{t+i-1}^o - \Phi \left( \frac{I_{t+i}^o}{K_{t+i-1}^o} \right) K_{t+i-1}^o \right\} \right] \]

\[ - \mathbb{E}_t \left[ \sum_{i=0}^{\infty} \Lambda_{t+i} \left\{ \prod_{j=0}^{i} \lambda_{t+j} \right\} \left\{ K_{t+i}^{o,book} - (1 - \delta) K_{t+i-1}^{o,book} - I_{t+i}^o \right\} \right]. \]

\( \Lambda_{t+i}^1, \Lambda_{t+i}^2, \Lambda_{t+i}^3, \text{and} \Lambda_{t+i}^4 \) for \( i = 0 \) to \( \infty \) are the Lagrangian multipliers. \( \prod_{j=0}^i \lambda_{t+j} \) is the term expressing the fact that between times \( t \) and \( t + i \), the saver has remained a saver and can optimize according to the discounted continuation utility. Notice \( \lambda_{t+i} \) from the perspective of the optimizing agent is always equal to 1.

The first order conditions with respect to current variables are:

\[ \frac{\partial \mathcal{L}}{\partial C_t^o} : \Lambda_1^2 C_t^o - \psi = \Lambda_1^3 (1 + \tau_t^c) P_t, \]

\[ \frac{\partial \mathcal{L}}{\partial N_t^o} : \Lambda_1^4 N_t^o = \Lambda_3^4 (1 - \tau_t^d) P_t W_t, \]

\[ \frac{\partial \mathcal{L}}{\partial B_{t+i} (t + i)} : Q_{t+i}^{(1)} = \mathbb{E}_t \left[ \frac{\Lambda_{t+i+1}}{\Lambda_t^3} \lambda_{t+i+1} + \frac{\Lambda_{t+i+1}}{\Lambda_t^3} (1 - \lambda_{t+i+1}) \right]. \]

Combining the first two equations, we get the wage demand equation of the optimizing household:

\[ W_t = \frac{1 + \tau_t^c}{1 - \tau_t^d} C_t^o N_t^o. \]

and \( \Lambda_{t+i+1}^3 \lambda_{t+i+1} + \Lambda_{t+i+1}^4 (1 - \lambda_{t+i+1}) \) gives us the pricing kernel of the economy between times \( t \) and \( t + 1 \).
Defining \( \lambda_{t+1}^\alpha V_{t+1} + (1 - \lambda_{t+1}^\alpha)U_{t+1} \) as \( \Xi_{t+1} \), the first order conditions with respect to next period variables are:

\[
\frac{\partial \mathcal{L}}{\partial c_{t+1}} = -E_t \left[ \beta E_{t+1} \left[ \frac{\psi_{t+1}}{\psi_t} \right] \frac{1}{1 - \gamma} \left( \frac{1 - \psi}{1 - \psi} \right) \left( \frac{1 - \gamma}{1 - \psi} \right) \Xi_{t+1} \right] - E_t \left[ \frac{\lambda_{t+1}^\alpha}{\lambda_t} \right] \Xi_{t+1} \psi_t \lambda_{t+1}^\alpha + E_t \left( \frac{\lambda_{t+1}^\alpha}{\lambda_t} \right) \Xi_{t+1} \psi_t \lambda_{t+1}^\alpha - E_t \left[ \frac{\lambda_{t+1}^\alpha}{\lambda_t} \right] \Xi_{t+1} \psi_t \lambda_{t+1}^\alpha.
\]

Rearranging the terms then combining these equations, we can derived the pricing kernel:

\[
\frac{\partial \mathcal{L}}{\partial V_{t+1}} = E_t \left[ \beta E_{t+1} \left[ \frac{\psi_{t+1}}{\psi_t} \right] \frac{1}{1 - \gamma} \left( \frac{1 - \psi}{1 - \psi} \right) \left( \frac{1 - \gamma}{1 - \psi} \right) \Xi_{t+1} \right] + E_t \left[ \frac{\lambda_{t+1}^\alpha}{\lambda_t} \right] \Xi_{t+1} \psi_t \lambda_{t+1}^\alpha.
\]

Using the identity from [20], we know that \( \frac{\lambda_t^\alpha}{\lambda_t} \) is equal to \( \frac{c_t}{(1 + \psi_t)\psi_t} \). Plugging it in to the equation...
above, we arrive at the nominal pricing kernel:

\[ M^S_{t,t+1} = \frac{N^1}{N_t} \lambda_{t+1}^o + \frac{N^2}{N_t} (1 - \lambda_{t+1}^o) = \beta \left( \frac{C_t^{o'}}{C_t'} \right)^{-\psi} \left( \frac{(1 + \tau_{t+1}^o)P_t}{(1 + \tau_{t+1}^o)P_t} \right) \frac{(\Xi_{t+1})^{1/\psi}}{E_t \left[ (\Xi_{t+1})^{1/\psi} \right]} \]  

and the real pricing kernel can be expressed as:

\[ M_{t,t+1} = M^S_{t,t+1} \left( \frac{P_{t+1}}{P_t} \right) = \beta \left( \frac{C_t^{o'}}{C_t'} \right)^{-\psi} \left( \frac{(1 + \tau_{t+1}^o)}{(1 + \tau_{t+1}^o)} \right) \frac{(\Xi_{t+1})^{1/\psi}}{E_t \left[ (\Xi_{t+1})^{1/\psi} \right]} \]  

\[ \text{D.2 Return Representation of the Pricing Kernel} \]

In order to understand the properties of the continuation utility \( \Xi \), we can write in the pricing kernel in term of the return on wealth of the optimizing agent, which can in turn be expressed in returns on the agent’s consumption and labor income. Start by defining

\[ \bar{V}_t = E_t \left[ \{ \lambda_{t+1}^o V_{t+1} + (1 - \lambda_{t+1}^o)U_{t+1} \}^{\frac{1 - \gamma}{\psi}} \right] , \]

then the value function becomes

\[ V_t = \left[ \frac{C_t^{o'}^{1-\psi}}{1 - \psi} - \frac{N^1}{1 + \omega} \right] + \beta \bar{V}_t^{\frac{1 - \psi}{\gamma}} . \]

Next, the goal is to manipulate \( \bar{V}_t \) into returns on wealth.

\[ \beta \bar{V}_t^{\frac{1 - \psi}{\gamma}} = \beta \bar{V}_t^{\frac{1 - \psi}{\gamma}} \bar{V}_t^{\frac{1 - \psi}{\gamma}} = \beta E_t \left[ \{ \lambda_{t+1}^o V_{t+1} + (1 - \lambda_{t+1}^o)U_{t+1} \}^{\frac{1 - \gamma}{\psi}} \bar{V}_t^{\frac{1 - \psi}{\gamma}} \right] \]

\[ = \beta E_t \left[ \{ \lambda_{t+1}^o V_{t+1} + (1 - \lambda_{t+1}^o)U_{t+1} \} \left( \frac{\lambda_{t+1}^o V_{t+1} + (1 - \lambda_{t+1}^o)U_{t+1}}{\bar{V}_t^{\frac{1 - \psi}{\gamma}}} \right) \right] \]

\[ = E_t \left[ M_{t,t+1} \left( \frac{C_t^{o'}}{C_t'} \right)^{\psi} \left( \frac{1 + \tau_{t+1}^o}{1 + \tau_{t+1}^o} \right)^{-1} \{ \lambda_{t+1}^o V_{t+1} + (1 - \lambda_{t+1}^o)U_{t+1} \} \right] . \]

Substituting out labor by putting in labor income through the wage demand equation, the value
function can be rewritten as the following:

\[
V_t = \frac{C_t^{\psi-1}}{1-\psi} - \frac{1}{1+\omega} \left[ \left( \frac{1-\tau^t}{1+\tau^t} \right) W_t N_t C^{\psi-\omega}_t \right] \\
+ \mathbb{E}_t \left[ M_{t+1} \left( C_{t+1}^{\psi} \right) \left( 1+\frac{\tau^t}{1+\tau^t} \right)^{-1} \left\{ \lambda^{\omega}_{t+1} V_{t+1} + (1-\lambda^{\omega}_{t+1}) U_{t+1} \right\} \right]
\]

\[
(1-\psi)C_t^{\omega} \psi V_t = \frac{C_t^{\psi-1}}{1-\psi} - \frac{1}{1+\omega} \left( \frac{1-\tau^t}{1+\tau^t} \right) W_t N_t
\]

\[
+ (1-\psi) \mathbb{E}_t \left[ M_{t+1} C_{t+1}^{\alpha} \left( 1+\frac{\tau^t}{1+\tau^t} \right)^{-1} \left\{ \lambda^{\omega}_{t+1} V_{t+1} + (1-\lambda^{\omega}_{t+1}) U_{t+1} \right\} \right]
\]

\[
(1-\psi)(1+\tau^t)C_t^{\omega} \psi V_t = (1+\tau^t)C_t^{\omega} - \bar{\nu}(1-\tau^t)L_t
\]

\[
+ (1-\psi) \mathbb{E}_t \left[ M_{t+1} C_{t+1}^{\alpha} (1+\tau^t) \lambda^{\omega}_{t+1} V_{t+1} \right]
\]

\[
+ (1-\psi) \mathbb{E}_t \left[ M_{t+1} C_{t+1}^{\alpha} (1+\tau^t)(1-\lambda^{\omega}_{t+1}) U_{t+1} \right]
\]

Notice the first expectation on the right-hand-side is the present value of the term on the left-hand-
side. Therefore, we can recursively substitute in the express on the right, including the second
expectation, into the first expectation:

\[
(1-\psi)(1+\tau^t)C_t^{\omega} \psi V_t = (1+\tau^t)C_t^{\omega} - \bar{\nu}(1-\tau^t)L_t
\]

\[
+ \mathbb{E}_t \left[ \sum_{s=1}^{\infty} M_{t+s} \left( \prod_{j=1}^{s} \lambda^{\omega}_{i+j} \right) \left\{ (1+\tau^{t+s})C^{\omega}_{t+s} - \bar{\nu}(1-\tau^{t+s})L_{t+s} \right\} \right]
\]

\[
+ \mathbb{E}_t \left[ \sum_{s=1}^{\infty} M_{t+s} \left( \prod_{j=1}^{s} \lambda^{\omega}_{i+j-1} \right) (1-\lambda^{\omega}_{t+s}) \left\{ (1-\psi)(1+\tau^{t+s})C^{\omega}_{t+s} \psi U_{t+s} \right\} \right].
\]

Using the identity:

\[
(1-\psi)(1+\tau^{t+s})C^{\omega}_{t+s} \psi U_{t+s}
\]

\[
= (1-\psi)(1+\tau^{t+s})C^{\omega}_{t+s} \psi \left[ \frac{C^{\psi-1}_{t+s}}{1-\psi} - \frac{1}{1+\omega} \left( 1+\frac{\tau^t}{1+\tau^t} \right) L_{t+s} C^{\omega-\psi}_{t+s} \right]
\]

\[
= (1+\tau^{t+s})C^{\omega}_{t+s} - \bar{\nu}(1-\tau^{t+s})L_{t+s},
\]

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the expectations on the right-hand-side above can be combined to derive the following expression:

\[
(1 - \psi)(1 + \tau_t^c)C_t^\omega \nu V_t = (1 + \tau_t^c)C_t^\omega - \bar{\nu}(1 - \tau_t^c)L_t
\]

\[
+ \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,j} \left\{ \prod_{j=1}^{s} \lambda_{t+j-1}^c \right\} (1 + \tau_{t+j}^c)C_{t+j}^o
\]

\[
- \bar{\nu} \mathbb{E}_t \sum_{j=1}^{\infty} M_{t,j} \left\{ \prod_{j=1}^{s} \lambda_{t+j-1}^c \right\} (1 - \tau_{t+j}^c)L_{t+j}
\]

\[
V_t = U_t + \frac{PV_t^c - \bar{\nu}PV_{t+1}^l}{(1 - \psi)(1 + \tau_t^c)C_t^\omega}.
\]

Comparing terms with the value function, \(V_t = \left[ \frac{C_{t+1}^\omega - \lambda_{t+1}^c}{1 - \psi} \right] + \beta \bar{\nu} \frac{1 - \psi}{t+\gamma} \), we conclude that the continuation utility can be expressed using the presents values:

\[
\bar{V}_t^{1-\psi} = \frac{1}{\beta} \frac{PV_t^c - \bar{\nu}PV_{t+1}^l}{(1 - \psi)(1 + \tau_t^c)C_t^\omega}.
\]

Furthermore,

\[
\lambda_{t+1}^o V_{t+1} + (1 - \lambda_{t+1}^o)U_{t+1} = \lambda_{t+1}^o U_{t+1} + \lambda_{t+1}^o \frac{PV_{t+1}^c - \bar{\nu}PV_{t+1}^l}{(1 - \psi)(1 + \tau_{t+1}^c)C_{t+1}^\omega} + (1 - \lambda_{t+1}^o)U_{t+1}
\]

\[
= U_{t+1} + \lambda_{t+1}^o \frac{PV_{t+1}^c - \bar{\nu}PV_{t+1}^l}{(1 - \psi)(1 + \tau_{t+1}^c)C_{t+1}^\omega}.
\]

Therefore, the continuation term in the pricing kernel becomes

\[
\left[ \frac{(2 + \lambda_{t+1}^o)(1 - \psi)(1 + \tau_t^c)C_t^\omega}{PV_t^c - \bar{\nu}PV_{t+1}^l} \right]^{\psi - \gamma} = \left[ \lambda_{t+1}^o V_{t+1} + (1 - \lambda_{t+1}^o)U_{t+1} \right]^{\psi - \gamma} = \left[ U_{t+1} + \lambda_{t+1}^o \frac{PV_{t+1}^c - \bar{\nu}PV_{t+1}^l}{(1 - \psi)(1 + \tau_{t+1}^c)C_{t+1}^\omega} \right]^{\psi - \gamma}
\]

\[
= \left[ \beta \left( \frac{C_{t+1}^o}{C_t^o} \right)^{\psi} \left( 1 + \tau_t^c \right) \frac{PV_{t+1}^c - \bar{\nu}PV_{t+1}^l}{PV_t^c - \bar{\nu}PV_{t+1}^l} \right]^{\psi - \gamma}
\]

\[
= \left[ \beta \left( \frac{C_{t+1}^o}{C_t^o} \right)^{\psi} \left( 1 + \tau_t^c \right) \frac{(1 + \tau_{t+1}^c)C_{t+1}^o - \bar{\nu}(1 - \tau_{t+1}^c)L_{t+1} + \lambda_{t+1}^o \frac{PV_{t+1}^c - \bar{\nu}PV_{t+1}^l}{PV_t^c - \bar{\nu}PV_{t+1}^l}}{PV_t^c - \bar{\nu}PV_{t+1}^l} \right]^{\psi - \gamma}
\]
where

\[
\tilde{PV}_{t+1}^c = \lambda_{t+1}^o PV_{t+1}^c
\]

\[
\tilde{PV}_{t+1}^l = \lambda_{t+1}^o PV_{t+1}^l.
\]

The above expression then becomes

\[
\begin{bmatrix}
\phantom{1-\frac{\bar{\nu}}{\bar{\nu}}}
\frac{1}{\bar{\gamma}}
\phantom{1-\frac{\bar{\nu}}{\bar{\nu}}}
\end{bmatrix}
= \left[ \beta \left( \frac{C_{t+1}}{C_t} \right) \right]^{\frac{1-\bar{\gamma}}{\bar{\gamma}}}
\]

\[
\times \left[ \frac{(1 + \tau_{t+1}^c)C_{t+1}^o + \tilde{PV}_{t+1}^c}{PV_{t+1}^c} \frac{PV_{t+1}^c}{PV_{t+1}^c - \bar{\nu}PV_{t+1}^l} + \frac{(1 - \tau_{t+1}^l)LI_{t+1} + \tilde{PV}_{t+1}^l}{PV_{t+1}^l} \frac{-\bar{\nu}PV_{t+1}^l}{PV_{t+1}^l - \bar{\nu}PV_{t+1}^l} \right]^{\frac{1-\bar{\gamma}}{\bar{\gamma}}}
\]

and the nominal pricing kernel in return representation is

\[
M_{t+1}^S = \left[ \beta \left( \frac{C_{t+1}}{C_t} \right) \right]^{\frac{1-\bar{\gamma}}{\bar{\gamma}}} \left( \frac{P_t}{P_{t+1}} \right) \left[ (1 - \text{share}_t) R_{t+1}^c + \text{share}_t R_{t+1}^l \right]^{\frac{1-\bar{\gamma}}{\bar{\gamma}}}
\]

### D.3 Optimal Investment

This section derives the investment decision of the representative saver, who owns all of the capital in the economy. I make one assumption regarding capital in the case of a switch to a spender from a saver: in the period immediately following the transition, the representative agent is no longer allowed to invest in new capital, and he/she will not obtain the benefits of having invested in capital in the previous period. This means the agent’s initial endowment after the switch will not include the after tax return on the physical capital and the tax shield of depreciation on the book value of capital. Furthermore, neither physical capital nor book value of capital can be carried over after the switch to the spender type. Instead, the capital will be transferred over to the incoming savers.

Rewrite the Lagrangian of the representative agent’s optimization problem from section A.1, after dropping the first three terms on utility functions since they do not involve investment or capital,
we have:

\[
\mathcal{L} = - \sum_{j=0}^{\infty} \lambda_{t+j}^{o} \left\{ \prod_{i=0}^{j} \lambda_{t+i}^{o} \right\} \left( (1 + \xi_{t+i}) P_{t+i} C_{t+i}^{o} + P_{t+i} R_{t+i}^{k} + \sum_{k=1}^{\infty} Q_{t+i}^{(k)} \right) \left[ B_{t+i} (t + i + k) - B_{t+i-1} (t + i + k) \right] \\
+ P_{t+i} \text{Lump} P_{t+i} - (1 - \xi_{t+i}) P_{t+i} N_{t+i}^{o} - (1 - \xi_{t+i}) P_{t+i} K_{t+i}^{o} - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) K_{t+i-1}^{o} \\
- B_{t+i-1} (t + i) - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) \\
- (1 - \xi_{t+i}) P_{t+i} W_{t+i} N_{t+i}^{o} - B_{t+i-1} (t + i) - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) \\
- (1 - \xi_{t+i}) P_{t+i} W_{t+i} N_{t+i}^{o} - B_{t+i-1} (t + i) - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) \\
- (1 - \xi_{t+i}) P_{t+i} W_{t+i} N_{t+i}^{o} - B_{t+i-1} (t + i) - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) \\
- (1 - \xi_{t+i}) P_{t+i} W_{t+i} N_{t+i}^{o} - B_{t+i-1} (t + i) - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) \\
- (1 - \xi_{t+i}) P_{t+i} W_{t+i} N_{t+i}^{o} - B_{t+i-1} (t + i) - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) \\
- (1 - \xi_{t+i}) P_{t+i} W_{t+i} N_{t+i}^{o} - B_{t+i-1} (t + i) - \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right)
\]

The first order conditions with respect to current investment is:

\[
\frac{\partial \mathcal{L}}{\partial P_{t+i}} = \lambda_{t+i}^{o} \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) + \frac{\lambda_{t+i}^{o}}{Q_{t+i}^{o}} 
\]

where the ratios of the Lagrangian multipliers are defined as the investment’s Q of capital and the investment’s Q of the book value of capital.

The first order conditions with respect to current capital is:

\[
\frac{\partial \mathcal{L}}{\partial R_{t+i}} = \lambda_{t+i}^{o} \Phi \left( \frac{I_{t+i}^{p}}{K_{t+i-1}^{o}} \right) 
\]

\[
\frac{\lambda_{t+i}^{o}}{Q_{t+i}^{o}} 
\]

\[
\frac{\lambda_{t+i}^{o}}{Q_{t+i}^{o}} 
\]

\[
\frac{\lambda_{t+i}^{o}}{Q_{t+i}^{o}} 
\]

\[
\frac{\lambda_{t+i}^{o}}{Q_{t+i}^{o}} 
\]

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Therefore, the first order condition becomes:

\[ M_{t,t+1}^{\text{noswitch}} = M_{t,t+1}^{\text{noswitch}} \left( \frac{P_{t+1}}{P_t} \right) = \beta \left( \frac{C_{t+1}^{\alpha}}{C_t^{\alpha}} \right) \left( \frac{(1 + \tau_t^j)}{(1 + \tau_{t+1}^j)} \right) \left( \frac{(V_{t+1})^\frac{1}{1-\gamma}}{E_t \left[ (V_{t+1})^\frac{1}{1-\gamma} \right]} \right)^{\gamma-\gamma} \]

Therefore, the first order condition becomes:

\[
\frac{\partial \Sigma}{\partial K_t^{\alpha}}: \quad Q_{t}^{\text{inv}} = \mathbb{E}_t \left[ M_{t,t+1}^{\text{noswitch}} \left( \frac{P_{t+1}}{P_t} \right) \lambda_t^{\alpha} \left( (1 - \tau_t^k)K_{t+1}^k \right) + Q_{t+1}^{\text{inv}} \left( (1 - \delta) + \Phi \left( \frac{P_{t+1}^o}{K_{t+1}^o} \right) - \Phi' \left( \frac{P_{t+1}^o}{K_{t+1}^o} \right) \frac{P_{t+1}}{K_{t+1}^o} \right) \lambda_t^{\alpha} \right] .
\]

The first order conditions with respect to current book value of capital is:

\[
\frac{\partial \Sigma}{\partial K_t^{\alpha, \text{book}}}: \quad \Lambda_t^\delta = \mathbb{E}_t \left[ \lambda_{t+1}^{\delta} \lambda_{t+1}^{\alpha} \tau_{t+1}^k P_{t+1} \delta + \lambda_{t+1}^{\delta} \lambda_{t+1}^{\alpha} (1 - \delta) \right] \\
\frac{\Lambda_t^\delta}{\Lambda_t^\delta P_t} = \mathbb{E}_t \left[ \left( \frac{\lambda_{t+1}^{\alpha}}{\lambda_{t+1}^{\alpha}} \right) \left( \frac{P_{t+1}}{P_t} \right) \tau_{t+1}^k \delta + \lambda_{t+1}^{\delta} \lambda_{t+1}^{\alpha} \right] \\
Q_{t}^{\text{inv,book}} = \mathbb{E}_t \left[ \left( \frac{\lambda_{t+1}^{\alpha}}{\lambda_{t+1}^{\alpha}} \right) \left( \frac{P_{t+1}}{P_t} \right) \left[ \tau_{t+1}^k \delta + Q_{t+1}^{\text{inv,book}} (1 - \delta) \right] \right] \\
= \mathbb{E}_t \left[ M_{t,t+1}^{\text{noswitch}} \lambda_{t+1}^{\alpha} \tau_{t+1}^k \delta + Q_{t+1}^{\text{inv,book}} (1 - \delta) \right] .
\]

### D.4 Monopolistic Producers and Price Rigidities with Labor and Capital

There is a dispersion of firms, denoted by \( j \), with identical production technology in the economy. With nominal price stickiness and monopolistic competition, each firm is faced with the fol-
lowing profit maximization problem:

$$\max_{P_t^*} \sum_{s=0}^{\infty} \alpha^s M^S_{t+s} \left\{ P_t^* Y_{t+s} - \underbrace{P_t Y_{t+s}}_{\text{real wage}} + \underbrace{R^k_{t+s}}_{\text{return on capital}} + \underbrace{K_{t+s}}_{\text{K}} \right\}$$

s.t. \( Y_{t+s} = Z_{t+s}K_{t+s-1}^\kappa N_{t+s}^{1-\kappa} \) \( (25) \)

$$Y_{t+s} = \left( \frac{P_t^*}{P_t} \right)^{-\eta} Y_{t+s}$$ \( (26) \)

$$P_t = \left[ \int_0^1 P_t(j)^{1-\eta} dj \right]^{1/\eta} = \left[ (1 - \alpha)P_t^{1-\eta} + \alpha P_{t-1}^{1-\eta} \right]^{1/\eta}. \quad (27)$$

Using Calvo (1983) pricing, a firm can choose to optimally adjust price to \( P_t^* \) with probability \((1 - \alpha)\) each period independent of the time elapsed between adjustments. The firm maximizes the present value of discounted profits in expectation of \( \alpha \). Individual firms take wage and the rental cost of capital as given in the competitive equilibrium. Furthermore, \((j)\) denotes a firm specific quantity and \( Z_t \) is the productivity that is uniform across all firms in the economy. Log productivity is an exogenous AR(1) process such that

$$z_{t+1} = \ln(Z_{t+1}) = \phi_z z_t + \sigma_z e_{z,t+1}.$$

The first step is to rewrite the objective function in terms of aggregate output, \( Y_{t+s} \), and the marginal cost of production, \( MC_{t+s} \). To calculate the marginal cost, first notice the marginal products are:

$$\frac{\partial Y}{\partial K} = \kappa ZK^{\kappa-1}N^{1-\kappa} = \kappa \frac{Y}{K} = R^k$$

$$\frac{\partial Y}{\partial N} = (1 - \kappa)ZKN^{-\kappa} = (1 - \kappa) \frac{Y}{N} = W.$$  

For the ease of exposition, I have dropped the time subscript here. Combining the above first order conditions, I obtain the standard equilibrium condition between capital and labor:

$$\frac{K}{N} = \frac{\kappa W}{1 - \kappa R^k}.$$
The cost of the firm is

\[
\text{Cost} = W \cdot N + R^k \cdot K = W \cdot N + \frac{k}{1 - \kappa} W \cdot N = \frac{1}{1 - \kappa} W \cdot N = \frac{1}{1 - \kappa} W \cdot Y \cdot \left(\frac{1 - \kappa}{\kappa}\right)^{\kappa} \cdot \left(\frac{R^k}{W}\right)^{\kappa},
\]

where the last equality uses the definition of the production function. The marginal cost, \(\frac{\partial \text{Cost}}{\partial Y}\), can be written as:

\[
MC = \frac{1}{1 - \kappa} W \cdot \left(\frac{1 - \kappa}{\kappa}\right)^{\kappa} \cdot \left(\frac{W}{1 - \kappa}\right)^{1 - \kappa}.
\]

The objective function of the representative firm becomes

\[
\max_{P_t^j} E_t \left[ \sum_{s=0}^{\infty} \alpha^t M^5_{t,s} \left\{ P_t^j \left( \frac{P_t^j}{P_{t+s}} \right)^{-\eta} Y_{t+s} \cdot \left( \frac{R_{t+s}^k}{W} \right)^{\kappa} \cdot \left( \frac{W_{t+s}}{1 - \kappa} \right)^{1 - \kappa} \cdot \left( \frac{P_t^j}{P_{t+s}} \right)^{-\eta} Y_{t+s} \right\} \right].
\]

The first order condition for firm \(j\) is:

\[
E_t \left[ \sum_{s=0}^{\infty} \alpha^t M^5_{t,s} \left\{ Y_{t+s}(j) - \eta P_t^j \left( \frac{P_t^j}{P_{t+s}} \right)^{-\eta-1} Y_{t+s} \cdot \left( \frac{R_{t+s}^k}{W} \right)^{\kappa} \cdot \left( \frac{W_{t+s}}{1 - \kappa} \right)^{1 - \kappa} \cdot \left( \frac{P_t^j}{P_{t+s}} \right)^{-\eta-1} Y_{t+s} \right\} \right] = 0
\]

\[
\Rightarrow E_t \left[ \sum_{s=0}^{\infty} \alpha^t M^5_{t,s} \left\{ Y_{t+s}(j) - \eta Y_{t+s}(j) \right. \right. = \eta MC_{t,s} \left( \frac{P_{t+s}}{P_t^j} \right) Y_{t+s}(j) \left\} \right] = 0
\]

\[
\Rightarrow E_t \left[ \sum_{s=0}^{\infty} \alpha^t M^5_{t,s} Y_{t+s}(j) P_t^j \right] = E_t \left[ \sum_{s=0}^{\infty} \alpha^t M^5_{t,s} \left( \frac{\eta}{\eta - 1} \right) MC_{t,s} P_{t+s} Y_{t+s}(j) \right].
\]

where \(\nu = \frac{\eta}{\eta - 1}\) is the frictionless markup in the absence of price adjustment constraint because the firms are monopolistic competitors in the economy.

I treat the left-hand-side and the right-hand-side of the equilibrium condition separately. The
idea is to rewrite the infinite sums in the expectations recursively. Starting with the left-hand-side:

\[
L.H.S. = E_t \left[ \sum_{j=0}^{\infty} \alpha^j M_{t,j+s}^J \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} Y_{t+s} P_t^*(j) \right]
\]

\[
= \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} Y_t P_t^*(j) \left[ \sum_{j=0}^{\infty} \alpha^j M_{t,j+s}^J \left( \frac{P_t}{P_{t+s}} \right)^{-\eta} \left( \frac{Y_{t+s}}{Y_t} \right) \right]
\]

\[
= \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} Y_t P_t^*(j) \left\{ 1 + E_t \left[ \alpha M_{t+1,j}^J \left( \frac{P_t}{P_{t+1}} \right)^{-\eta} \left( \frac{Y_{t+1}}{Y_t} \right) \right] \sum_{s=1}^{\infty} \alpha^{-1} M_{t+1,j+s}^J \left( \frac{P_{t+1}}{P_{t+s}} \right)^{-\eta} \left( \frac{Y_{t+s}}{Y_{t+1}} \right) \right\}
\]

Now the right-hand-side:

\[
R.H.S. = E_t \left[ \sum_{j=0}^{\infty} \alpha^j M_{t,j+s}^J \nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa) \left( \left( \frac{1}{Z_{t+s}} \right) R_t^J W_t^{1-\kappa} \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} Y_{t+s} P_t \right) \right]
\]

\[
= \nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa)} \left( \frac{1}{Z_{t+s}} \right) R_t^J W_t^{1-\kappa} \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} Y_t P_t
\]

\[
\times E_t \left[ \sum_{j=0}^{\infty} \alpha^j M_{t,j+s}^J \left( \frac{Z_t}{Z_{t+s}} \right) \left( \frac{R_{t+s}^J}{R_t^J} \right)^{\kappa} \left( \frac{W_{t+s}}{W_t} \right)^{1-\kappa} \left( \frac{P_{t+s}}{P_{t}} \right)^{-\eta} \left( \frac{Y_{t+s}}{Y_{t}} \right) \right] \]

\[
= \nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa)} \frac{R_t^J W_t^{1-\kappa}}{Z_t} \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} Y_t P_t
\]

\[
\times \left\{ 1 + \alpha E_t \left[ M_{t+1,j}^J \left( \frac{Z_t}{Z_{t+1}} \right) \left( \frac{R_{t+1}^J}{R_t^J} \right)^{\kappa} \left( \frac{W_{t+1}}{W_t} \right)^{1-\kappa} \Pi_{t+1}^{1+\eta} \left( \frac{Y_{t+1}}{Y_t} \right) J_{t+1} \right] \right\}
\]

Finally, combine L.H.S. and R.H.S. and eliminate \( \left( \frac{P_t^*(j)}{P_t} \right)^{-\eta} Y_t \) outside of the expectations, I have

\[
\left( \frac{P_t^*(j)}{P_t} \right) F_t = \nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa)} \frac{R_t^J W_t^{1-\kappa} J_t}{Z_t}
\]

\[
\Rightarrow \left[ \frac{1}{1-\alpha} \left( 1 - \alpha \left( \frac{1}{\Pi_t} \right)^{1-\theta} \right) \right]^{\frac{1}{\eta}} F_t = \nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa)} \frac{R_t^J W_t^{1-\kappa} J_t}{Z_t},
\]

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where the second equation uses the relationship in equation (27), and $F_t$ and $J_t$ are recursively defined as

\[ F_t = 1 + \alpha E_t \left[ M_{t,t+1} \Pi^\eta_{t+1} \left( \frac{Y_{t+1}}{Y_{t}} \right) F_{t+1} \right] \]

\[ J_t = 1 + \alpha E_t \left[ M_{t,t+1} \left( \frac{Z_t}{Z_{t+1}} \right)^\kappa \left( \frac{W_{t+1}}{W_{t}} \right)^{1-\kappa} \Pi^1_{t+1} \left( \frac{Y_{t+1}}{Y_{t}} \right) J_{t+1} \right]. \]

### D.5 The Term Structure of Interest Rates

The decomposition of nominal bond yields consists of real yields, expected inflation, and inflation risk premium. In closed form, I have:

\[ i_t^{(n)} = r_t^{(n)} + \frac{1}{n} \left\{ E_t[\pi_{t,t+n}] + \text{cov}_t(m_{t,t+n}, \pi_{t,t+n}) - \frac{1}{2} \text{var}_t(\pi_{t,t+n}) \right\}, \]

where the conditional covariance of the real pricing kernel between times $t$ and $t + n$ with inflation during the same period gives us the compensation for inflation risk for holding $n$-period to maturity nominal bonds. To understand inflation risk premium in the current model, I study this covariance term.

While inflation risk premium is the compensation the investors require from the government for holding nominal bonds, term premium is the compensation for risk the investors require in return for holding long-term bonds over short-term bonds. Typically, I can extract components of the average spread between bonds of different maturities by the following:

\[ E \left[ i_t^{(n)} - i_1 \right] = \frac{1}{n} \left\{ \sum_{k=1}^{n-1} (n-k) E \left[ \text{cov}_t \left( m_{t,t+1}, i_t^{(n-k)} \right) \right] - \frac{1}{2} \sum_{k=1}^{n-1} (n-k)^2 E \left[ \text{var}_t \left( i_t^{(n-k)} \right) \right] \right\}, \]

where the covariance between the 1-period nominal pricing kernel and nominal interest rate is the term premium. Long-term bonds are worse at hedging consumption risk when compared to short-term bonds if the marginal rate of consumption substitution between times $t$ and $t + 1$ is high while long-term bonds are expected to have cheaper prices at time $t + 1$. Therefore, the investor requires a risk premium at time $t$ for holding long-term debt thus generating positive term premium.

### E Model Solution and Calibration

I solve the model by second-order perturbation around the steady state. Due to the large number of endogenous variables in the model, perturbation is the most efficient algorithm in producing the
policy functions in the solution. The computational program Dynare is an add-on to Matlab, and it calculates analytical first and second moments of the variables in the model. I then calibrate the model parameters such that the model matches a selected number of the observed macroeconomic and financial variables in the quarterly U.S. data.

As typical practice, I start the calibration by fixing some of the parameters to standard values commonly used in the DSGE literature. Table 3 reports the main parameter values of the benchmark model. The time preference parameter $\beta$ is set to match the level of short-term real interest rate in the model, which translate to a steady state real short rate of, 1%, annually. The inverse of the elasticity of intertemporal substitution, $\psi$, is set to 2.8, within the range of typical values used in the macro literature. Notice this value implies the EIS to be around 0.35, which is less than the minimal value required, 1, to generate long-run risk dynamics in consumption growth à la Bansal and Yaron (2004). The risk aversion parameter, $\gamma$, is set to 11. Since the representative agent in the model can earn labor income as a mean to smooth consumption, his/her attitude toward risk is different than those who do not supply labor. Following Swanson (2012), I adjust the risk aversion parameter by taking into account the labor margin using the closed-form formula $\psi + \omega \nu + \gamma - \psi - \frac{1}{1 - \psi}$. The representative saver’s true coefficient of relative risk aversion is therefore 3.45. To the best of my knowledge, no other paper in the term structure literature can generate sizable bond risk premia in general equilibrium to match data employing a level of risk aversion as low as the current model. In their benchmark calibration, Rudebusch and Swanson (2012) use the coefficient of relative risk aversion of 75 to match the yield curve even after employing Epstein-Zin-Weil recursive preferences and long-run economic risk in the model. Piazzesi and Schneider (2006) use 59 for the coefficient of relative risk aversion in their partial equilibrium model with recursive preferences in order to match the slope of the yield curve. Finally, van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramirez (2012) estimate a DSGE model with recursive preferences and find the data-implied risk aversion is between 40.78 and 84.78 depending on the set of observables used in the estimation. Even after adjusting for the labor margin, the estimated coefficient of relative risk aversion through their model is not low enough to be in the range commonly accepted in the RBC literature. The last preference parameter to be specified is the Frisch elasticity of labor supply. I set the inverse of this parameter, $\omega$, to be 0.18 in order to match the difference in volatilities of output and aggregate consumption.

Next, I calibrate the set of parameters governing the savers-spenders share and the probability of switching types. The average fraction of spenders at the beginning of each period is set to 50%, which is consistent with the value used by Mankiw (2000) and Gali, Valles, and Lopez-Salido (2007) and empirically verified by others. The parameter that sets the mean probability of switching types, $\theta_1$, is 4, which transforms into an target probability for the representative saver to remain a saver each period at 98.2%. The autocorrelation of $\theta_1$ is set to 0.56, and the sensitivities of $\theta_1$ on output and debt are 2.8 and 3.2, respectively. These numbers strike a balance between keeping
the volatilities of output and consumption low in the model while ensuring the correlation between output and the pricing kernel to stay negative.

Monetary policy is conducted through the use of a Taylor rule in the model, where the central bank adjusts the nominal short-term interest rate as a function of inflation and the output gap as well as the short rate from the previous period. The constant in the Taylor rule is set to 2% to match the level of the nominal short rate. The Taylor rule coefficient on inflation is set to 1.65 to match the volatility of inflation, and it satisfies the Taylor principle which stipulates that the nominal short rate has to have a greater than one-to-one response to inflation in order for the inflation path to not explode. The Taylor rule coefficient on the output gap is 0.9, within the range of typical values used in the literature. Again, $\rho_x$ helps to ensure that the correlation between output and the pricing kernel is negative. The autocorrelation coefficient of the nominal short-term interest rate is 0.8, close to the value used by Rudebusch and Swanson (2012). I use this parameter to control the volatility of the short rate. Finally, the inflation target is 1.33% per quarter, and this allows me to match the level of the average inflation in the model to data.

Fiscal policy consists of four taxes and government spending in the benchmark model. The four taxes are distortionary taxes on consumption, labor income and return on capital as well as a lump sum tax to ensure that a stationary debt process exists. In the absence of distortionary taxes, Leeper (1991) has shown that the response of lump sum tax needs to satisfy $\left|\frac{1}{\beta} - \rho_b\right| < 1$ in order for the government to guarantee debt repayment. The lump sum tax rule has a constant of 0.0487 with a small response to real debt, at 0.001, just enough to ensure the non-explosive debt process. These values are very similar to those employed by Fernández-Villaverde, Güerrón-Quintana, Kuester, and Rubio-Ramírez (2012). Furthermore, table [4] reports the parameter values governing the fiscal policy rules for the government spending-to-output ratio and the distortionary tax rates. These numbers are taken directly from Fernández-Villaverde et al., where they conduct Bayesian estimation of the system of four fiscal rules after imposing $\phi_{g,y/b} < 0$ and $\phi_{\tau,y/b} > 0$ using U.S. data from 1970 Q1 to 2010 Q2. For the sake of teasing out the mechanism underlying the link between fiscal policy and bond risk premia, I ignore stochastic volatilities in the fiscal rules and just use the estimated mean of volatilities as the standard deviation of the fiscal shocks.

The following parameters are all standard, taken directly from existing literature. The maturity structure parameter is set to 0.5 following Cochrane (2001). Under a geometrically declining debt structure, this means half of total debt outstanding is short-term and due every period. On productivity, the rate of depreciation on capital is 0.021 while the capital adjustment parameter, $\zeta$, is 0.7. Lower the $\zeta$, the more costly it is to adjust investment from the existing level. Both numbers are used by Kaltenbrunner and Lochstoer (2010) in the LRR I model with transitory productivity shocks. The capital share of output, $\kappa$, is 0.33. Finally, standard in the New-Keynesian models, the price rigidity parameter, $\alpha$, is 0.66. This means every period, two thirds of the firms in the economy are not able to adjust their prices to the optimal level. The higher the $\alpha$, the stickier the nominal prices are. The
price markup parameter resulting from monopolistic competition is 6, resulting in a markup charged by the firms of 20%. The transitory productivity shock and the monetary policy shock follow simple first order autoregressive processes with autocorrelation coefficients of 0.98 and 0.145, respectively. The volatility of the productivity shock is set to 10 basis points per quarter to match the volatility of output, and the volatility of the monetary policy shock is equal to one basis point per quarter to match the volatility of inflation.

The calibration is performed to fit the model implied unconditional volatilities of output, consumption as well as the first order autoregressive coefficients of output and consumption to the corresponding moments in the U.S. data from 1961 to 2007. Output and consumption data are from the St. Louis Federal Reserve’s database (FRED II) and are detrended by the HP filter. Furthermore, the average levels and standard deviations of inflation and the nominal short rate are also matched to the U.S. data for the same time period. Inflation is calculated by using the GDP price deflator series obtained from FRED II. Interest rate data is from the New York Federal Reserve Bank. For the financial variables, I match the average term premium for 10-year to maturity Treasury bonds taken directly from Kim and Wright (2005). For comparison purposes, I also show the unconditional means of the slope of the term structure and the one-quarter holding period excess return of the 10-year bond. The outcome of the calibration is reported in table 5. The model does a decent job in matching the main characteristics of the data, especially with regards to the volatilities of output, consumption, inflation and the interest rate. It does generate too high of autocorrelations in output and consumption. There is tension between the autocorrelations of output and consumption and the volatilities of inflation controlled by the parameters governing the switching probabilities. While the model underreports the average slope of the yield curve and the average access return, it perfectly matches the average 10-year bond risk premium, at 1.06 percent per annum. To the best of my knowledge, no existing DSGE model can make this claim employing such a low labor-adjusted CRRA.

F Additional Analysis

F.1 Preference and Switching Parameters

Figures 8 and 9 are the comparative statics of the benchmark model constructed by varying the parameters governing the representative saver’s utility function and the probability of switching types. Figure 8 focuses on the preference parameters: the time preference ($\beta$), the inverse of the Frisch labor supply elasticity ($\omega$), the risk aversion ($\gamma$) and the inverse of elasticity of intertemporal substitution ($\psi$). Figure 9 focuses on the parameters governing the law of motion of $\dot{\lambda}^o$: the unconditional mean of $\dot{\lambda}^o$ ($\theta_\lambda$), the autocorrelation ($\phi_\lambda$), its dependence on output ($\phi_{\lambda,y}$) and on the
debt-to-output ratio ($\phi_{\lambda,b}$). Recall that $\hat{\lambda}^o$ is transformed through the logistic function into $\lambda^o$ in the model. The left column of each graph shows the annualized model-implied average term premium of a 10-year to maturity nominal bond ($E[TP_{40}]$), and the right column shows the representative saver’s model-implied unconditional probability of remaining as the saver type ($E[\lambda^o]$) each period (the same as the fraction of saver expected to obtain continuation utility). Each subplot displays the surface of the unconditional means by adjusting two parameter values together.

From panel A in figure 8, the inverse of of Frisch elasticity of labor supply ($\omega$) has a strong impact on the average 10-year risk premium without affecting the average switching probability significantly. By increasing $\omega$ from 0.18 to 0.3, the average risk premium goes up by about 40% while the fraction of savers expected to remain as savers decrease slightly. Given the equilibrium wage, higher $\omega$ means the representative saver dislikes providing labor so the agent relies more on Treasury bonds and investment for consumption, driving up the risk premium. The impact of the time preference parameter ($\beta$) is similar. As the representative saver becomes more patient, the average risk premium increases to about 1.1% per annum and the probability to not switch lowers to 91.4%, holding $\omega$ fixed. This is straightforward: as the agent values future consumption more and more, the required compensation for holding a risky asset increases. Panel B in figure 8 demonstrates the crucial relationship between the inverse of EIS and risk aversion. As expected, as risk aversion increases, the average bond risk premium monotonically increases as well. At the lower range of $\gamma$, in particular when $\gamma$ falls below the inverse of EIS, the average risk premium actually turns negative. Under Epstein-Zin-Weil recursive utilities, the preference for early resolution of uncertainty depends on the curvature coefficient of the return on wealth term in the pricing kernel, namely $\psi - \gamma$. If $\psi - \gamma$ is positive while holding $\psi$ to be greater than 1, then high future consumption is preferred by the representative saver, and higher return on wealth lowers the marginal utility, causing the average risk premium to flip signs. Furthermore, the fraction of savers expected to obtain continuation utility is a decreasing function in $\gamma$. Conditional on the realization of the shock, the higher the probability of staying as a saver type ($\hat{\lambda}^o$), the stronger the response of the pricing kernel and the higher the covariance with long-term yields. This is the main mechanism behind the large bond risk premia generated by the benchmark model. However, the higher the $\hat{\lambda}^o$, the probability of a future switch happening also diminishes. As a result, unconditionally speaking, the higher average $\lambda^o$ is associated with the lower average risk premium seen by varying $\gamma$ in panel B. Finally, $\psi$ seems to influence the unconditional switching probability without significantly impact the average risk premium.

The comparative statics of the saver’s switching probability parameter, $\hat{\lambda}^o$, are presented in figure 9. Generally speaking, the inverse relationship between the average risk premium and the unconditional probability to remain the saver type is apparent. The righthand plot of panel A shows that $\hat{\lambda}^o$ is a decreasing function in both the target value of $\hat{\lambda}^o$ ($\theta_{\lambda}$) and its persistence ($\phi_{\lambda}$). Unconditionally, higher average $\lambda^o$ means the representative saver is less concerned with a switch to
the spender type at any moment in time, thus making future wealth less risky. The lefthand plot of
panel A in figure 9 confirms this intuition: the average bond risk premium is an increasing function
in both $\theta_\lambda$ and $\phi_\lambda$, just the opposite of the plot for $\lambda^o$ to the right. Figure 9 panel B repeats the ex-
ercise by adjusting the coefficient loading of $\hat{\lambda}^o$ on output ($\phi_{\lambda,y}$) and the debt-to-output ratio ($\phi_{\lambda,b}$).
From the righthand plot, as $\hat{\lambda}^o$ responds more strongly to output and government borrowing, the
unconditional likelihood to switch to the spender type $(1 - \lambda^o)$ increases, which in turn drives up the
average 10-year risk premium in the lefthand plot. Moreover, as $\phi_{\lambda,y}$ and $\phi_{\lambda,b}$ become too large, the
chance of a switch to the spender type increases exponentially and reaches unrealistic levels around
40%. Therefore, even though a direct estimation of these parameters is not feasible, the comparative
statistics does provide some boundaries in the calibration procedure such that the model performs
within reason.

F.2 The Role of Investment and Capital

Investment and capital accumulation are essential ingredients in the benchmark model in order
to generate large average bond risk premium because investment contributes heavily to the representa-
tive saver’s future wealth. The saver’s intertemporal budget constraint is setup such that if a switch
to the spender type happens, the agent immediately loses the accumulated capital stock and the
rental income from owning that capital. Table 6 reports the summary statistic of the no-investment
model, which eliminates investment and capital from the benchmark model. Column (1) shows that
the model economy is less volatile in the no-investment model, and all three proxies for bond risk
premium are positive and insignificant. Indeed, the average term premium of a 10-year Treasury
bond is 0.32 basis points, in contrast to the 1.068% for the benchmark model. Like the no-switching
model, the positive bond risk premium in the model without investment comes almost entirely from
the transitory productivity shock. Shocks to the share of government spending and the tax rate on
consumption contribute to a small portion of the unconditional compensation for risk, while the
labor income tax rate shock produces negative average risk premium by itself.

Figure 6 plots the impulse responses of the no-investment model following a positive govern-
ment spending-to-output shock and a negative transitory productivity shock. The reactions to these
shocks are largely standard. The government spending shock raises output, labor supply and real
wage, but lowers consumption. Tax revenue and government borrowing increase while primary sur-
plus decreases. The representative saver’s return on consumption claim and return on labor income
respond negatively to the spending shock in the no-investment model. In the derivation of the pricing
kernel in the technical appendix, the return on consumption claim is defined as:

$$R_{t+1}^{\text{consumption claim}} = \frac{(1 + \tau_{t+1})C_{t+1} + \lambda_{t+1}^o PV_{t+1}^c}{PV_t^c},$$

21
where $PV_c$’s are the present value of future consumption at times $t$ and $t+1$. When a positive shock to the share of government spending ($g_{t+1}$) is realized in period $t+1$, consumption of the representative saver ($C_{t+1}^o$) decreases just like in the benchmark case. However, unlike the benchmark case in which the present value of future consumption is sensitive to the $g_{t+1}$ shock, the reaction by $PV_{t+1}$ to the same shock is much weaker in magnitude without capital’s contribution to future wealth. As a result, the impulse response of the return on consumption claim to the government spending-to-output shock in the no-investment model is driven by the negative response of $C_{t+1}^o$ as opposed to the positive response of $\lambda_{t+1}^o$. The negative impulse response of the return on labor income ($R_{t+1}^{\text{labor income}}$) in figure 6 can be explained by similar reasoning.

The combined effect of the negative impulse responses of the returns on consumption claim and labor income dominates the negative reaction of the representative saver’s consumption growth, and the nominal pricing kernel ($m_{\text{nominal}}$) decreases following the realization of the positive government spending shock. Contrasting figure 6 to figure ??, the shock to the government spending-to-output ratio generates a much more variable stochastic discount factor with the desirable sign in a model with investment and capital accumulation. On the other hand, the model economy and the nominal pricing kernel react in the same fashion, both in direction and in magnitude, to the negative productivity shock regardless if capital and investment are present.

### F.3 The Role of Price Rigidities

The New-Keynesian framework is the workhorse of modern monetary economics. Early works in the literature have demonstrated that price rigidities faced by monopolistic competitive firms is the key for monetary policy to have real effects on the economy. The benchmark model presented in this paper is built upon the New-Keynesian model with an active monetary policy and non-Ricardian fiscal policy. I am able to quantify the relative contributions of monetary policy and fiscal policy on bond risk premia in this setting. In this section, I shut off price rigidities in the benchmark model to examine how the mechanism underlying large and positive bond risk premia is affected when monetary policy becomes neutral.

Summary statistics of the no-rigidities model are displayed in table 7. Column (1) shows that the unconditional volatilities of the endogenous variables in the no-rigidities model are lower while the average tax rates and switching probabilities remain much the same. The average term premium

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8 Although not shown here, the likelihood of remaining as the saver type ($\lambda_{t+1}^o$) does indeed increase following the realization of the shock to the share of government spending in the on-investment model as evidenced by increases in output and debt.

9 Clarida, Gali, and Gertler (1999) is one example among others. Too many to cite in completeness.
is increased by 9 basis points from the benchmark specification. Column (2) to (6) report that the government spending-to-output shock and the return on capital tax rate shock are the main determinants of the volatilities of the endogenous variables. More importantly, they account for large shares of the average 10-year risk premium at roughly 40% each, close to the levels produced under the benchmark specification. The productivity shock again contributes very little to the second moments and the average term premium.

Impulse responses of the no-rigidities model are presented in figure 7. In general, a pattern similar to the benchmark model emerges following each of the three shocks to government spending, productivity and the tax rate on return of capital. The fiscal policy shocks are much more substantial than the productivity shock not only on fiscal variables, like tax revenue and total debt, but also on real variables, like output, investment, consumption and labor hours. The fiscal policy shock drives government borrowing and the saver’s likelihood of switching types. Nominal price rigidities seem to have very little effect on the reaction of the representative saver’s return on wealth, as \( R_{\text{consumption claim}} \) and \( R_{\text{labor income}} \) both increase noticeably following the fiscal policy shocks. The response of the nominal pricing kernel to the fiscal policy shocks is much stronger than the response to the productivity shock and on par with its counterpart in the benchmark model.
### Tables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.9976</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse of EIS</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>Coefficient of risk aversion</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>Inverse of Frisch labor supply elasticity</td>
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</tr>
<tr>
<td>CRRA</td>
<td>Swanson (2012) adjusted risk aversion</td>
<td>3.4475</td>
</tr>
<tr>
<td><strong>Switching Probability</strong></td>
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<td></td>
</tr>
<tr>
<td>$\theta_{\mu}$</td>
<td>Mean percentage of spenders in the economy</td>
<td>0.5</td>
</tr>
<tr>
<td>$\theta_{\lambda}$</td>
<td>Mean of saver to spender switching parameter</td>
<td>4</td>
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<tr>
<td>$\phi_{\lambda}$</td>
<td>Autocorrelation of the switching parameter</td>
<td>0.56</td>
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<tr>
<td>$\phi_{\lambda,y}$</td>
<td>Dependence of the switching parameter to output</td>
<td>2.8</td>
</tr>
<tr>
<td>$\phi_{\lambda,b}$</td>
<td>Dependence of the switching parameter to debt</td>
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<tr>
<td>$\bar{i}$</td>
<td>Constant in Taylor rule</td>
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<tr>
<td>$\rho_\pi$</td>
<td>Taylor rule coefficient on inflation</td>
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<tr>
<td>$\rho_x$</td>
<td>Taylor rule coefficient on output gap</td>
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<td>$\rho_i$</td>
<td>Nominal short rate autocorrelation</td>
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<td>$\pi^*$</td>
<td>Inflation target</td>
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<td>Lump sum transfer rule constant</td>
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<tr>
<td>$\rho_b$</td>
<td>Lump sum transfer rule coefficient on real debt</td>
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<tr>
<td>$\phi$</td>
<td>Maturity structure parameter</td>
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<tr>
<td><strong>Productivity</strong></td>
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<tr>
<td>$\delta$</td>
<td>Depreciation rate of capital</td>
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</tr>
<tr>
<td>$\zeta$</td>
<td>Capital adjustment cost parameter</td>
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<tr>
<td>$\kappa$</td>
<td>Capital share of output</td>
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<td><strong>Price Rigidities</strong></td>
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<tr>
<td>$\alpha$</td>
<td>Percentage of non-optimizing firms</td>
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<tr>
<td>$\eta$</td>
<td>Price markup parameter</td>
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<tr>
<td>$\nu$</td>
<td>Markup due to monopolistic competition</td>
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Table 3: Calibrated Parameter Values I
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transitory Productivity Shock</strong></td>
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<tr>
<td>$\phi_z$</td>
<td>Autocorrelation</td>
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<tr>
<td>$\sigma_z$</td>
<td>Volatility</td>
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<tr>
<td><strong>Monetary Policy Shock</strong></td>
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<td>$\phi_u$</td>
<td>Autocorrelation</td>
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<tr>
<td>$\sigma_u$</td>
<td>Volatility</td>
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<td><strong>Government Spending Rule</strong></td>
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<td>$\phi_g$</td>
<td>Autocorrelation</td>
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<tr>
<td>$\phi_{g,y}$</td>
<td>Coefficient on output</td>
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<tr>
<td>$\phi_{g,b}$</td>
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<td>$\sigma_g$</td>
<td>Log volatility</td>
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<td>$\theta_g$</td>
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<td><strong>Consumption Tax Rate Rule</strong></td>
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<tr>
<td>$\phi_{\tau_c}$</td>
<td>Autocorrelation</td>
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<td>$\phi_{\tau_c,y}$</td>
<td>Coefficient on output</td>
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<tr>
<td>$\phi_{\tau_c,b}$</td>
<td>Coefficient on real debt</td>
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<td>$\theta_{\tau_c}$</td>
<td>Unconditional mean</td>
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<td><strong>Labor Income Tax Rate Rule</strong></td>
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<td>$\phi_{\tau_l}$</td>
<td>Autocorrelation</td>
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<td>$\phi_{\tau_l,y}$</td>
<td>Coefficient on output</td>
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<td>$\phi_{\tau_l,b}$</td>
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<td>$\theta_{\tau_l}$</td>
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<td><strong>Capital Income Tax Rate Rule</strong></td>
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<td>$\phi_{\tau_k}$</td>
<td>Autocorrelation</td>
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<td>$\phi_{\tau_k,y}$</td>
<td>Coefficient on output</td>
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<td>$\phi_{\tau_k,b}$</td>
<td>Coefficient on real debt</td>
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<td>$\theta_{\tau_k}$</td>
<td>Unconditional mean</td>
<td>0.3712</td>
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</table>

Table 4: **Calibrated Parameter Values II**
## Table 5: Data and Model-Implied Descriptive Statistics 1961 – 2007

Only the matched macroeconomic moments were used for the purpose of the calibration. $y$ is output, $c$ is consumption and $\pi$ is inflation. $i$ is the nominal short rate or the yield on a one-quarter to maturity bond. $i^{(40)}$ is the yield of a 40-quarter to maturity bond, so $i^{(40)} - i$ is effectively the slope of the nominal yield curve. $EX^{(40)}$ is the one-quarter realized holding-period excess return over the short rate of the same long-term nominal bond while $TP^{(40)}$ is the term premium of the long-term bond taken directly from Kim and Wright (2005). $\tau^c$, $\tau^f$ and $\tau^k$ are effective tax rates. $\mu$ is the fraction of spenders. $\lambda^o$ and $\lambda^r$ are probabilities to NOT switch types for the savers and spenders, respectively. All values are expressed in percentages. Inflation and financial variables are annualized.

<table>
<thead>
<tr>
<th>Unconditional Moments</th>
<th>U.S. Data</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td><strong>Macroeconomic Moments</strong></td>
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</tr>
<tr>
<td>$SD[y]$</td>
<td>1.46</td>
<td>1.4675</td>
</tr>
<tr>
<td>$SD[c]$</td>
<td>1.19</td>
<td>1.2575</td>
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<tr>
<td>$AR(1)[y]$</td>
<td>0.86</td>
<td>0.9787</td>
</tr>
<tr>
<td>$AR(1)[c]$</td>
<td>0.87</td>
<td>0.9131</td>
</tr>
<tr>
<td>$E[\pi]$</td>
<td>3.73</td>
<td>3.7065</td>
</tr>
<tr>
<td>$E[i]$</td>
<td>6.14</td>
<td>6.1564</td>
</tr>
<tr>
<td>$SD[\pi]$</td>
<td>2.42</td>
<td>2.4791</td>
</tr>
<tr>
<td>$SD[i]$</td>
<td>3.36</td>
<td>3.2252</td>
</tr>
<tr>
<td><strong>Financial Moments</strong></td>
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<td></td>
</tr>
<tr>
<td>$E[i^{(40)} - i]$</td>
<td>1.43</td>
<td>1.0221</td>
</tr>
<tr>
<td>$E[EX^{(40)}]$</td>
<td>1.76</td>
<td>0.6387</td>
</tr>
<tr>
<td>$E[TP^{(40)}]$</td>
<td>1.06</td>
<td>1.0680</td>
</tr>
<tr>
<td><strong>Fiscal Moments</strong></td>
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<td></td>
</tr>
<tr>
<td>$E[\tau^c]$</td>
<td>7.75</td>
<td>6.4865</td>
</tr>
<tr>
<td>$E[\tau^f]$</td>
<td>22.44</td>
<td>15.667</td>
</tr>
<tr>
<td>$E[\tau^k]$</td>
<td>37.12</td>
<td>34.097</td>
</tr>
<tr>
<td>$E[\mu]$</td>
<td>-</td>
<td>53.955</td>
</tr>
<tr>
<td>$E[\lambda^o]$</td>
<td>-</td>
<td>91.624</td>
</tr>
<tr>
<td>$E[\lambda^r]$</td>
<td>-</td>
<td>99.534</td>
</tr>
<tr>
<td></td>
<td>(1) No-Investment</td>
<td>(2) g Only</td>
</tr>
<tr>
<td>----------------</td>
<td>------------------</td>
<td>------------</td>
</tr>
<tr>
<td>$SD[y]$</td>
<td>0.7739</td>
<td>0.6371</td>
</tr>
<tr>
<td>$SD[c]$</td>
<td>1.1106</td>
<td>0.7454</td>
</tr>
<tr>
<td>$SD[w]$</td>
<td>0.4995</td>
<td>0.0280</td>
</tr>
<tr>
<td>$AR(1)[y]$</td>
<td>0.9463</td>
<td>0.9315</td>
</tr>
<tr>
<td>$AR(1)[c]$</td>
<td>0.9963</td>
<td>0.9993</td>
</tr>
<tr>
<td>$SD[\pi]$</td>
<td>0.1484</td>
<td>0.0814</td>
</tr>
<tr>
<td>$SD[i]$</td>
<td>0.2124</td>
<td>0.1076</td>
</tr>
<tr>
<td>$E[i^{(40)}]$</td>
<td>0.0028</td>
<td>0.0010</td>
</tr>
<tr>
<td>$E[XR^{(40)}]$</td>
<td>0.0078</td>
<td>-0.0010</td>
</tr>
<tr>
<td>$E[TP^{(40)}]$</td>
<td>0.0032</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

Table 6: **No-Investment Model Summary Statistics for Different Shock Specifications**

The table reports quarterly model-implied statistics of selected variables. The values for inflation, the nominal rate, and risk premia are annual rates. Means and standard deviations are reported in percentage terms. Column (2) is the model with only government spending shocks. Column (3) is the model with only transitory productivity shocks. Column (4) is the model with only shocks to the consumption tax rate rule. Columns (5) is the model with only shocks to the labor income tax rate rule.
### Table 7: No-Rigidities Model Summary Statistics for Different Shock Specifications

The table reports quarterly model-implied statistics of selected variables. The values for inflation, the nominal rate, and risk premia are annual rates. Means and standard deviations are reported in percentage terms. Column (2) is the model with only government spending shocks. Column (3) is the model with only transitory productivity shocks. Column (4) is the model with only shocks to the consumption tax rate rule. Columns (5) is the model with only shocks to the labor income tax rate rule. Column (6) is the model with only shocks to the capital income tax rate rule. The monetary policy shock is inert under the flexible price setting, thus not reported.

<table>
<thead>
<tr>
<th>No-Rigidities</th>
<th>(2) g Only</th>
<th>(3) z Only</th>
<th>(4) τc Only</th>
<th>(5) τl Only</th>
<th>(6) τk Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SD[y]$</td>
<td>1.3428</td>
<td>1.0019</td>
<td>0.1206</td>
<td>0.2189</td>
<td>0.4959</td>
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<tr>
<td>$SD[c]$</td>
<td>1.0724</td>
<td>0.7781</td>
<td>0.2006</td>
<td>0.1953</td>
<td>0.4539</td>
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<tr>
<td>$SD[k]$</td>
<td>6.7305</td>
<td>4.3555</td>
<td>1.3993</td>
<td>1.2583</td>
<td>2.4863</td>
</tr>
<tr>
<td>$SD[w]$</td>
<td>3.0008</td>
<td>1.9294</td>
<td>0.4683</td>
<td>0.5699</td>
<td>1.1047</td>
</tr>
<tr>
<td>$AR(1)[y]$</td>
<td>0.9716</td>
<td>0.9629</td>
<td>0.9372</td>
<td>0.9825</td>
<td>0.9793</td>
</tr>
<tr>
<td>$AR(1)[c]$</td>
<td>0.9861</td>
<td>0.9817</td>
<td>0.9617</td>
<td>0.9973</td>
<td>0.9973</td>
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<tr>
<td>$AR(1)[inv]$</td>
<td>0.9831</td>
<td>0.9871</td>
<td>0.9959</td>
<td>0.9818</td>
<td>0.9814</td>
</tr>
</tbody>
</table>

| $E[\pi]$      | 3.7944     | 2.3975     | 1.8564       | 1.9379       | 2.3680       | 2.6319       |
| $SD[\pi]$     | 2.2224     | 1.5782     | 0.2482       | 0.3693       | 0.7817       | 1.2727       |
| $SD[i]$       | 3.0587     | 2.1090     | 0.3749       | 0.5251       | 1.1124       | 1.8040       |

| $E[i^{(40)}]$ | 1.1016     | 0.4092     | 0.0173       | 0.0522       | 0.2174       | 0.4055       |
| $E[XR^{(40)}]$| 0.7878     | 0.1793     | 0.0405       | 0.0556       | 0.1869       | 0.3255       |
| $E[TP^{(40)}]$| 1.1564     | 0.4300     | 0.0189       | 0.0547       | 0.2261       | 0.4268       |
Figure 6: No-Investment Model Shock Comparison
Figure 7: No-Rigidities Model Shock Comparison
Figure 8: Comparative Statics of Preference Parameters

Comparative statics of the model implied unconditional mean of the 10-year nominal bond risk premium (TP40) and the average fraction of savers NOT switching types each period ($\lambda^o$). Panel A is constructed by varying time preference ($\beta$) and the inverse of Frisch elasticity of labor supply ($\omega$). Panel B is constructed by varying the risk aversion ($\gamma$) and the inverse of elasticity of intertemporal substitution ($\psi$). The $z$-axes are reported in percentages.
Comparative statics of the model implied unconditional mean of the 10-year nominal bond risk premium (TP40) and the average fraction of savers NOT switching types each period ($\lambda^o$). These parameters govern the law of motion of $\hat{\lambda}^{o,r}$ in the logistic functions. Panel A is constructed by varying the autocorrelation ($\phi_{\lambda}$) and the unconditional mean ($\theta_{\lambda}$) of $\hat{\lambda}^{o,r}$. Panel B is constructed by varying the feedback on the debt-to-GDP ratio ($\phi_{\lambda,b}$) and the feedback on output ($\phi_{\lambda,y}$). The z-axes are reported in percentages.

Figure 9: Comparative Statics of Switching Probability Parameters