ABSTRACT.

This paper examines some of the economic and econometric issues that arise in attempting to measure the degree of concentration in an industry and its dynamic evolution. We offer new measures of concentration over aggregated periods of time and provide a sound statistical basis for inferences. Concentration is one aspect of the problem of measuring “market power” within an industry. Modern economic analysis of antitrust issues does not focus only on the level of concentration, but still must examine the issue carefully. We contrast concentration at a point in time with a dynamic profile of change in the distribution of shares in a given market. Our methods are demonstrated with an application to the US steel industry.

Keywords: Industrial Concentration, antitrust, market power, mobility, Steel, tests, dynamic profiles.

1 Introduction

The analysis of antitrust claims in government and privately brought cases is a complex matter. Antitrust issues are first and foremost issues of whether or not a firm exercises market power. One important component of the market power puzzle is the degree of concentration within an industry. The purpose of this paper is to introduce some new methods to quantify and describe the level of concentration within an industry. We pay particular attention to how one may develop concentration profiles that reveal (sometimes) fast changing market structures. Applied economists have understood for some time that measuring market power is not a one dimensional issue. Even
before the advent of the so-called “New Empirical Industrial Organization” (henceforth NEIO) in the early 1980s, economists understood well that one could not simply look at any one aspect of industrial behavior in a given industry, and draw definite conclusions about the level of competitiveness therein. Bresnahan(1989) refers to the traditional empirical approach to analyzing competitiveness in a given market as the Structure, Conduct and Performance Paradigm (or the SCPP). That is, the traditional approach has been to define the relevant product and geographic markets, and then to examine structure by looking at the degree of concentration in the market and to look at pricing behavior to see if the rm(s) is (are) indeed pricing their outputs close to marginal cost. The implicit assumption in this body of work is that marginal cost is observable and measurable and that a reduced form analysis of structure and performance on cross-section data is sufficient, cf. Appelbaum (1979,1982) for an initial critique of this approach. Under this approach, the Lerner(1934) index has been a well known benchmark test of market power. The index takes the form

\[ L = \left[ P - MC \right] = \frac{P}{1+\varepsilon}; \quad (1) \]

where \( \varepsilon \) is the elasticity of demand. Following Blair and Esquibel (1996), this well known formula is obtained by noting the relation between marginal cost \( MC \), and price, as,

\[ MC = P(1 - \frac{1}{\varepsilon}); \quad (2) \]

The implicit assumption here is that the monopolist would select output where \( MR = MC \), then (2) follows easily. Finally, under a Cournot equilibrium this formula can be recast as,

\[ \left[ P - MC \right] = P = S_i \varepsilon; \quad (3) \]

Here, \( S_i \) is the ith rm’s market share and \( \varepsilon \) is the absolute value of the elasticity of demand, cf. Church and Ware (2000, p. 239). Equation (3) shows that under the SCP strategy, knowledge of market share is an important element in ascertaining the degree of market power within a given market. In fact, the same authors show that,

\[ HHI = P \sum_{i=1}^{n} \left( \frac{P - MC}{P} \right); \quad (4) \]
where HHI is the Hirfandahl-Hirschman Index (see below). Thus, as commonsense would suggest, the level of concentration in the industry can be an important component of quantifying market power in the SCP framework if the Lerner Index is the benchmark.

The NEIO approach emphasizes the fact that in general, economic marginal cost is not observable. In addition, each industry has its own nuances which distinguish it from others and a “conduct parameter” is an unknown to be estimated, not assumed in a cross-section model, cf. Bresnahan (1989, pp. 952-953). Bresnahan further notes that the NEIO approach focuses on the use of an econometric model for an individual industry, not on its reduced form and using data over time. The NEIO approach has been applied in practice in several applications. Ellison (1994) builds on work by Porter (1983) to show that demand for a given product (they focus on railroads) can be assumed to be log-linear in price and takes the form,

\[
\log(Q) = a_0 + a_1 \log(P) + a_2 \log(L);
\] (5)

in this model \(a_1\) is the elasticity of own price demand, . The supply relationship can take the form,

\[
P(1 + S_i w_i = a_1) = MC;
\] (6)

where \(S_i\) is the market share of the \(i\)...rm and \(w_i\) is a “conduct” parameter. As in Church and Ware (2000, p. 441), let

\[
\mu = S_i w_i;
\] (7)

Then,

\[
P(1 + \mu = a_1) = MC;
\] (8)

Thus, within the NEIO framework, the elasticity of demand for the product and a ...rm’s market share are important determinants of its conduct and, potentially, market power within that industry. To summarize, under the SCP framework, market power can be quantified as,

\[
L = \mu^{HHI};
\] (9)
while under the NEIO framework, market power is formulated as,

\[ L = \mu^{HHI}; \tag{10} \]

where as before, \( \mu \) is the conduct parameter.

In a different application, Maasoumi et al. (2002) identify several indicia of competition in analyzing market power, as follows:

(i) relative price performance
(ii) pricing behavior
(iii) productivity
(iv) demand elasticities
(v) contestability
(vi) elasticity of supply
(vii) financial performance

It may be verified that all of their criteria can be impacted by the degree of concentration in the market under scrutiny. Taylor and Zona (1997) list similar criteria and add quality and advertising. Regardless of one’s beliefs about how to econometrically ascertain market power, a firm’s market share, and ultimately the degree of concentration in an industry, are important considerations in the determination of market power in that industry. Modern antitrust economists will likely agree that concentration should not be the only factor looked at in determining the level of competitiveness in a market. But to ignore concentration as one important dimension of market power is simply bad science.

We introduce a more rigorous methodology to quantify the degree and dynamics of concentration in a given industry for a relevant product market. Section two below reviews the literature on measuring concentration in a market. Here we provide an axiomatic characterization of concentration measures. Section three presents new concepts and methods, especially for dynamic analysis of aggregated market shares. Section four presents a case study of the steel industry and section five concludes.
2 Measuring Concentration

Since the pioneering work of Bain (1951, 1956) economists have examined the structure of an industry by exploring the share of the market that individual firms in that industry possessed. The most common measure has been a simple concentration index. The measure looks at the share of sales (or share of output or share of industry employees) of the top 2, 4 or 6 firms. The most common component has been the Herfindahl-Hirschman index. The HHI, alluded to earlier, may be defined as follows:

Let there be \( N \) firms in an industry or a relevant market with shares \( S_i; 1 = 1; 2; \ldots; N \): Then,

\[
\text{HHI} = \sum_{i=1}^{N} S_i^2; \tag{11}
\]

The HHI has been used widely, and as a guideline by the Department of Justice in assessing whether mergers and acquisitions “significantly” increase HHI and constitute a threat to competition in markets. Since in many applications one practically knows the whole population of firms, sampling variability is often neglected. For instance, the DOJ guidelines provide actual amounts of increase (100 points, say) in the HHI which would raise red flags. No statistical significance is reported. This may be problematic when historical analysis is conducted of a market that has experienced change, especially with changing numbers of participants and change in market concentration/conditions. For instance, there are markets, such as steel, that have evolved from being relatively highly concentrated into being competitive, mostly due to entry by new importers/exporters and technological change. The relevant market here will have to be the international market with increasingly many smaller operators. This would raise sampling variability questions which need to be attended to in looking at “statistically significant” historical changes.

There are many other measures of concentration that can compete with the HHI, and may be more suited for detailed analyses and may possess superior properties. For instance, some merger or entry events may only change certain parts of the distribution, the tails or the centre. Different indices of concentration put different weights over different parts of the distribution and may give completely contradictory evidence. This lack of robustness may be dealt with in two ways. One is to report a larger class of concentration measures which reflect this sensitivity to concentration
in different parts of the share distribution. We do this below by proposing a class of Generalized Entropy indices of concentration and their statistical distributions. Another approach is to avoid this kind of index based, "complete rankings" and consider "partial but uniform" orderings that evaluate concentration over large classes of evaluative functions.

Considerable advances have been made in greater understanding of measures of "equality" or concentration in related fields, such as income/wealth inequality. It appears that these developments have not had much of an impact in the area of industrial organization and market analysis. In particular, it is understood that variance-type indexes, like HHI, have serious and unnecessary limitations which can hamper a proper understanding of a market. These limitations arise from several considerations. One is the limitation of "variance" as a full measure of flatness/dispersion in a distribution. Radically different distributions may have the same "variance", or HHI. This is particularly true of distributions with very different tail properties. Note that it is the tails of distributions which may be the target of policy, and subject to change. Yet many indices of "dispersion", like variance, may miss all such changes. GINI is another example of a concentration index which is relatively oblivious to changes in the tails of distributions.

Indeed, it is useful for policy makers and analysts to declare the "weights" they attach to a reduction in concentration over various parts of a distribution. For instance, do we care more about reducing the relative size between the largest firms and small firms, or within groups of large (small) firms, or between/within regions? The reason our answers to these questions should be more explicit is that whatever index is used, it will implicitly and inadvertently decide the answer to these questions. So one may unknowingly subscribe to weight systems that would not be desirable when made explicit, or would not accord with policy goals expressed in different terms.

Other problems may arise from the limited ability of some measures of concentration to adequately or appropriately decompose and dissect the degree of concentration as measured by HHI into economically interesting firm groups or regions, say. This too is important for analysis of specific markets and sub-markets.

Below we highlight some of these shortcomings and offer an analysis of alternative measures which may be used in empirical analysis.
Note that by definition $1/N < \text{HHI} < 1$, where $N$ is the number of firms in an industry (HHI is often reported multiplied by 100 or 1000, however). The maximum concentration occurs when one firm has all the sales, output, etc. and that the measure takes a value of 10,000. Minimum occurs when each firm has an equal share of $1/N$. It is also implicit that the larger the number of firms, $N$, the less concentration is indicated by HHI. Thus entry of substantially equal sized firms, all else being equal, benefits competition by reducing HHI. This is desirable. Also, it is known that the more equal the firm sizes the smaller is the HHI. This too is regarded as procompetitive. HHI also satisfies an important and desirable "transfers" property to be discussed below. Nevertheless, it is not an "ideal" index of industrial concentration. We develop this concept in the next section.

2.1 Alternative measures of concentration: An axiomatic approach.

It would be helpful to think in terms of a "Market Objective Function" (MOF), much like a welfare function, explicitly representing our preferences with regard to the distribution of sales or employment. This will allow us to discuss the relationship between MOFs and any index of concentration. We should then be better able to assess the latter’s desirability.

Consider MOFs which are increasing in sales (outputs, assets, etc) and are averse to concentration. Let $x$ be the sales variable, $\bar{x}$ its mean, and $x_e$ the "equal equivalent sale"; i.e. the level of sales which if given to every firm would leave MOF at the same level as for a given distribution of sales. It is clear that $x_e < \bar{x}$ so long as there is any concentration, and a measure of divergence between these two would indicate the degree of "loss" in MOF due to concentration. Following Atkinson (1970) and Kolm (1969), it can be argued that,

$$I(x) = 1 - \frac{x_e}{\bar{x}};$$

may be a good measure of "relative" inequality.

The MOF approach does not identify a unique concentration index, per se, even when a particular MOF is agreed upon. But an axiomatic derivation technique that identifies GE, for example, can be used and is instructive. By borrowing from functional analysis first developed in "information theory", see Maasoumi (1993), one is forced to put down an explicit set of properties (axioms)
which may be desirable from the point of view of MOFs. Using these axioms as explicit constraints on the function space one then obtains the appropriate concentration index. Bourguignon (1979) and Shorrocks (1980, 1984) consider the “fundamental welfare axioms” of symmetry, continuity, Principle of Transfers, and additive decomposability. These will identify GE as the desirable scale invariant family of concentration measures.

Axiom 1. The concentration index (function) is symmetric in sales/outputs.

This is equivalent to anonymity which requires that only sales (outputs) matters not the identity of the ..rms.

Axiom 2. Principle of transfers.

This requires that concentration decrease if we redistribute sales/output from a single larger ..rm to a smaller one, leaving their respective rankings and all the other ..rms’ positions unchanged.

Axiom 3. Continuity.

This requirement is relatively harmless, helping in the mathematical derivations and in comparing different population of ..rms. Note that this allows for the possibly non-practical zero concentration.

Axiom 4. Invariance to scalar multiplication.

This is a serious limitation as it restricts attention to “relative” concentration. It implies mean invariance; doubling every ..rm’s output would leave concentration unchanged. It is satisfied by HHI. Questions of “efficiency”, such as the overall size of the market, growth, etc, are not accommodated by HHI and other relative measures. “Absolute”, and “mean absolute” concentration measures can do so but have their own limitations. It may be that the policy maker is relatively less concerned with concentration in a fast growing market stage. This has been argued by the proponents of ..rms in the technology area in recent years. Then more concentration is tolerated when total sales/output is increasing than otherwise. Such complex preferences are better handled by rankings obtained from Generalized Lorenz and Stochastic Dominance. For a discussion of these issues see Maasoumi (1998) where Stochastic Dominance and other ranking techniques are discussed.

The class of indices satisfying Axioms 1-4 is very large. Also, any further axioms are less likely to
command consensus. In fact, any further requirements must be justified by plausible considerations of such things as policy, empirical necessity, and practical interest. One such requirement is:

Axiom 5. Additive decomposability (aggregation consistency).

This requirement says that total concentration must be the sum of a "between group" component, obtained over group means, and an additive component which is a weighted sum of "within group" concentration values. This kind of decomposability is very useful for controlling and dealing with heterogeneity of firms, regions, market areas, and as a means of unambiguously identifying the areas of concentration and those firms that are involved.

In their various incarnations, Axioms 1-5 together identify the GE as the "ideal" family of indices. For a weighted random vector \( X = (X_1; \ldots; X_n)^0 \) with weights \( w = (w_1; \ldots; w_n)^0 \) the GE concentration measure is defined as

\[
I_{\alpha} (X) = \frac{1}{\alpha(\alpha+1)n} \sum_{i=1}^{n} \frac{w_i}{\bar{w}} \left( \frac{X_i}{\bar{X}} \right)^{\alpha+1} 1_{\alpha} \geq 0; 1; \]

(13)

where \( \bar{X} = \frac{1}{n} \sum_i w_i X_i \) and \( \bar{w} = \frac{1}{n} \sum_i w_i = n \). Usually (as in our application) the weights are the reciprocal of inclusion probabilities. This family includes Herfindahl's, variance of logarithms, square of the coefficient of variation, and for \( \alpha = 0; 1 \) this index converges to the first and second Theil measures of information, respectively:

In the field of inequality measurement collections of axioms have been identified which "justify" inequality indices. One of the most popular and enduring inequality indices is the Gini ratio given by:

\[
G = (2 = x) \int_{0}^{1} x[F; 1 = 2] dF; \]

(14)

Using the above formulae, the MOF corresponding to each measure is readily identified. The MOF for Gini depends on \( F \), the distribution of outputs/sales, and hence involves inter-firm comparison of "welfare". Such comparisons may be inevitable, for strategic or other reasons, especially in developing socially acceptable lower bounds for concentration in specific markets; zero concentration is merely a mathematical point of departure that may be undesirable.
If the useful additive decomposability requirement of axiom 5 is imposed, such measures as Gini
and variance of logarithms are disqualified. The latter two measures provide ambiguous decom-
positions of overall inequality by population subgroups; in the context of inequality see Shorrocks
(1984). It has been shown that Theil’s second measure (gamma = -1) provides the most unam-
biguous answer to such fundamental questions as: How much of the overall concentration is due
to the concentration within the r-th group? For example, type of product, technology, location,
size itself, may characterize groups. Having a good idea about the incidence of concentration is an
essential pre-requisite for devising well-directed and appropriate remedial action. It is also essen-
tial in establishing lower bounds for concentration that reflect acceptable differences on the basis
of appropriate market conditions. Such additive decomposability and “aggregation consistency”
criterion, requiring that concentration increases if one or more Ir increase (I(b) constant), are vi-
olated by Gini and HHI! We shall see further supporting arguments in favor of requiring additive
decomposability in empirically relevant applications.

Theil’s two measures were further studied in Maasoumi and Theil (1979) and Maasoumi (1989b)
with a view to determine their characteristics in terms of the moments of distributions. Nagar-type
approximations were developed around the lognormal distribution in much the same way as Nagar
and Edgeworth expansions are developed in econometrics around the normal distribution. This
development is helpful in comparing them with HHI, and may be considered when estimation and
inference issues are presented below. Briefly, assuming the existence of the rst four moments, the
following small-$\frac{\gamma}{4}$ expansions were given by Maasoumi and Theil (1979). Let log $x = z$, $E z = \mu$,
(var $z = \frac{\gamma}{4}$, $^3 \mu = E (z ; -1)^3 = \frac{\gamma}{3}$) as the skewness, and $^4 \mu = E (z ; -1)^4 = \frac{\gamma^2}{4}$ as the kurtosis of the
log output distribution. Assuming the existence of the rst four moments, they obtained:

$$I_0 = -\frac{1}{2} \frac{\gamma}{4} \{1 + \frac{3}{3} \frac{\gamma}{4} \mu_1 + \frac{1}{3} \frac{\gamma}{4} \mu_2 + o(\frac{\gamma}{4})\};$$

(15)

$$I_1 = -\frac{1}{2} \frac{\gamma}{4} \{1 + \frac{3}{3} \frac{\gamma}{4} \mu_1 + \frac{1}{3} \frac{\gamma}{4} \mu_2 + o(\frac{\gamma}{4})\};$$

(16)

Note that, when $z$ has a lognormal distribution, both indices equal $-\frac{1}{2} \frac{\gamma}{4}$. HHI can be shown to
be a simple function of $\frac{\gamma}{4}$, but not of the higher moments. Thus it can fail with departures from
lognormality. Interestingly, modern diffusion theories in the area of output growth, tend to point
to a lognormal distribution for output. See Rutherford (1951, Econometrica), Fatemi (2000) and her references. But these theories seem to be influenced by mathematical expediency. The above approximate formulae can be used when the underlying distribution is not known. They allow us to see that positive skewness and leptokurtosis increase concentration, and that $I_0$ is more sensitive to positive skewness (high sales/output groups) and fat tails (large extreme sales groups) than $I_{-1}$.

These entropy measures appear more soundly justified and useful. It is relatively easy to implement them. Not surprisingly, Theil and others have previously suggested the idea of using entropy as a measure of concentration. As shown in Ebrahimi et al (1999), entropy is a much richer function of all the moments of a distribution, and more closely identifies it than any single moment such as variance or HHI. For a policy maker and in litigation, it would be a matter of great concern to ignore major differences between markets, or a market over time, simply by using HHI or other variance measures.

It is misleading to consider states of a market at only single points in time. Transitory conditions may mislead and become difficult to disentangle when looking at several periods/situations. As we noted in our discussion of the NEIO above, it is desirable to consider market concentration over several periods, and to develop a dynamic concentration profile that tells us how it has evolved, and, importantly in the area of market power analysis, whether it is stable.

This too can be done here inspired by Maasoumi’s own work in the area of income mobility, see Maasoumi and Zandvakili (1986, 1991), and Maasoumi and Trede (2001).

3 Generalized Entropy Concentration Profiles

It is misleading to consider states of a market at only single points in time. Transitory conditions may mislead and become difficult to disentangle when looking at several periods/situations. As we noted in our discussion of the NEIO above, it is desirable to consider market concentration over several periods, and to develop a dynamic concentration profile that tells us how it has evolved, and, importantly in the area of market power analysis, whether it is stable.

This too can be done following the concepts of mobility as in Maasoumi and Zandvakili (1986,
Let \( X_{it} \) denote sales/output of rm \( i \) in period \( t \); and \( X_t = (X_{1t}; X_{2t}; \ldots; X_{Nt})^0 \) in period \( t = 1; \ldots; T \). Let \( S_i = S_i(X_{i1}; \ldots; X_{iT}) \) be the ‘permanent’ or “aggregate” sales of rm \( i \) over \( T \) periods. Of course, one can define the aggregates over periods 1 to \( T^0 \) and develop a mobility profile as \( T^0 \) approaches \( T \). Then

\[
S = (S_1; \ldots; S_n)^0
\]

is the vector of aggregate sales for a chosen time frame. Following Maasoumi (1986) the following type of aggregation functions are justified on the basis that they minimize the generalized entropy distance between \( S \) and all of the \( T \) “sales” distributions:

\[
S_i = \frac{\hat{A}_t X_t}{\sum_{t=1}^{T} \hat{A}_t X_t} \Bigg|_{i=1} \quad \text{; } \hat{A}_t \in \mathbb{R}^0; i \in 1
\]

\[
Y_t = \frac{X_t \hat{A}_t}{\sum_{t=1}^{T} X_t \hat{A}_t} \Bigg|_{t=1} \quad \text{; } \hat{A}_t \in \mathbb{R}^0; i \in 1
\]

where \( \hat{A}_t \) are the weights attached to sales in period \( t \), \( \sum_{t=1}^{T} \hat{A}_t = 1 \). The elasticity of substitution of sales/output across time is constant at \( \gamma = 1 + \lambda \). The case \( \lambda = 1 \) corresponds to perfect inter-temporal substitution which subsumes Shorrocks’ analysis for certain weights. This case is also the most common formulation of the “permanent income” concept in economics and used by Burkhauser and Poupore (1997). In this context we can think of the concept of “permanent output”, or “expected sales”.

Mobility is measured as the ratio of “long run” concentration occurring when the period of examination is extended, and a measure of short run concentration. The latter may be represented by any one period of interest, or a weighted average of the single period concentrations. We might think of this as a notion of “competition enhancing” mobility, a welfare theoretic base in favor
of large, non-concentrated markets\textsuperscript{1}. The extension of the time interval is meant to reflect the
dynamics and smooth out the transitory or business cycle effects in the industry. Shorrocks (1978) proposed the following mobility measures:

\[ M = 1 \prod_{t=1}^{T} \frac{I(S)}{I(X_t)} \]

where \( I(\phi) \) is the “inequality” measure. For convex inequality measures \( I(\cdot) \), \( 0 < M < 1 \) is easily verified when \( S \) is the linear “permanent output” function. For other aggregator functions see Maasoumi and Zandvakili (1990). A priori, there would be no reason for an analyst to give unequal weights to different years under study. Nevertheless, Shorrocks (1978) suggests the ratio of year \( t \) income to total income over the \( T \) periods as suitable values for \( \gamma \)'s. We consider both weighting schemes here\textsuperscript{2}.

3.1 The asymptotic distribution

Cowell (1989) noted that the GE measures of inequality are functions of the moments of the income variable. Similarly, we note that the GE mobility measures can be written as functions of moments of (transformed) income variables. Let \( ^{\gamma}(Y) = \int_{0}^{y} y dF(y) \) for any variable \( Y \). Then, for \( \phi \in (0; 1) \)

\[ M_{\gamma} = M_{\gamma}(^{\gamma}(X_1); \ldots; ^{\gamma}(X_T); ^{\gamma}(S); ^{\gamma}(X_{T+1}); \ldots; ^{\gamma}(X_{T+\gamma}); ^{\gamma}(S_{\gamma+1})) \]

\[ = 1 \prod_{t=1}^{T} \frac{^{\gamma}(X_{t+1})}{^{\gamma}(X_t)} \frac{^{\gamma}(S+1)}{^{\gamma}(S)} \]

\textsuperscript{1}This is particularly so in the case of econometric studies of “wage dispersion” in which statistical causes of dispersion are usefully identified, but welfare-theoretic motivation is lacking in regards to “dispersion” as a measure of “mobility”, or inequality.

\textsuperscript{2}In other work with PSID data Maasoumi and Zandvakili (1986, 1990) have studied different weights, including Principal Component weights and unequal subjective weights. They...and these weights are inconsequential for the qualitative inferences and rankings.
and

\[ M_0 = M_0(1(X_1); \ldots; 1(X_T); 1(S); 1(X_1 \ln(X_1)); \ldots; 1(X_T \ln(X_T)); 1(S \ln(S))) \]

\[ = 1_i \sum_{t=1}^T \frac{\frac{1}{i}(S \ln(S)) \cdot \ln^{(1)}(X_t)}{i(X_t)} \cdot \ln^{(1)}(X_t) \]

\[ M_{i1} = M_{i1}(1(X_1); \ldots; 1(X_T); 1(S); 1(\ln(X_1)); \ldots; 1(\ln(X_T)); 1(\ln(S))) \]

\[ = 1_i \sum_{t=1}^T \frac{\ln^{(1)}(S) \cdot \ln^{(1)}(X_t)}{i(\ln(X_t))} \cdot \ln^{(1)}(X_t) \]

Let \( \hat{\eta} \) be a vector of moments and \( M = M(\hat{\eta}) \) a differentiable function. Then, for general processes the following asymptotic distribution is well known for \( \hat{\eta} \), a (weighted) estimator of \( \eta \), e.g., see Amemiya's (1985):

\[ s \frac{N}{1 + \frac{v_w^2}{w}} (\hat{\eta} - \eta) \overset{\text{d}}{\sim} N(0; \hat{\Sigma}) \]

with \( \hat{\Sigma} = \text{Cov}(\hat{\eta}) \) and \( v_w^2 \) the coefficient of variation of the weights \( w \). By an application of the Delta method it follows that:

\[ \frac{p}{N} M(\hat{\eta}) \left( 1 + \frac{v_w^2}{w} \right)^{\frac{1}{2}} \mu^{\frac{1}{2}} \overset{\text{d}}{\sim} N(0; 1) \]

where

\[ \frac{1}{2} \mu^{\frac{1}{2}} = \frac{\mu}{\hat{\Sigma}} \frac{\hat{\Sigma}}{\hat{\Sigma}} \]

with \( \frac{\hat{\Sigma}}{\hat{\Sigma}} \) denoting the derivative vector. Of course, \( \hat{\Sigma} \) is usually unknown and has to be substituted by a suitable estimator \( \hat{\Sigma} \); also, the gradient will have to be evaluated at \( \hat{\eta} \). However, this does not alter the asymptotic distribution. See, for example, Amemiya (1985). The partial derivatives and the asymptotic covariance matrices mentioned above are derived in Maasoumi and Trede (2001). Bootstrap methods are attractive alternatives here.
4 The U.S. Steel Industry: A Case Study

Maasoumi, Prowse and Slottje (2002) have recently analyzed the U.S. steel industry with respect to its competitive behavior. The industry is interesting because it has clearly evolved from one that was heavily concentrated, to one that is now quite competitive.

Consider the HHI values for a recent ten year span in the steel industry:

$$HHI = [0.13296695, 0.11994628, 0.12208988, 0.12133361, 0.14649663, 0.14370595, 0.15858994, 0.16174695, 0.20259183, 0.14396202].$$

This measure shows an initial steady but modest annual rise to a peak of .20, and a subsequent decline to the average value of .1439. These values are all modest by the DOJ standards, but there is sufficient annual variability in them to raise the question we have raised in this paper: At which point in time is the level of concentration representative? Clearly, this is a rapidly changing industry, much like the modern “high tech” sectors. Nor does HHI offer a robust picture of these changes relative to different weights attributed to different sized firms in the industry. GE measures, and the proposed M measure profiles are suited to this application.

Below we offer some of the representative results for the US steel industry based on the M-concentration profiles. In Table 1 we report “inequality”/concentration in each year, for several members of the GE family ($\alpha = -1, 0$), and the concentration levels for 2-10 years of aggregated sales based on several aggregation functions ($S_{\bar{\cdot}}; \sim = -1, 0$). A notable quality of these values is that they are accompanied by standard errors that afford the reader an approximate idea of asymptotic significance of the values and the differences across years. Based on these cases, and five other combinations of $\alpha$ and $\bar{\cdot}$ values, several representative concentration profiles are depicted in Figures 1-9.
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Figure 1:
These graphs depict how $M$ rises from low values for one and two-year “aggregations”, to higher, sometimes very close to the maximum value of 1, as the number of aggregated years approaches 10. There is clear evidence of significant “mobility” and dispersion in this industry over this period.

5 Conclusions

We have proposed and demonstrated new methods for measuring concentration, for analysing concentration profiles as the period of analysis is expanded, and provided a case study where the
Figure 3:
statistical significance of our inferences is also reported. Our analysis was based on indices that had been developed elsewhere by Shorrocks and Maasoumi and Zandvakili. These offer "complete" rankings of concentration situations in a given market/industry. The methods are especially suited for fast changing and volatile "hitech" industries.

There is a related and growing interest in "uniform" partial ordering of distributed outcomes, welfare states (income distributions, poverty levels), and in program evaluation. "Complete" ordering is done when one employs specific indices of concentration, inequality or poverty in welfare, mean-variance analysis in finance. The most popular uniform order relations are the Stochastic Dominance (SD) relations of various orders, based on the expected utility paradigm. It is our plan to use these techniques in the context of industrial concentration. Recently, Linton, Maasoumi and Whang (2002) have proposed a "subsampling" procedure for estimating the critical values of a test of first and second order dominance due to McFadden (1989). Their method is based on subsampling bootstrap, produces a consistent test, allows for correlation amongst the random variables and for the observations to be autocorrelated over time. They also allow the variables themselves to be residuals from some estimated model. This can be important if one wants to adjust for certain effects before comparing distributions.

6 Bibliography


Church, J. and Ware, R. ” Identifying and Measuring Market Power.” Industrial Organization 12: 423-456


