News Shocks and Inflation: Lessons for New Keynesians*

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Abstract

News about future increases in Total Factor Productivity (TFP) lead to a large and persistent drop in both inflation and the Federal funds rate. We show that a DSGE model with nominal rigidities and a standard parametrization along the lines of Smets and Wouters (2007) is unable to replicate these responses. We then analyze through a sequence of four 'lessons' why the model fails and how to make progress in improving the model's fit. The first lesson is that standard interest rate policy rules are not the cause of the failure. The second lesson is that even if the parameters are reestimated so as to match as closely as possible the impulse responses of macro aggregates to TFP news, the model fails in delivering reasonable dynamics. The third lesson is that augmenting the model with features designed to reduce the sensitivity of wage and price markups to TFP news leads to a substantial improvement in fit. The fourth and final lesson is a caveat to the partial success of lesson three: while we can modify the parametrization to improve the model’s fit to TFP news shocks, this comes at the cost of severely deteriorating the model’s performance with respect to monetary policy shocks. TFP news shocks therefore represent an important empirical challenge for the New Keynesian business cycle literature.

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1 Introduction

The macroeconomic literature has witnessed a resurgence of the idea that news about future changes in fundamentals are an important driver of economic activity. Originally proposed by Pigou (1927), the idea has been resuscitated empirically by Beaudry and Portier (2004, 2006) who argue that anticipations of future increases in productivity drive long-run variations in Total Factor Productivity (TFP) and account for a substantial portion of business cycle fluctuations in real aggregates. More recently, Barsky and Sims (2011), Kurmann and Otrok (2013) and Barsky, Basu and Lee (2014) document that a positive TFP news shock generates a slow persistent increase in real aggregates that is accompanied by a sharp and long-lasting decline in inflation.

In this paper, we investigate whether New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models along the lines of Smets and Wouters (2007) can generate the joint dynamics of real aggregates and inflation to a TFP news shock. We first show that the model parameterized in accordance with Smets and Wouters’ (2007) estimates fails to generate the joint dynamics to a TFP news shock. We then analyze through a sequence of four 'lessons' why the model fails and how to make progress in improving the model’s fit.

In addition to real aggregates and inflation, our analysis includes the Federal funds rate and the slope of the term structure (i.e. spread between long and short-rate government bonds) as integral variables. This choice is motivated by our earlier work (Kurmann and Otrok, 2013) where we document that the same TFP news shock triggers a large and persistent drop in the Federal Funds rate and a substantial steepening of the yield curve. Since monetary policy is a central part of the New Keynesian model and its transmission to long rates through expected future short rates matters crucially for aggregate demand and therefore inflation, it is important to explore the extent to which the model can simultaneously match these interest rate dynamics. Moreover, since the slope is a forward-looking variable that apparently contains significant information about private sector expectations, incorporating it in our analysis should sharpen inference about TFP news shocks.

Figure 1 provides a preliminary look at the ability of a New Keynesian DSGE model to fit the responses of real aggregates, inflation and interest rates to a positive TFP news shock. The solid black lines and grey confidence intervals display the empirical impulse responses to a positive TFP news shock computed from the VAR identification procedure proposed by Barsky and Sims (2011) based on post-war U.S. data. The dotted red lines show the theoretical impulse responses of the New Keynesian model of Smets and Wouters (2007)
calibrated to the posterior means of their Bayesian estimation.

Figure 1: Impulse responses to TFP news shock implied by VAR (solid black lines and grey 68% confidence intervals) and by DSGE model calibrated to Smets-Wouters (2007) estimates (dotted red lines).

We leave the details of the VAR identification procedure and the DSGE model until later. But it is clear from the figure that the model with this parametrization fails miserably. In the VAR, a positive news shock leads to a gradual increase in TFP; generates a small temporary downturn in economic activity before turning to robust positive growth; and implies a sharp prolonged drop in inflation. Monetary policy accommodates this with a more than proportional drop in the Federal funds rate and the spread between long and short rates increases. In the model, by contrast, output, investment and employment do not move on impact and the subsequent increase is well below the empirical counterpart.
Furthermore, inflation, the Federal Funds rate and the long-short spread barely react to the TFP news shock and in fact move in the opposite direction of the data. The problem then, is not just a quantitative one, but a problem with the sign of the impulse response functions going in the wrong direction.

This preliminary evaluation is admittedly a bit unfair to the model since Smets and Wouters’ (2007) estimation does not include TFP news shocks. Nonetheless, Figure 1 is instructive of just how far we will need to move to reconcile the model with the data. To give the model a fairer evaluation, we use a Bayesian limited-information estimation procedure to search for the combination of model parameters that matches the VAR impulse responses as closely as possible. That is, we ask if there exists a parameterization of the model that can match the VAR impulse response functions. This exercise produces several useful lessons.

The first lesson is that conditional on the observed dynamics of inflation and output in the VAR, standard interest rate rules prescribing an aggressive systematic reaction to inflation provide a good explanation for the response of the Federal funds rate to TFP news shocks. We consider this a positive result because it implies that the model has the potential to simultaneously explain the response of the Federal funds rate to other shocks. At the same time, the result implies that the main problem for the New Keynesian models consists in generating the joint dynamics of real aggregates and inflation in response to a TFP news shock. That is, the model is feeding the ‘wrong’ responses of output and inflation into the ‘right’ interest rate rule.

For the second lesson, we reestimate the entire model so as to fit the VAR evidence as closely as possible. The Bayesian estimation approach naturally lends itself to impose priors so that the slopes of the New Keynesian wage and price curves implied by the model remain within the bounds established by the empirical literature. On the positive side, we find that the impulse responses of inflation and the term structure are now of the right sign. On the negative side, we find that quantitatively, the model’s responses both in terms of real macro aggregates and in terms of inflation still fall well short of their VAR counterparts. In addition, the parameter estimates suggest little to no investment adjustment cost and highly variable capital utilization. Both of these estimates are far outside the range estimated elsewhere in the DSGE literature.

Lesson three attempts to improve the performance of the model by introducing working capital as in Christiano, Eichenbaum and Evans (2005) and preferences with limited wealth effect on labor supply as in Jaimovich and Rebelo (2009) and Schmidt-Grohe and Uribe (2010). These two additions reduce the sensitivity of the of price and wage markups, respectively, to TFP news shocks thereby helping the model in generating a larger and more
persistent drop in inflation. While these additions improve the performance of the model, our quantitative analysis shows that they are not sufficient to match all features of the data. Nevertheless, the two additions provide interesting insights. In particular, while Jaimovich and Rebelo (2009) and Schmidt-Grohe and Uribe (2010) introduced their preferences in a purely real business cycle models to generate comovement in real aggregates on impact of the TFP news shock, we find that these same preferences help dampen the upward pressure on wages and hence inflation in New Keynesian models.

Our fourth lesson adds an important caveat to the partial success of lesson three. This caveat is that the estimated model does a very poor job in matching the empirical responses to a monetary policy shock, which is one of the more prominent shocks in the business cycle literature. Indeed, the very features that allow us to match the responses to a TFP news shock – very flexible capacity utilization and absence of investment adjustment costs – are the opposite of the features that help the model in matching the responses to a monetary shock.

Based on these lessons, we argue that TFP news shocks represent a challenge for the New Keynesian business cycle literature. This challenge is all the more important because our VAR estimates indicate that TFP news shocks account for 30% to 50% of unpredictable movements in inflation and the Federal funds rate. Moreover, as the impulse responses in Figure 1 indicate, the drop of the Federal funds rate in response to a TFP news shock exceeds the drop in inflation, implying that U.S. monetary policy responds aggressively by lowering the real rate. As Christiano, Ilut, Motto and Rostagno (2008, 2010) show theoretically, this drop in real interest rates in response to a TFP news shock is hard to generate in a purely real business cycle model. By contrast, they show that in a New Keynesian context, standard monetary policy rules prescribe exactly such a drop in real interest rates, thus implying that monetary policy may fuel inefficient booms in times of news about future productivity increases. Of course, this conclusion about the pitfalls of standard monetary policy rules hinges critically on the New Keynesian DSGE model to generate macroeconomic dynamics in line with the empirical evidence. Hence, resolving the counterfactual response of inflation to TFP news shocks in the New Keynesian model should be of prime interest.

Our findings contrast with Barsky, Basu and Lee (2014) who argue that a standard DSGE model can account for the drop in inflation in response to a TFP news shock. The reason for the difference in conclusions lies in what we view as a reasonable parameterization of the model. In particular, we discipline the choice of parameters by matching the model to jointly fit the observed dynamic responses of real aggregates, inflation and interest rates to TFP news shock. We do not believe this is too high of a standard to ask of a model that
has become the workhorse for much of monetary policy making.

The rest of the paper is organized as follows. Section 2 reviews the VAR identification procedure for identifying news shocks. Section 3 presents the DSGE model and lesson one. Section 4 presents our empirical approach and lessons two and three. Section 5 contains lesson 4. Section 5 shows that our results for the term structure are robust to an alternative model of the term structure that incorporates time varying term premia. Section 6 concludes.

2 VAR evidence of TFP news shocks

We first describe the VAR identification of TFP news shocks by Barsky and Sims (2011) that we adopt in this paper. We then discuss the data used in the estimation and report results.

2.1 Identifying TFP news shocks

To build intuition for the VAR identification of TFP news shocks, consider the following moving average process for the logarithm of TFP

\[
\log TFP_t = v(L)\epsilon^\text{current}_t + d(L)\epsilon^\text{news}_t, \tag{1}
\]

where \(\epsilon^\text{current}_t\) and \(\epsilon^\text{news}_t\) are uncorrelated innovations; and \(v(L)\) and \(d(L)\) are lag polynomials with the restriction that \(d(0) = 0\). This restriction together with the exogeneity assumption on TFP defines \(\epsilon^\text{current}_t\) as a traditional contemporaneous TFP shock and \(\epsilon^\text{news}_t\) as a TFP news shock; i.e. while \(\epsilon^\text{current}_t\) is revealed and affects TFP in \(t\), \(\epsilon^\text{news}_t\) is revealed in \(t\) but affects TFP only in \(t+1\) or later. This formalization of TFP as the sum of two components, one capturing the contemporaneous shocks to productivity and the other capturing the slow diffusion of new technologies, receives empirical support form a host of microeconomic evidence (e.g. Rotemberg, 2003 and references therein).

To identify the news shock, Barsky and Sims (2011) include a measure of TFP in a VAR with a selection of macroeconomic variables. The exogeneity assumption of TFP, which is a basic tenent of business cycle modeling, implies that the two shocks \(\epsilon^\text{current}_t\) and \(\epsilon^\text{news}_t\) explain all of the variations in TFP. Together with \(d(0) = 0\) this means that if TFP is ordered first without loss of generality, the contemporaneous TFP shock \(\epsilon^\text{current}_t\) is identified as the shock associated with the first column of the matrix \(\hat{A}\) obtained from a Cholesky decomposition of the VAR residuals.\(^1\) The news shock \(\epsilon^\text{news}_t\) then corresponds to the innovation that explains

\(^1\)To see this, let the VAR be described by \(Y_t = C(L)u_t\), where \(u_t\) denotes the vector of estimation residuals
all remaining variation in TFP conditional on being orthogonal to $\varepsilon_t^{\text{current}}$. This identification is implemented by extracting the shock that maximizes the amount of the forecast error variance (FEV) of TFP over a given forecast horizon $k$ to $\tilde{k}$, with the side constraint that none of the FEV of TFP at forecast horizon $k = 0$ is explained.\footnote{Barsky and Sims’ (2011) identification approach has several desirable features. First, the approach allows but does not require that either the contemporaneous TFP shock or the TFP news shock or both have a permanent impact on TFP (i.e. $v(1) = 1$ and/or $d(1) = 1$ in the above notation). Second, the approach does not make any restriction about common trends in the different VAR variables. Third, because it is a partial identification method that only makes assumptions about TFP, the approach can be applied to VARs with many variables without imposing additional and potentially invalid assumptions about other shocks.}

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Before continuing, it is instructive to compare the identification procedure to alternative approaches proposed by Beaudry and Portier (2006) and Beaudry and Lucke (2010). In both cases, TFP news shocks are identified by the restriction that $d(0) = 0$ (as in our procedure) and a set of auxiliary short- and long-run restrictions that fully identify all structural shocks of the VAR. This approach has important drawbacks. First, identification of TFP news shocks requires the researcher to take a stand on all other shocks in the VAR. As a result, TFP news shocks are entirely conditional on auxiliary assumptions about other shocks. As more variables are added to the system, the number of auxiliary restrictions required for identification increases disproportionately, thereby compounding the problem. Second, long-run restrictions often suffer from important robustness issues with respect to common trend assumptions. See for example Fisher’s (2010) discussion of Beaudry and Lucke (2010).\footnote{Worse, common trend properties in the data imply that certain long-run restrictions may be redundant. As Kurmann and Mertens (2014) show, this is the case for the higher-dimension VARs used by Beaudry and Portier (2006), leaving their TFP news shock underidentified with variance-covariance matrix $\Sigma$. Then, let $u_t = \hat{\tilde{e}}_t$ be the mapping between VAR prediction errors and structural shocks $\hat{\tilde{e}}_t$ implied by the Cholesky decomposition $\hat{\tilde{A}}' = \Sigma$. Since $\hat{\tilde{A}}$ is lower-triangular, the only shock that can have an immediate effect on the first variable in the VAR (i.e. TFP) is the first element of $\hat{\tilde{e}}_t$.}

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$$q_{\text{news}} = \arg \max_q e_{TFP}' \left[ \sum_{k=\tilde{k}}^{k-1} \sum_{l=0}^{k-1} C_l \hat{\tilde{A}} q q' \hat{\tilde{A}}' C_l' \right] e_{TFP}$$

$$s.t. e_{TFP}' q = 0 \text{ and } q' q = 1$$

where $e_{TFP}$ is a selection vector; and $C_l$ are the vector moving average matrices of the VAR lag polynomial $C(L)$. See the appendix of Kurmann and Otrok (2012) for details.}

and the reported results meaningless. For these reasons, we prefer the partial identification procedure of Barsky and Sims (2011), which by definition can handle an arbitrary number of variables.

2.2 Data

As in Kurmann and Otrok (2013), we specify a VAR that combines term structure and macroeconomic variables. For the term structure data we use two time series. The first is the Federal Funds rate. The second is the term spread, measured as the difference between the 60-month Fama-Bliss unsmoothed zero-coupon yield from the CRSP government bonds files and the Federal Funds rate. We use the Federal Funds rate as the short term rate because the DSGE model examined below does not differentiate between the monetary policy rate and the short-end of the Treasury yield curve (e.g. a 3-month bill rate).

For the macroeconomic data we use a measure of TFP, output, investment, consumption, employment and inflation. The measure of TFP is a quarterly version of the series constructed by Basu, Fernald and Kimball (2006) as updated by Fernald (2012). This series exploits first-order conditions from a firm optimization problem to correct for unobserved factor utilization and is thus preferable to a simple Solow residual measure of TFP. The macro aggregates are all logged, population-adjusted and where applicable in real chain-weighted terms. For inflation, we use the growth rate of the GDP deflator.

All of the macroeconomic series are obtained in quarterly frequency from the FRED database of the St. Louis Fed. The term structure and stock market data are available in daily and monthly frequency. We convert them to quarterly frequency by computing arithmetic averages over the appropriate time intervals. Inflation and term structure data are reported in annualized percent. All remaining variables are reported in natural logs; and the real aggregates are population adjusted. The sample period is 1959:2-2007:3. We choose this end date as the DSGE model does not account for the zero-lower bound.

2.3 Results

The VAR is estimated in levels with 4 lags of each variable, an intercept term, but no time trend. To improve precision, we impose a Minnesota prior (see Hamilton 1994, page 360) on the estimation and compute error bands by drawing from the posterior. To identify TFP news, we proceed as in Barsky and Sims (2011) and Kurmann and Otrok (2013) and set the...

\footnote{Basu, Fernald and Kimball (2006) also make use of industry level data to correct for differences in returns to scale. Since this industry level data is available only on an annual basis, our quarterly TFP measure does not include this returns to scale correction. See Barsky and Sims (2011) for details.}
forecast horizon over which the maximization criteria is applied to $k = 0$ to $\tilde{k} = 40$ quarters. The results are robust to longer forecast horizons and a stationary VAR specifications instead of one in levels.

The black solid lines in Figure 1 show the impulse responses to a TFP news shock, which are essentially the same as those reported in Kurmann and Otrok (2013). By definition, TFP does not react on impact of the shock. Thereafter, TFP increases gradually to what appears a permanently higher level. Output, consumption and investment also increase gradually to a new higher level while employment slowly reverts towards its initial level. On impact of the shock, consumption increases significantly whereas output, investment and employment decline first. Finally, both inflation and the Federal Funds rate drop markedly on impact and remain persistently below their initial value for 15 to 20 quarters. The spread in turn jumps up on impact, implying a substantial steepening of the yield curve, before returning to its initial level after about 10 quarters.

3 A New Keynesian model with TFP news shocks

The New Keynesian DSGE model we adopt is essentially the one described in Smets and Wouters (2007). In this section we briefly review this well known model. We then add to this model a formulation for TFP that can account for the VAR evidence on TFP news shocks. Next, we describe how we append an expectations hypothesis model for long term bond pricing. We use this bond price to construct the model-implied slope of the term structure. We end the section with Lesson 1, on the role of the specification of the monetary policy rule.

3.1 Model

The macro block of the model is essentially the one presented in Smets and Wouters (2007) and contains the following real and nominal frictions: (i) infrequent nominal price and wage setting that allows for indexation to lagged inflation; (ii) habit persistence in consumption; (iii) investment adjustment costs; (iv) variable capital utilization; and (v) fixed costs of production. Details about these frictions and the derivation of the model are available in Smets and Wouters (2007) and a separate appendix to this paper. The only major difference with respect to Smets and Wouters (2007) specification is that TFP in our specification has

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5 The difference is that the current data set is longer and that the VAR includes employment as a variable.
a stochastic trend driven by news shocks. Specifically, we model the demeaned log of TFP as the sum of two components

$$ \log TFP_t = z_t + x_t, $$

(2)

where $z_t$ denotes a persistent but transitory component that is driven by contemporaneous innovations to TFP

$$ z_t = \rho \varepsilon_{t-1} + \sigma_z \varepsilon_{t}^{current} $$

(3)

and $x_t$ denotes the stochastic trend part that is driven by news

$$ \Delta x_t = (1 - \rho_x) \Delta x + \rho_x \Delta x_{t-1} + \sigma_{x} \varepsilon_{t-1}^{news}. $$

(4)

Both (3) and (4) are special cases of the more general specification in (1) and are chosen because they match the evolution of TFP to the contemporaneous shock and the news shock in Kurmann and Otrok (2013) very closely. In particular, their VAR evidence indicates that contemporaneous TFP shocks have a persistent but transitory effect only; and that news about future TFP begins to diffuse one quarter after the shock hits. Notice that the specification of the contemporaneous TFP component $z_t$ does not matter for the impulse responses to a news shock and is included here for completeness.

Monetary policy is described by the same interest rate rule as in Smets and Wouters (2007):

$$ R_t = \rho R_{t-1} + (1 - \rho) [\pi_t + \theta_{\gamma_{\text{gap},t}} + \Delta \gamma_{\text{gap},t}] + \theta \gamma_{\text{gap},t} + \varepsilon^R_t, $$

(5)

where $R_t$ denotes the Federal Funds rate, $\pi_t$ inflation; $y_{\text{gap},t}$ the output gap and $\Delta y_{\text{gap},t}$ the growth rate of the output gap (all in log deviations from their respective steady states); and $e^R_t$ an exogenous monetary policy shock. The output gap is defined as the difference between actual output and potential output if there were no nominal price and wage rigidities.

The slope of the term structure is the difference between the yield on a short term and long term bond yield. Since we work with a loglinearized version of the model, term premia are constant. Even if we did not loglinearize the model, term premia variations for the type of preferences with habit persistence used in the Smets and Wouters (2007) model would be negligible.

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6 Other minor differences compared to Smets and Wouters (2007) are that consumption and leisure enter utility additively (i.e. the degree of intratemporal risk aversion is one); consumption habit is internal instead of external; and the cost of variable capital utilization is modeled through the depreciation rate as in King and Rebelo (2000). Except for the internal habit assumption, which is discussed further in the results section, these differences do not matter for any of the results.

7 Even if we did not loglinearize the model, term premia variations for the type of preferences with habit persistence used in the Smets and Wouters (2007) model would be negligible.
Hypothesis (EH); i.e. the long bond is the sum of expected changes in the short rate

\[ R_t^L = \frac{1}{T} \sum_{i=0}^{T-1} E[R_{t+i}/I_t] \]  

(6)

where the \( E[R_{t+i}/I_t] \) denote expectations of future short rates as implied by the information set available at time \( t \). In the DSGE model, these short-rate expectations can be backed out from the rational expectations solution. The slope then, is this interest rate less the short term interest rate controlled by the central bank. In Appendix A we show how to construct the empirical counterpart for the VAR.

3.2 Lesson 1: Standard interest rate rules capture the Federal funds rate response to a TFP news shock

In Figure 1 we documented that the interest rate response in the Smets-Wouters model to a news shock is a modest increase. In the same figure we showed that in the VAR monetary policy appears to accommodate the news shock with an expansionary response. A natural question then, is whether or not the type of interest rate rule in (5) is compatible with the VAR responses of the Federal Funds rate, inflation and the output gap to a TFP news shock. To answer this question we feed the inflation, output and output gap responses to a news shock from the VAR through three interest rate rules and plot their implied response for interest rates. The first rule is the rule in Smets-Wouters. The second use the same form as Smets-Wouters but estimates the parameters of (5) to maximize the fit of the VAR response of the Federal Funds rate. The third is a simple rule used in the literature:

\[ R_t = 0.5R_{t-1} + (1 - 0.5)[2E_t \pi_{t+1} + 1\Delta y_t]; \]  

(7)

In two of these rules we need a measure of the output gap from the VAR. Since potential output, which together with actual output defines the output gap, is not directly observable, we need to construct it from data in the VAR. We do this by defining the log of potential output as

\[ y_t^p = \phi y_{t-1}^p + (1 - \phi) trend_t, \]  

(8)

where \( trend_t \) is proportional to the VAR response of TFP, with the factor of proportionality defined as the ratio of the long-run response of output to TFP. This implies that monetary policy recognizes that the news shock shifts productivity to which potential output reacts sluggishly, as implied by \( \phi \). The specification of potential output in (8) is admittedly ad-hoc.
and may not necessarily correspond to the definition of potential output in the DSGE model (i.e. equilibrium output in the absence of nominal rigidities). We find, however, that the output gap response implied by our preferred estimation of the model looks similar in sign and shape to the output gap estimated using the above definition.

Based on the specification of potential output in (8), we obtain the following parameterization of (5) fits the VAR response of the Federal Funds rate closest

\[
R_t = 0.42 R_{t-1} + (1 - 0.42) [2.03 E_t \pi_{t+1} + 0.59 y_{gap,t}] + 1.96 \Delta y_{gap,t}
\]

(9)

with \( \phi = 0.55 \). The red line with dots in Figure 2 displays the implied response of the Federal Funds rate with this specification.

Figure 2: Impulse response of the Federal funds rate to a TFP news shock according to the VAR (black solid line and 68% confidence interval); the Smets-Wouters interest rate rule calibrated to their estimates (blue dotted line); the Smets-Wouters interest rate rule estimated to fit the Federal funds rate response in the VAR (red solid line); and the alternative interest rate rule in inflation and output growth (green dashed line).
As Figure 2 shows, the best fitting rule closely tracks the VAR response of the Federal Funds rate (black solid line and grey 68% confidence intervals). Also note that when estimating (5), we fixed $\theta_\pi$ to the median estimate of 2.03 in Smets and Wouters. The reason for this is that for sufficiently large values of $\theta_\pi$, different combinations of $\{\rho, \theta_\pi, \theta_{\text{gap}}, \theta_{\Delta\text{gap}}\}$ provide a very similar fit as the one given by the estimates in (9). This weak identification issue should, in fact, be seen as a positive result rather than an issue. It implies that (5) fits well for an entire combination of monetary policy parameters. Part of the explanation for the lack of identification is the fact that the output response to the shock is zero on impact—so a wide range of response parameters can be consistent with this.

If we use the interest rate rule as estimated by Smets and Wouters we do fairly well in terms of fit. This response is shown by the blue dotted line in Figure 3. The main problem with this response is that on impact the federal funds rate does not drop enough. This is due to the Smets-Wouters estimates which emphasis the gradual nature of the response of the Federal Funds are to most shocks. The news shock, though, drives the Federal Funds rate down immediately with a strong response.

The lesson from this exercise is that the VAR response of the Federal Funds rate to a TFP news shock is reasonably well approximated by an interest rate rule that responds aggressively to inflation as prescribed by Taylor (1993). This in turn has the important implication that the real short rate falls in response to the news shock. For our DSGE model we conclude that the rule in the model does not cause the failure in Figure 1. Instead, the dynamics for inflation and output generated by the DSGE must be the source of the problem. That is, we have the 'right' rule but the model is generating the 'wrong' dynamics for the variables entering the rule—in particular inflation.

4 Lessons from re-estimating the DSGE model

As noted in the introduction, Smets and Wouters’ (2013) estimation of the model does not include a TFP news shock. In this section we re-estimate the model parameters conditional on a TFP news shock to see to what extent a reparameterization of the model can improve the fit. We first describe our estimation approach, and then develop two lessons on how the model can be altered to improve fit.

4.1 Calibration and estimation

Given that the model has many parameters, a limited information estimator is unlikely to identify all of them precisely. We therefore partition the parameters into two groups. The
first group consists of parameters that we can calibrate to match long-run moments of the data or that are of little consequence for the dynamics of real aggregates and inflation in response to a TFP news shock. The second group of parameters is estimated to match the impulse responses of the different variables in the VAR to a TFP news shock.

Table 1 presents the set of calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
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<tr>
<td>$\alpha$</td>
<td>Elasticity of production to labor</td>
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<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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<td>$\delta$</td>
<td>Depreciation rate</td>
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<tr>
<td>$\phi_p$</td>
<td>Gross markup in goods market</td>
<td>1.1</td>
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<tr>
<td>$\epsilon_p$</td>
<td>Curvature of Kimball aggregator in goods market</td>
<td>10</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Gross markup in labor market</td>
<td>1.5</td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>Curvature of Kimball aggregator in labor market</td>
<td>10</td>
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</tbody>
</table>

Where applicable, values are reported for a quarterly frequency. The first four parameters imply a labor share of 0.675 in line with Gollin (2002); an average annualized quarterly real interest rate of 2.34% as measured in our data; an annual depreciation rate of 10 percent; and an average markup for final goods producers of 10% as reported by Basu and Fernald (1997). The curvature parameters on the Kimball aggregators and the gross markup in the labor market are set as in Smets and Wouters (2007). As we will discuss below, they are not identified separately. Finally, the growth rate of output and TFP (not reported in Table 1) are set to match the average growth rate of the two variables in the data (1.86% and 1.29% annually for the 1959-2007 sample).

The second group of parameters is estimated using a Bayesian limited information estimator (Kim, 2002) that minimizes the distance between the model-implied IRFs to a news shock and the empirical counterparts from the VAR. The impulse matching estimation follows in the spirit of Christiano, Eichenbaum and Evans (2005) who estimate model parameters to match VAR impulse response functions to a monetary shock. Instead of using a classical minimum distance estimator, we adopt a Bayesian approach as proposed by Christiano, Trabandt and Walentin (2010). Two considerations motivate this approach. First, the Bayesian approach allows to naturally incorporate prior information from the literature about the range of reasonable values of certain parameters. Second, as we learned in earlier iterations of the paper, a classical minimum distance estimator would be subject to several weak identification problems that complicate the analysis without affecting the main conclusions of
Denote by $\hat{\Psi}$ the vector of empirical IRFs to a news shock obtained from the VAR. Likewise, denote by $\Psi(\zeta)$ the same vector of IRFs implied by the DSGE model, where $\zeta$ contains all the structural parameters to be estimated. Our estimate of $\zeta$ will in principal be the parameters that minimizes the distance between $\hat{\Psi} - \Psi(\zeta)$. In practice more weight will be given to those impulses that are more precisely estimated and the parameters will be pushed in the direction of prior beliefs.

A Bayesian estimation of this model will be an analysis of the posterior distribution of the parameters—the product of the likelihood function and the prior. Our first step then is to build the likelihood function implied by the impulse response function matching objective. Letting $\zeta_0$ denote the true values of these parameters, the asymptotic distribution of $\hat{\Psi}$ is

$$\sqrt{T}(\hat{\Psi} - \Psi(\zeta_0)) \sim N(0, V(\zeta_0, \eta_0, T))$$

where $\eta_0$ is the true value of the shocks and $V()$ is the true variance-covariance matrix of the data ($\hat{\Psi}$). In practice we will need to estimate $V()$, which we will return to shortly. Note that we are treating $\hat{\Psi}$ as the data in the likelihood function. Given the asymptotic distribution of $\hat{\Psi}$ then the likelihood function is given by

$$f(\hat{\Psi}|\zeta, V(\zeta_0, \eta_0, T)) = \left(\frac{1}{2\pi}\right)^{N/2} |V(\zeta_0, \eta_0, T)|^{-1/2} \exp\left[-\frac{1}{2}(\hat{\Psi} - \Psi(\zeta))V(\zeta_0, \eta_0, T)(\hat{\Psi} - \Psi(\zeta))^T\right]$$

The posterior then is the likelihood multiplied by the prior, $p(\zeta)$:

$$f(\zeta|\hat{\Psi}, V(\zeta_0, \eta_0, T)) \propto f(\hat{\Psi}|\zeta, V(\zeta_0, \eta_0, T)) p(\zeta)$$

where we use proportionality as we have dropped the marginal density of the data ($\hat{\Psi}$) in the denominator as we will not need it. Simulation from the posterior requires the Metropolis-Hastings algorithm that is now standard in the full information likelihood literature. Simulating from the posterior is straightforward—given a candidate draw for $\zeta$ the evaluation of the posterior requires simply an evaluation of the multivariate normal and the posterior distributions, which all are straightforward to evaluate.

To construct an estimate of $V()$ we follow Christiano, Trabandt and Walentin (2010) and construct a bootstrap estimate for $V()$ which we denote $\tilde{V}$. In principal $\tilde{V}$ contains the population variance covariance matrix of $\hat{\Psi}$. To approximate this matrix we we use the estimated VAR and residuals from the VAR to construct a set of M bootstrap realizations.
of the impulses response functions. The sample analogue of the population VCV matrix is then
\[ \mathbf{V} = \frac{1}{M} \sum_{i=1}^{M} (\mathbf{\Psi}_i - \bar{\mathbf{\Psi}})(\mathbf{\Psi}_i - \bar{\mathbf{\Psi}})' \]  
(13)

As is well known this matrix is inefficient in small samples. We make the same choices as Christiano, Trabandt and Walentin (2010) and first use only the diagonal of this matrix. Second, we downweight lags at longer horizons using the weighting function:
\[ \left[ 1 - \frac{|j|}{n} \right] \]  
(14)

where \( n \) is the maximal horizon of the impulse response function and \( j \) the horizon of the impulse being weighted.

We implement the estimation in two steps. In a first step, we estimate the two parameters in (4) governing the response of TFP to a news shock by matching as closely as possible the empirical response of TFP in the VAR. We obtain \( \rho_x = 0.92 \) and \( \sigma_x = 0.0004 \); i.e. a highly persistent growth process hit by small innovations. As Figure 1 in the introduction shows, (4) thus parameterized provides an excellent description of the response of TFP to a news shock. In the second step we estimate the remaining parameters using the Bayesian limited information procedure using the VAR responses of the different real aggregates, inflation, the Federal funds rate and the long-short spread as implied by the EH (as computed in Appendix) as targets. For each of these impulse responses, we include the entire 40 quarter horizon in the estimation criteria.

Table 2 reports the prior distributions of the different parameters that we estimate. In a first instance, we follow Smets and Wouters (2007) and specify 'loose priors' for all parameters. Based on the estimates thus obtained, we then specify 'tight priors' for the Calvo parameters on price and wage re-optimization so as to achieve estimates of the resulting New Keynesian price and wage curve in line with the established literature.

4.2 Lesson 2: A reasonable parameterization of the model fits poorly

Table 3 reports means and 5%-95% bounds of the posterior distribution of the different parameters. As a reference point, the first column reports the posterior means of Smets and Wouters' (2007) full-information estimation that we used to parameterize the model in Figure 1. The second column, labelled loose prior estimates, reports estimates resulting from
our re-estimation of the model based on the loose priors described in Table 2. The Calvo price parameters $\kappa_p = 0.73$ and $\omega_p = 0.20$ are close to the values estimated by Smets and Wouters (2007) and other empirical studies on the New Keynesian Phillips curve (NKPC). By contrast, the estimation pushes the posterior mean of the Calvo wage parameters to very high values, implying an extreme degree of nominal wage rigidity (an estimated frequency of wage reoptimization of $1 - \kappa_w = 0.03$ per quarter) and a degree of indexation for non-reoptimized wages to past inflation of $\omega_w = 0.84$.

The estimation also has strong implications for investment adjustment cost and variable capital utilization. The investment adjustment cost parameter is driven to its lower bound of $S^* = 0$ (i.e. no adjustment costs in the vicinity of the steady state). For technical reasons that have to do with convergence of the Metropolis-Hastings algorithm, we therefore fix $S^* = 0$. The parameter governing the variability of capital utilization $\sigma_u$ is also estimated close to its lower bound of 0, which implies that capital utilization is roughly proportional to the rental rate of capital (for $\sigma_u = 0$, depreciation would increase linearly with utilization). The remaining parameters are relatively close to the estimates of Smets and Wouters (2007) with the exception of the inverse labor supply elasticity $\eta$, which is estimated to be lower (but is very imprecisely estimated), and the persistence of the interest rate rule $\rho_R$, which is also estimated to be lower.

To understand these estimates, it is instructive to consider the New Keynesian price and wage curves that the model implies. In particular, consider first the New Keynesian Phillips curve (NKPC) which results from the loglinearized price setting block of the model

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} + \pi_3 m_c_t$$

(15)

where $\pi_t$ denotes inflation; $m_c_t$ denotes real marginal cost of firms; and $\pi_1 = \frac{\omega_p}{1 + \beta \omega_p}$, $\pi_2 = \frac{\beta}{1 + \omega_p}$ and $\pi_3 = \frac{1}{1 + \beta \omega_p} \frac{(1 - \kappa_p)(1 - \beta \kappa_p)}{\kappa_p} \frac{1}{\phi_p - 1}$. For ease of illustration, let us assume that $\omega_p = 0$ in which case $\pi_1 = 0$ and the NKPC becomes purely forward-looking; i.e.

$$\pi_t = \pi_3 \sum_{i=0}^{\infty} \beta^i E_t m_c_{t+i}.$$  

(16)

For inflation to experience a large and persistent drop in response to a TFP news shock, it has to be the case that (i) the present value of expected real marginal cost terms declines and (ii) the slope parameter $\pi_3$ is relatively large. Conditional on the calibrated values for $\beta$, $\phi_p$ and $\epsilon_p$ in Table 1, the estimates of $\kappa_p = 0.73$ and $\omega_p = 0.20$ imply $\pi_3 = 0.043$, which is
on the higher side of values reported in the literature but not unreasonable. Real marginal cost, in turn, can be expressed in log-linearized form as

$$mc_t = \alpha w_t + (1 - \alpha)(r^k_t - u_t) - tfp_t$$

(17)

where \(w_t\) denotes the real wage; \(r^k_t\) the rental rate of capital; and \(u_t\) capital utilization. Since TFP increases only gradually one period after the news shock hits, the present value of expected real marginal cost declines only if the real marginal cost is $\leq$ at (i.e. wages and the utilization adjusted rental rate react little to increases in aggregate demand) and the change in aggregate demand is small (or negative) on impact of the shock. This explains the extreme estimates for the Calvo wage parameters, the capital utilization parameter, and the investment adjustment cost parameter. Specifically, the New Keynesian wage curve implied by the model can be expressed as

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_t\pi_{t+1} + E_t w_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} + w_4 (mrs_t - w_t)$$

(18)

where \(mrs_t\) denotes the marginal rate of substitution of workers; and \(w_1 = \frac{1}{1+\beta}\), \(w_2 = \frac{1+\beta \kappa_w}{1+\beta}\), \(w_3 = \frac{\omega_w}{1+\beta}\) and \(w_4 = \frac{1}{1+\beta} \frac{(1-\kappa_w)(1-\beta \kappa_w)}{(\phi_w-1)\kappa_w} \frac{1}{(\phi_w-1)\kappa_w+1}\). The higher the wage rigidity \(\kappa_w\) parameter, the less responsive the real wage is from departures from the labor supply curve (i.e. \(mrs_t \neq w_t\)). In turn, the more variable capital utilization, the less sensitive is \(r^k_t - u_t\). And the smaller the investment adjustment cost, the smaller the increase in aggregate demand. Indeed, as previous papers have argued in a real business cycle context (e.g. Jaimovich and Rebelo, 2009), investment adjustment cost provides firms with a strong incentive to increase investment on impact of the TFP news so as to smoothen investment dynamics in the future when productivity is higher. In the present context, this would lead to upward pressure on aggregate demand and thus marginal cost and inflation. Moreover, investment according to our VAR results does not increase on impact.

The parameter estimates for wage rigidity, capital utilization and investment adjustment cost are far from the values usually found in the literature. In particular, the estimate of the Calvo parameter estimate of \(\kappa_w = 0.97\) is problematic for a model intended to explain inflation dynamics because it implies that nominal wages are essentially never reoptimized. Moreover, conditional on the calibrated values for \(\beta, \phi_w\) and \(\epsilon_w\), the resulting wage curve slope equals \(w_4 = 0.0001\), which is orders of magnitude smaller than what is usually found in

\(^8\)Smets and Wouter’s (2007) posterior mean estimates imply \(\pi_3 = 0.022\); Eichenbaum and Fisher’s (2008) GMM estimation and Kurmann’s (2007) VAR-based estimation both imply values of about 0.025.
the literature. To address this issue, we impose tighter priors on the parameters of the price and wage adjustment probabilities $\kappa_p$ and $\kappa_w$. These priors, labelled 'tight priors' in Table 2, are closely centered on the Smets-Wouters estimates, as those estimates are typical of the literature. We could of course also restrict the investment adjustment cost parameter and capital utilization parameter, but this would just drive us back to the results in Figure 1. Instead we ask how well the model fits the VAR evidence if we impose that the parameters governing nominal rigidities to stay within a reasonable range.

The third column of Table 3, labelled 'tight prior estimates' reports the resulting estimates. The posterior mean of $\kappa_p$ is 0.62, which is consistent with the unconstrained estimate. The estimate for $\kappa_w$ is now 0.76, which is in line with the estimate of Smets and Wouters (2007) and others in the literature. The tight prior specification thus accomplishes our goal of having reasonable nominal parameter estimates. In turn, the parameters governing variable capital utilization and investment adjustment cost remain close or at 0 for the aforementioned reasons.

Figure 3 plots the impulse responses of the model conditional on these tight prior estimates and compares them with the VAR counterparts. While the fit of the model is better than for the original Smets-Wouters estimates, the performance of the model is still poor. Consumption and output never reach the level implied by the VAR; investment and employment stay negative for far too long; inflation and the Federal funds rate barely decline on impact and have an overall negative response that is small relative to the data; and the

\footnote{For example, Smets and Wouters' (2007) posterior mean estimates imply $w_4 = 0.032$.}
spread moves in the wrong direction on impact.

Lesson two is therefore that once we impose a prior that the nominal rigidities be reasonable in magnitude, the model cannot match the quantitative responses of real aggregates, inflation and interest rates to a TFP news shock. Moreover, other parameters of the model remain very far from the estimates in Smets and Wouters (2007). If we were to constrain these parameters as well, the fit would deteriorate further. In particular, investment adjustment cost, which Smets and Wouters (2007) highlight as one of the important ingredients for the fit of their model, is driven to zero. This is in fact a very robust result that occurs with many other configurations of the other parameters. Modern New Keynesian DSGE models as proposed by Smets and Wouters (2007) thus fail to match the quantitative response of both macro and term structure variables to a TFP news shock.
4.3 Lesson 3: Working capital and preferences with limited short-run wealth effects improve the fit

We now seek to improve the fit by augmenting the model with different features while keeping the tight priors on $\kappa_p$ and $\kappa_w$. In particular, given the above discussion about the determinants of inflation and wage dynamics, we are looking for features that flatten the marginal cost curve.

Our first model alteration is to add working capital as in Christiano, Eichenbaum and Evans (2005). With this formulation firms must borrow to pay their wage bill. This borrowing cost ($R_t$) enters the real marginal cost curve as follows:

$$mc_t = \alpha(R_t + w_t) + (1 - \alpha)(r^k_t - w_t) - tfp_t$$

The fact that the Federal Funds rate falls after the news shock then lowers marginal cost, helping to explain the fall in inflation.

Our second modification is to preferences. We adopt a version of the Jaimovich-Rebelo (2009) preferences which eliminated the wealth affect on labor supply. This will help limit the upward pressure on wages after a news shock (which increases wealth). We use the formulation of these preferences adopted by Schmidt-Grohe and Uribe (2010) which adds internal habit persistence. The preferences are given by

$$u(C_t, N_t) = \frac{(C_t - bC_{t-1} - \phi N_t S_t)^{1-\gamma} - 1}{1 - \sigma}$$

$$S_t = (C_t - bC_{t-1})^\gamma S_{t-1}^{1-\gamma}$$

For $\gamma = 1$ preferences reduce to standard King-Plosser-Rebelo preferences in consumption and leisure with habit persistence. For $\gamma \to 0$, the influence of short-term wealth effects on labor supply shrinks to zero, reducing the sensitivity of the marginal rate of substitution and therefore wages, marginal cost and inflation to aggregate demand increases.

The fourth column of Table 3, labelled ‘tight prior estimates of augmented model’ shows the estimates resulting from this estimation. Due to the tight prior on $\kappa_w$, its posterior mean remains close to the one estimated by Smets and Wouters (2007). If the prior on $\kappa_w$ was looser, the estimation would push the posterior mean close to 1 as before. The parameters governing variable capital utilization and investment adjustment cost remain estimated close to or at zero. The parameter governing short-term wealth effects is driven towards zero, as expected, since this limits upward pressure on wages to the greatest extent possible.

\[\text{In the estimation, we set } \sigma = 1 \text{ as this does not substantially affect the quantitative results.}\]
Figure 4 shows the fit of the augmented model conditional on these estimates.

The fit is considerably better than the fit of the original Smets-Wouters model conditional on the tight prior estimates in Lesson 2. In particular, the model now generates appropriate amplification of real aggregates although the very low estimate of the wealth effect parameter $\gamma$ implies that employment returns to its initial value only in the limit. The drop in inflation and the Federal funds rate on impact is markedly more important although still insufficient.
As a result, the EH part of the spread responds too little on impact.

The third lesson is therefore that the introduction of preferences with limited short-term wealth effect on labor supply and working capital considerably help improve the fit of the model. One may therefore be tempted to claim at least partial success. Note however, that many of the parameters are still at odds with the literature. As a foreshadow to the final lesson, recall that these results have been obtained conditional on zero investment adjustment cost and highly variable capital utilization. Both of these features are important ingredients of medium-scale DSGE models to fit salient other business cycle facts and in particular the response of the model to monetary policy shocks, as documented by Christiano, Eichenbaum and Evans (2005).

5 Lesson 4: The estimated model fails to generate empirically plausible dynamics to monetary policy shock

A robust feature of our three sets of parameter estimates is that investment adjustment costs are driven to zero. This has a number of problematic implications. First, with zero investment adjustment cost, the model implies a constant value of the firm and therefore absence of fluctuations in stock prices. Second, (close to) zero investment adjustment costs stands in sharp contrast with Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007) and Christiano, Trabandt and Walentin (2010) among many others who find that investment adjustment costs are essential to fit the data with respect to other shocks.

For our last lesson we investigate how well our best-fitting news shock model (from lesson 3) does in explaining the response to a monetary shock. We choose a monetary shock because it is a prominent shock in the DSGE literature. Moreover, the New Keynesian model was built in large part to understand the effects of monetary policy. If the positive results from Lesson 3 come at the cost of losing the ability to explain the dynamics in response to a monetary policy shocks, then this raises an important challenge for the model.

Figure 5 depicts the monetary policy shock from the model and from a VAR identification
of monetary shocks.

![Monetary policy shock](image)

**Figure 5**: Impulse responses to a monetary policy shock of VAR (black solid lines and grey 68% confidence intervals) and of tight prior estimation of Smets-Wouters model (dotted red lines).

The VAR consists of the same variables as before and is estimated using the same Minnesota prior. The black line denote the posterior median while the gray bands show the 68% coverage intervals. The dotted red line is the response to a monetary shock of the modified New Keynesian model with parameters estimated in Lesson 3. The results are fairly striking. Output, investment and employment increase sharply on impact but then immediately return to close to their initial levels. That is, there is no delayed hump shaped response that is a key features of VAR responses to a monetary shock. Consumption does have a slightly hump shaped response, but the 'hump' is very small. Inflation moves very little and in the
opposite direction of the data; i.e. the model does not generate the price puzzle on impact. For the Federal funds rate, the impact response is by construction of the shock that hits the model consistent with the VAR evidence. But the Federal funds rate then immediately reverts into positive territory. As a result, the slope responds too much on impact, before also reversing sign after one period.

Lesson 4 then is that while we can make progress in matching the dynamic responses of real aggregates, inflation and interest rates to a TFP news shock, it comes at the cost of severely deteriorating the fit of the model to the monetary policy shock. A key reason for this failure is the absence of investment adjustment cost and the highly variable capital utilization.

6 Robustness: An Alternative Model of the Slope

One possible issue with our results is that we impose the expectations hypothesis of the term structure, thereby ignoring time varying term premia which are known to be important in the literature. To address this issue we add an asset pricing block to the model that consists of a no-arbitrage affine pricing model as proposed, for example, by Ang and Piazzi (2003) and adapted to a DSGE context by in Hordahl et al. (2007). To start, notice that the Rational Expectations equilibrium of the linearized New Keynesian DSGE model can be expressed as linear state-space system

\begin{align}
Y_t &= \phi_Y + \Phi_Y S_t \\
S_t &= \phi_S + \Phi_S S_{t-1} + G\varepsilon_t.
\end{align}

where the \(n_y \times 1\) vector \(Y_t\) contains the endogenous variables; the \(n_s \times 1\) vector \(S_t\) contains the states; and the \(n_{\text{shock}} \times 1\) vector \(\varepsilon_t\) contains the \(i.i.d.\) innovations to the exogenous shocks that we assume multivariate normal \((0, I)\).\footnote{There are \(n_k < n_s\) endogenous states (i.e. predetermined endogenous variables), which are ordered first in \(S_t\). Hence, \(G\) is a \(n_s \times n_{\text{shock}}\) matrix with zeros in the upper \(n_k \times n_{\text{shock}}\) block and a matrix with the exogenous shocks’ standard deviations in the appropriate places of the lower \((n_s - n_k) \times n_{\text{shock}}\) block.}

The short rate (i.e. the Federal Funds rate in our model) is part of the macro system and therefore included in the linear state-space system; i.e.

\[R_t = \delta_0 + \delta'_1 S_t,\]

where \(\delta_0\) and \(\delta'_1\) contain the appropriate elements of \(\phi_Y\) and \(\Phi_Y\), respectively. The nominal
yield on a $T$-period discount bond is defined as

$$R_t^T = -\frac{\log P_t^T}{T},$$

(22)

where $P_t^T$ is the period-$t$ price of the bond with $P_t^0 = 1$. Under no arbitrage, this price satisfies

$$E_t \left[ M_{t+1}^T P_{t+1}^{T-1} \right] = P_t^T,$$

(23)

where $M_{t+1}^T$ is the nominal pricing kernel. Following Ang and Piazzesi (2003) and many others in the latent factor no-arbitrage literature, we assume that the logarithm of this pricing kernel is described by

$$\log M_{t+1}^T = -R_t - \frac{1}{2} \Lambda'_t \Lambda_t - \Lambda'_t \varepsilon_{t+1},$$

(24)

where the $n_{\text{shock}} \times 1$ vector $\Lambda_t$ denotes the market price of risk associated with the different shocks in $\varepsilon_t$. Similar to Hordahl et al. (2007), these risk factor are assumed to follow an affine process in the states

$$\Lambda_t = \Lambda_0 + \Lambda_1 S_t,$$

(25)

with the $k_x \times 1$ vector $\Lambda_0$ defining average risk; and the $n_{\text{shock}} \times n_s$ matrix $\Lambda_1$ defining how risk varies depending on the state of the economy. Given (19)-(25), bond prices can be computed recursively as linear functions of $S_t$ that can be decomposed into fluctuations due to expected future short rates (i.e. the Expectations Hypothesis) and time variations in term premia (see the appendix for details).

As shown by Wu (1996) and Bekaert, Cho and Moreno (2010), the pricing kernel implied by linearized DSGE models with homoscedastic innovations represent a special case of the formulation in (24) with $\Lambda_0$ a function of the different structural parameters of the DSGE model and $\Lambda_1 = 0$. In other words, the linearized DSGE model implies that risk and therefore term premia are constant. To allow for time-variation in term-premia we let elements of $\Lambda_1$ be non-zero. This potentially involves estimating the entire $n_{\text{shock}} \times n_s$ matrix $\Lambda_1$, which is large for our DSGE model. To make the estimation manageable, we impose two restrictions. The first restriction is that we let risk vary only with respect to two macro variables: expected inflation $E_t \pi_{t+1}$ and expected consumption growth $E_t \Delta c_{t+1}$. This allows us to express risk associated with the news shock as

$$\Lambda_t^{\text{news}} = \Lambda_{0,\text{news}} + \Lambda_{1,\pi}^{\text{news}} E_t \pi_{t+1} + \Lambda_{1,\Delta c}^{\text{news}} E_t \Delta c_{t+1},$$

(26)
where $\Lambda_{1,\pi}^{news}$ and $\Lambda_{1,\Delta c}^{news}$ tell us how the price of risk with respect to the news shock reacts to changes in expected inflation and expected consumption growth, respectively. Both of these variables can be constructed as linear functions of the state vector $S_t$ using the state-space solution of our DSGE model in (19)-(20). The second restriction follows naturally from our limited information estimator in that the only exogenous shock we consider is the news shock. As the appendix shows in detail, the two restrictions together imply that $\Lambda_{1,\pi}^{news}$ and $\Lambda_{1,\Delta c}^{news}$ are the only additional free parameters to estimate. This imposes considerable discipline on our estimation (by comparison Ang and Piazzesi, 2003 estimate a total of 13 different coefficients in their formulation of $\Lambda_t$).

There are two important features of our modelling of risk. First, our formulation of time-varying risk can be motivated by the consumption-based asset pricing literature, which says that changes in the conditional covariances between inflation and consumption growth are an important driver of term premia variations (e.g. Piazzesi and Schneider, 2006). Generating sufficiently large time-variations in term premia by deriving $M_{t+1}^S$ explicitly from preferences in the context of a non-linear DSGE model has proven to be very challenging. Our formulation should therefore be considered as a basic test of whether variations in risk as a linear function of two macro variables are capable of generating quantitatively large term premia fluctuations. Second, the macro dynamics of our model as described by the state-space system in (19)-(20) are independent of time-variation in risk. Since risk depends on the macro states, however, the joint estimation of both macro and term structure dynamics imposes discipline on the parameters of the macro model.

As a final estimation, we add the impulse responses of the observed spread and long rate to the objective and reestimate the entire model including the two risk parameter. The last column of Table 2 reports estimation results and Figure 6 shows the fit of the model relative

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12See Rudebusch and Swanson (2009, 2011) or Binsbergen et al. (2010) for recent attempts.
to the VAR evidence.

Figure 6: Impulse responses to a TFP news shock of VAR (black solid lines and grey 68% confidence intervals) and of tight prior estimation of augmented Smets-Wouters model with affine bond pricing (dotted red lines).

As is apparent from these results, the parameter estimates of the loglinearized model barely change and therefore, all of the above lessons remain intact. The posterior means for the two risk parameters are large and significantly different from zero, generating sizable although insufficient variations in term premia on impact of the news shock.
7 Conclusion

To be added
References


Table 2: Prior distribution of estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Distribution</th>
<th>Loose prior</th>
<th>Tight prior</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. dev.</td>
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<td>Inverse labor supply elasticity</td>
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<tr>
<td>( \omega_w )</td>
<td>Degree of wage indexation</td>
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<td>( \theta_{\Delta y^*_{gap}} )</td>
<td>Output gap growth response</td>
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<td>Normal</td>
<td>0.00</td>
<td>200</td>
</tr>
<tr>
<td>( \lambda_1,_{\Delta y} )</td>
<td>Risk loading on expected consumption growth</td>
<td>Normal</td>
<td>0.00</td>
<td>200</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Smets-Wouters parametrization</td>
<td>Loose prior estimates</td>
<td>Tight prior estimates</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>-------------------------------</td>
<td>-----------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Short-term wealth effect</td>
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<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse labor supply elasticity</td>
<td>1.83</td>
<td>[0.15,2.70]</td>
<td>1.59</td>
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<tr>
<td>$\kappa_p$</td>
<td>Probability of price non-adjustment</td>
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<tr>
<td>$\omega_p$</td>
<td>Degree of price indexation</td>
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<td>0.20</td>
<td>0.69</td>
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<tr>
<td>$\xi_w$</td>
<td>Probability of wage non-adjustment</td>
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<td>0.97</td>
<td>0.76</td>
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<td>Degree of wage indexation</td>
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<td>0.84</td>
<td>0.95</td>
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<tr>
<td>$b$</td>
<td>Habit persistence</td>
<td>0.71</td>
<td>0.65</td>
<td>0.55</td>
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<tr>
<td>$\sigma_u$</td>
<td>Capital utilization parameter</td>
<td>0.54</td>
<td>0.08</td>
<td>0.07</td>
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<td>$S^*$</td>
<td>Investment adjustment cost</td>
<td>5.48</td>
<td>[0.04,0.12]</td>
<td>[0.03,0.10]</td>
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<tr>
<td>$\rho_R$</td>
<td>Persistence of interest rate rule</td>
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<td>0.45</td>
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<tr>
<td>$\theta_\pi$</td>
<td>Inflation response</td>
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<td>2.03</td>
<td>1.11</td>
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<tr>
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<td>Output gap response</td>
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<td>0.11</td>
<td>0.39</td>
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<tr>
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<tr>
<td>$\lambda_{1,\Delta w_c}$</td>
<td>Risk loading on expected consumption growth</td>
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Notes: Estimates refer to posterior means and 5%-95% bounds in brackets. The posterior distribution is obtained using the Metropolis-Hastings algorithm.
8 Appendix A: Expectations Hypothesis in the VAR

Our VAR framework allows us to decompose the reaction of the long rate into variations due to term premia and expectations about future short rates (i.e. the Expectations Hypothesis). This decomposition is useful because the linearized DSGE model can only match the EH part of the news IRF. We can decompose the yield on a $T$-period yield $r^T_t$ (in our case the 60-month yield) as

$$R^T_t = \frac{1}{T} \sum_{i=0}^{T-1} E[R_{t+i}/I_t] + t_{p_t},$$

where the $E[R_{t+i}/I_t]$ denote expectations of future short rates as implied by the VAR based on information $I_t$; and $t_{p_t}$ denotes term premia. This type of decomposition has been used widely in the term structure literature. Notable examples are Cambpell and Shiller (1991) or more recently Diebold, Rudebusch and Aruoba (2006) and Evans and Marshall (2007).

The following figure shows the decomposition of the slope and long rate response into Expectations Hypothesis part and term premia part.

While term premia react significantly to TFP news shocks, these variations make up less than half of the total response of the long rate and the spread; i.e. the Expectations Hypothesis accounts for a large part of the response of the term structure to news shocks. This implies that a linearized DSGE model with homoscedastic innovations, which by definition makes term premia constant, should at least in principle be able to capture a large part of movements in the slope and the long end of the term structure to TFP news shocks. Time-variations in term premia remain of course important to analyze and we do so at the end of the paper.