Buy Now, Search Later: A Model of Low-Price Guarantees With Post-Purchase Search

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Abstract

A common feature of low-price guarantees is that they allow consumers to postpone bargain-hunting until after the purchase. This paper addresses a number of questions concerning the adoption pattern of price-matching and price-beating guarantees with post-purchase search and their impacts on market prices. It is shown that low-price guarantees are offered by low-cost firms, and are associated with relatively low prices. All firms weakly reduce their prices in the presence of low-price guarantees, and firms offering low-price guarantees usually have incentives to cut their prices. These results are in sharp contrast with the traditional view on these policies as collusive practices.

Keywords: Low-Price Guarantee, Price-Matching, Post-Purchase Search, Guided Search, Price Competition.

JEL classification numbers: D43, L13, M3.
1. Introduction

Theoretical literature on low-price guarantees (LPGs) tends to view these promises to match or beat a lower competitor’s price as anti-competitive practices. Focusing on price-matching guarantees (PM), researchers suggest that LPGs facilitate tacit collusion due to the effect a firm’s guarantee has on the competitors’ incentives to undercut the firm’s price. Since a price-matching firm cannot be “undersold,” its rivals cannot attract the firm’s consumers by pricing lower. This reasoning underlies the tendency of competing firms to increase their prices in the presence of price-matching firms. For a survey of the literature, see a recent paper by Arbatskaya et al. (2004).

The current paper contributes to recent studies in economics and marketing that challenge the traditional views on LPG policies as collusive practices.\(^1\) Corts (1997) views LPGs as means of price discrimination between informed and uninformed consumers and shows that price effects of LPGs are ambiguous. The model examined here is novel in that firms may adopt LPGs that allow consumers to search for a lower price after they make their purchases. Such LPGs are widespread, according to Arbatskaya et al. (2004) – across a variety of industries, over 90 percent of price-beating and about 22 percent of price-matching guarantees explicitly specify the length of time consumers are allowed to search for a lower price after purchase. LPGs with post-purchase search are adopted on a great variety of products, e.g. automobile tires, cars, consumer electronics, computers, clothes, jewelry, drugs, furniture, decorations, and sports and office products. The post-purchase search period can be for 2, 7, 14, 30, 60 days and even 1 year, with a 30-day search period being

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\(^1\) Price-matching policies lose their collusive potential when consumers have to incur hassle costs to obtain refunds (Hviid and Shaffer, 1999) or when, instead of price-matching, firms beat competitors’ advertised prices (Corts, 1995).
the most common. The post-purchase search characteristic of LPGs, yet unexplored in
the economics literature, may have important implications for market competitiveness. A
number of questions concerning the adoption pattern of low-price guarantees and their effects
on prices are addressed. Which firms adopt LPGs? Can they signal low prices? Are LPGs
always anti-competitive?

While most of the existing theoretical models predict that firms offering LPGs have higher
prices, empirical evidence suggests that, overall, LPG firms do not have significantly different
prices than other firms in the market. Furthermore, LPGs explicitly allowing post-purchase
search tend to be associated with lower prices.\(^2\) Similarly, market observers associate LPGs
with a competitive weapon that indicates a firm’s willingness to compete on price. Intro-
duction of LPGs is interpreted as a sign of continuing price wars.\(^3\) Experimental evidence
also demonstrates that consumers believe price-matching firms to have lower prices (Jain
and Srivastava, 2000).

I propose a simple model in which LPG policies with post-purchase search affect the way

\(^2\)In Arbatskaya et al. (1999), whether a firm announces a price-matching or price-beating guarantee does
not have a significant effect on a price advertised for P185/75R14 automobile tires. The anti-competitive
potential of LPGs is further tested in Arbatskaya et al. (2003). The paper is based on paired price quotes
from LPG and non-LPG firms on identical automobile tires advertised in a U.S. Sunday newspaper in the
same city on the same day. The empirical analysis indicates that price-beating firms usually do not facilitate
rivals’ prices but the collusive potential of price-matching guarantees cannot be ruled out. An alternative
way to organize the data is to look at whether a firm would match or beat a lower price found by a consumer
after purchase. None of 44 price-matching firms in the data explicitly allow post-purchase search while more
than 2/3 of 103 price-beating firms would give refunds for 30 days after purchase. Remarkably, in pairs
where LPG firms allow post-purchase search they set a higher price in 4 of 71 cases (5.6%). At the same
time, firms which offer LPGs without post-purchase search set a higher price in 45 of 76 cases (59.2%). The
evidence suggests that the post-purchase search characteristic could be an important determinant of whether
an LPG firm has incentives to set a low price.

\(^3\)For example, a low-price guarantee offered by Staples states: If, within 14 days of a Staples purchase,
a customer finds a lower price on an identical item somewhere else, Staples will pay 110 percent of the
difference. The guarantee is a part of an “in-your-face message about everyday low prices.” (Boston Globe,
January 11, 2002.) In an aggressive marketing campaign of the fourth quarter of 2001, Buy.com claimed to
have the “Lowest Prices on Earth,” and offered a price-matching guarantee. (PR Newswire, September 25,
2001.)
consumers search: LPGs attract the consumers informed about which firms offer to match or beat lower prices. In accordance with the empirical evidence, LPGs are associated with relatively low prices in the equilibrium. Low-cost firms adopt LPGs and charge low prices while high-cost firms do not adopt the guarantees and charge high prices. The reason why high-cost firms do not adopt LPGs is that it is too costly for such firms to offer refunds. Importantly, LPG policies with post-purchase search are usually pro-competitive.

In the LPG games studied in the literature, an adoption of a price-matching guarantee usually constitutes a weakly dominant strategy. The firm loses nothing from promising to meet competition and it can gain through the relaxing effect the guarantee has on its competitors. Therefore, universal adoption of low-price guarantees is usually a part of the equilibrium. In practice, although LPGs are widely observed they are not ubiquitous. Consistent with evidence from a variety of markets, universal adoption of LPGs is generally not a part of the equilibrium in the model presented below.

In the next section, I present a model describing markets in which firms can offer low-price guarantees with post-purchase search. In the LPG game, firms simultaneously choose LPG policies and then prices. Section 3 analyzes pricing policies of LPG and non-LPG firms

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4 Price-matching and price-beating policies can not only affect where consumers search but also the intensity of search before and after purchase. I return to this issue in Section 4. The “buy-now” effect of LPGs is discussed in Arbatskaya et al. (2004) as a possible rationale for LPG adoption and the explanation for observed patterns in data on LPGs. Consumers who consider buying at a store and are not fully informed about prices in other stores may be reluctant to buy without an assurance that they receive (or can receive) a competitive deal. By providing price protection, low-price guarantees encourage consumers to buy on the spot and undertake the search for a lower price later. In the benchmark model, this is assumed to be the case – consumer are only engaged in post-purchase search. In reality, consumers are, in fact, being persuaded to buy immediately by promises to match lower advertised prices for an extended period after purchase. See Boston Globe, February 16, 1997. In an article titled “Low-price guarantee creates confidence,” the executives of Six Continents Hotels and Cendant Corp. explain that they implement LPGs to encourage guests to book with confidence (Hotel & Motel Management, September 16, 2002). Experiments on the effect of price-matching policies on purchase decisions support the idea that consumers are likely to discontinue search when they encounter a store with price protection (Srivastava and Lurie, 2001).

5 For strongly asymmetric markets, one of the firms in a duopoly market may not commit to price-matching because its rival has no incentive to undercut the firm’s price, as in Logan and Lutter (1989).
in such markets, evaluates price effects of LPG adoption, and provides comparative statics results. It also studies the LPG adoption decision of a firm, as dependent on the firm’s costs. Finally, Section 3 describes the pure-strategy perfect-Bayesian Nash equilibrium to the LPG game in which LPGs are associated with lower prices and compares it to the outcome in markets where LPG policies are prohibited. Section 4 illustrates the properties of the equilibrium with an example and discusses the alternative formulations of the model. Concluding remarks follow in Section 5.

2. The Model

Consider a market in which \( n \) firms produce a homogeneous product at constant marginal costs \( 0 \leq c_1 < c_2 < \ldots < c_n \). Firms are numbered 1 through \( n \) according to their costs, with firm 1 having the lowest cost. Firms simultaneously and independently choose their LPG policies (a promise to match or beat a lower price, if consumers were to find it at a competing firm after purchase) and then prices.\(^6\) While none of consumers are informed about prices prior to visiting stores, they differ with respect to the information they possess about firms’ LPG policies and the cost of claiming refunds. While the uninformed do not know which firms offer LPGs, the informed have this information and can use it when choosing where to search. The total population of consumers is normalized to one, and the percentage of the informed is denoted by \( \alpha \in (0, 1) \).

Consumers have an individual demand, \( d(p) \), for the product; the demand is twice continuously differentiable and downward-sloping on \([0, r]\), where \( r \) is a choke price \( r \equiv \inf\{p : \)

\(^6\)This set up corresponds to environments where LPG policies are more long-term policies than prices. It better fits with the assumption that LPGs are observed by some consumers while prices are not observed prior to search.
$d(p) = 0\}$. Assume that per-consumer profit, $\Pi(p_i) \equiv (p_i - c_i)d(p_i)$, is strictly concave ($\partial^2 \Pi / \partial p_i^2 < 0$) and obtains the unique maximum at $p_i^m$, the monopoly price of firm $i$; $p_i^m < r$. Denote the associated profit as $\Pi_i^m = \Pi(p_i^m)$. Due to the assumptions on the profit and demand functions, the monopoly price of a firm is increasing in the firm’s marginal cost. Therefore, monopoly prices of firms are ordered in the same way as their marginal costs, $p_1^m < p_2^m < ... < p_n^m$. Consumer surplus is defined by $S(p_i) = \int_{p_i}^{r} d(p) dp$.

In the benchmark model, I assume that consumers do not engage in any pre-purchase search; they simply buy from the first store they visit. After purchasing the product, the informed can learn about lower prices available elsewhere, either by searching other stores or by receiving advertisements. It is assumed that the informed can at a cost become fully informed about prices set by firms in the market. However, the exact nature of search technology does not affect the main results, as is discussed in Section 4.

Consumers have a number of reasons to postpone price search until after purchase. Price search is usually more expensive before purchase because the value of a product to consumers depreciates over time, as is obvious in the case of emergency purchases (a flat tire) or impulse buys. In practice, a large proportion of purchases are unplanned and, hence, are made without any prior research. When consumers make multiple purchases during one shopping trip, they may be unsure whether certain products are reasonably priced. After purchase, consumers may receive a Sunday newspaper with price advertisements, learn from a knowledgable friend, or inquire at an information gathering agency. In fact, consumers do

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7Section 4 shows that the analysis can be easily extended to allow the uninformed and the informed to have different demand functions, $d^U(p_i)$ and $d^I(p_i)$. The Appendix provides details for the case where $d^I(p_i)$ is perfectly inelastic.

8Such consumer behavior is indeed rational under conditions specified in Section 3. Section 4 discusses the consequences of relaxing the assumption.
not have to engage in active search – they may incidently come to know about a lower price available elsewhere. The purchase itself can induce other people to share information about their purchase experiences for the same product.

The informed have to incur a search cost (hassle cost) to obtain refunds due under an LPG. The search cost includes the cost of getting informed about actual prices in the market, the cost of making sure that all restrictions on the product, competitors, and timing of price offers are satisfied, as well as the cost of returning to the store and obtaining the refund. For the empirical evidence on the restrictions placed by firms offering LPGs, see Arbatskaya, et al. (2004). Let $F(s)$ be the cumulative distribution of search costs for the informed that has a continuous positive density $F'(s)$ on its support $[\underline{s}, \bar{s}]; \underline{s} \geq 0$. The distribution of firm production costs and consumer search costs are common knowledge. As one might expect, consumers will be more likely to search for a lower price when the refunds are large.

Consumers purchasing the product at LPG firm $i$ can later ask for a refund if they find the product available at a lower price elsewhere. The per-unit refund is a linear function of the firm’s price, $p_i$, and the lowest competitor’s price denoted by $p_0$; $p_i \geq p_0$. The refund function $R(p_i) = A + \beta p_i - \gamma p_0$ encompasses all the types of LPGs documented in Arbatskaya, et al. (2004). Generic statements of the LPGs with the corresponding refund functions are summarized in Table 1.
Table 1. Refund Functions

<table>
<thead>
<tr>
<th>LPG Type</th>
<th>Guarantee wording</th>
<th>Refund Function</th>
</tr>
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<tbody>
<tr>
<td>PM</td>
<td>If after purchasing from us, you will find a lower competitor price we’ll match it.</td>
<td>$R(p_i) = p_i - p_0$</td>
</tr>
<tr>
<td>PBΔ</td>
<td>we’ll beat it by 110% of the difference in prices.</td>
<td>$= 1.1(p_i - p_0)$</td>
</tr>
<tr>
<td>PB%</td>
<td>we’ll beat it by 10%.</td>
<td>$= p_i - 0.9p_0$</td>
</tr>
<tr>
<td>PB$</td>
<td>we’ll beat it by $100.</td>
<td>$= 100 + p_i - p_0$</td>
</tr>
</tbody>
</table>

The key feature of a firm’s LPG refund function is that the refund is large when the purchase price is high and the lowest known price is low. Although the analysis of LPG markets would allow for any of these LPG types, I use the refund function $R(p_i) = \beta(p_i - p_0)$, which directly encompasses the two most prominent types of LPGs – price-matching and price-beating by a percentage of the difference in prices; $\beta = 1$ corresponds to price-matching and $\beta > 1$ to price-beating guarantees. The informed who claim the refund $R(p_i) = \beta(p_i - p_0)$ obtain the discount (effective) price $\tilde{p}_i = p_i - \beta(p_i - p_0)$. Since $\tilde{p}_i = p_0 - (\beta - 1)(p_i - p_0)$, the effective price is $p_0$ for price-matching guarantees and less than $p_0$ for price-beating guarantees. The effective price is weakly decreasing in $p_i$: $\frac{\partial \tilde{p}_i}{\partial p_i} = -(\beta - 1) \leq 0$. Finally, I assume that firms choose whether to adopt a common type of LPG so that all adopted LPGs have the same refund function.

Figure 1 illustrates how the game unfolds. First, each firm learns its cost and chooses whether or not to offer an LPG. Second, firms and the informed consumers observe firms’ LPGs. Third, firms simultaneously choose prices. Fourth, consumers choose a firm to visit (an LPG firm or a non-LPG firm) and learn its price; the informed consumers learn their costs of searching for a lower price and claiming the refund. Fifth, consumers make their decisions.
purchase decisions. Finally, the informed decide whether or not to incur the cost of learning market prices and claiming the refund.\footnote{In an alternative setup of the model, the uninformed could learn about LPGs when visiting a store. Hence, although they select a firm at random, they could claim a refund if they buy from an LPG firm. This makes the adoption of an LPG less desirable but the main results would go through.}

A perfect-Bayesian Nash equilibrium to the LPG game is characterized by i) firms’ decisions on whether to offer LPGs; ii) firms’ prices, conditional on the LPG adoption decisions; iii) the informed consumers’ beliefs about equilibrium prices at LPG and non-LPG firms; iv) the informed consumers’ decision to visit an LPG or non-LPG firm; and v) the informed consumers’ decision to collect information and claim a refund, depending on their search cost. In the equilibrium, each decision-maker acts optimally and consistently with the equilibrium behavior of other agents and their beliefs; the beliefs themselves are well-founded.

3. The Analysis

The main goal of this section is to characterize the perfect-Bayesian Nash equilibrium in which low-price guarantees are associated with relatively low prices. In the equilibrium,
firms adopt LPGs to attract the informed consumers, who indeed prefer to buy from LPG stores because these stores are expected to have lower prices and they offer an option of claiming refunds.

The analysis of the LPG game will proceed in few steps. First, the pricing behavior is analyzed in markets with no LPGs and in markets with an exogenous LPG adoption pattern, and the price effects of LPG adoption are examined. Then, I look at firms’ incentives to adopt LPGs. Finally, the equilibrium in the LPG game is described and compared to the outcome attained were LPGs banned.

3.1. Pricing Behavior For An Exogenous LPG Adoption Pattern

Markets With No LPGs

Consider a market in which consumers are unaware of prices and pre-purchase search is prohibitively costly. Consumers have to choose a store at random; each of $n$ firms ends up selling to the $(1/n)$th fraction of consumers. Firm $i$’s profits are

$$\Pi_i^0(p_i) = \frac{1}{n}\Pi(p_i)$$

Firm $i$ charges its monopoly price, which depends on the firm’s marginal cost. The equilibrium prices in the market with no LPGs are denoted $p_i^0 = p_i^{m0}$ and profits are $\Pi_i^0 = \Pi_i^{m0}/n$.

It has been assumed that consumers buy at the first store they encounter. Consumers would not engage in pre-purchase search when the expected gain from search is lower than the pre-purchase search cost. Since the expected gain is at most the difference in consumer surplus at the lowest and the highest monopoly prices, when this difference is below the pre-purchase search cost, no pre-purchase search is conducted.
Pricing In Markets With LPGs

In markets with no LPGs, firms charge monopoly prices. However, when a firm offers an LPG, the firm’s price-setting incentives change. Charging the monopoly price is no longer profitable for the firm when it implies costly refunds. An LPG firm reduces the cost of offering LPG by decreasing its price.\(^{11}\) Intuitively, not only refund values are lower at lower prices but also the fraction of the informed consumers who claim refunds decreases.

Consumers rationally use information they have in deciding where to search. In the equilibrium where LPGs direct informed consumers to better offers, the informed consumers search and buy from one of LPG firms at random. In markets with LPGs, a firm’s profit function depends on whether the firm offers an LPG and whether it has the lowest price in the market. First, the profits of a non-LPG firm in a market where at least one firm has an LPG are

\[
\Pi^N(p_i) = \frac{1 - \alpha}{n} \Pi(p_i)
\]

where \((1 - \alpha)\) is the measure of the uninformed. In an equilibrium, the non-LPG firm sets its monopoly price, \(p_i^N = p_i^m\) and receives profits \(\Pi^N_i = (1 - \alpha) \Pi_m^m / n\).

Next, I derive the profit of an LPG firm. The informed choose to gather price information and collect the refund from the LPG store they bought the product from when the gain from post-purchase search exceeds the cost, \(s\). The gain from search, denoted \(\Delta S\), is the difference

\(^{11}\text{The cost of servicing LPGs can be high. Sainsbury spent 20 million pounds on its price-matching policy that applies to 1500 items. (Independent On Sunday, October 3, 1999.)}\)
in the consumer surplus at the discount (effective) price \( \tilde{p}_i(p_i) \equiv p_i - R(p_i) \) and the full price \( p_i \)

\[ \Delta S(p_i) \equiv S(\tilde{p}_i) - S(p_i) = \int_{\tilde{p}_i(p_i)}^{p_i} d(p)dp \] (3)

Since firm \( i \)'s effective price \( \tilde{p}_i \) is weakly decreasing in \( p_i \), the gain from search is increasing in \( p_i \) when \( p_i < r \); \( \partial (\Delta S(p_i))/\partial p_i = d(p_i) + (\beta - 1)d(\tilde{p}_i(p_i)) > 0 \). A proportion \( G(p_i) \equiv F(\Delta S(p_i)) \) of the informed consumers search after purchase, where \( F \) is the distribution of search costs.\(^\text{12}\) For \( s < \Delta S(p_i) < \pi \), some but not all of the informed engage in post-purchase search when firm \( i \)'s price is \( p_i \). The propensity to claim refunds weakly increases as firm \( i \)'s price increases since \( \partial G(p_i)/\partial p_i = (\partial F/\partial (\Delta S)) \cdot (\partial (\Delta S)/\partial p_i) \geq 0 \).

The profits of an LPG firm, given that \( m \) firms in the market have LPGs, are

\[ \Pi^G(p_i) = \frac{1 - \alpha}{n} \Pi(p_i) + \frac{\alpha}{m} \Pi(p_i)(1 - G(p_i)) + \frac{\alpha}{m} \Pi(\tilde{p}_i(p_i))G(p_i) \] (4)

where \( G(p_i) \) is the proportion of the informed who claim refunds. An LPG firm derives its profit from the uninformed, the informed who do not activate the firm’s LPG and the informed who ask for refunds after purchase.

Firm \( i \)'s profit can be written as

\[ \Pi^G(p_i) = \left( \frac{1 - \alpha}{n} + \frac{\alpha}{m} \right) \Pi(p_i) - \frac{\alpha}{m} G(p_i) \Delta(p_i) \] (5)

where \( \Delta(p_i) \equiv \Pi(p_i) - \Pi(\tilde{p}_i(p_i)) \) is the difference in per-customer profits at the full and discount prices. The first term in (5) includes the profit received from the uninformed and

\(^{12}\)If the informed had a unit demand for the product, their gain is simply the value of the refund, \( \Delta S(p_i) = R(p_i), \) and \( G(p_i) = F(R(p_i)) \). However, when demand is downward-sloping, the gain from search exceeds the value of refund at the regular price \( \Delta S(p_i) > R(p_i)d(p_i) \) because the quantity purchased at the discount price \( \tilde{p}_i \) is higher.
the benefit from the LPG signal that attracts the informed consumers. The second term is the cost of offering the LPG – the losses from giving refunds to the informed who ask for them. The second term can be interpreted as a loss from setting a price higher than the lowest price in the market. It is zero when there are no informed consumers, when no informed consumer asks for refunds, or when the firm has the lowest price.

A decrease in a firm’s price affects the firm’s profits in a number of ways. On the one hand, a lower than monopoly price directly reduces the firm’s profits. On the other hand, the lower price implies lower refunds and a smaller proportion of the informed who ask for them. Therefore, a firm faces a trade-off between charging more to the uninformed and informed consumers who do not ask for refunds and paying more in refunds to those the informed who activate the LPG. Intuitively, the prospect of future search by the informed can discipline the firm offering an LPG.

The following first-order condition provides a necessary condition for \( p_i \) to be optimally chosen:

\[
\left( \frac{\partial \Pi^G}{\partial p_i} \right) = \left( \frac{1 - \alpha}{n} + \frac{\alpha}{m} \right) \frac{\partial \Pi}{\partial p_i} - \frac{\alpha}{m} \left( \frac{\partial G}{\partial p_i} \Delta + G \frac{\partial \Delta}{\partial p_i} \right) = 0
\]  

Equation (6) implicitly defines the equilibrium price of firm \( i, p^G_i \), as a function of the lowest price in the market and the number of firms adopting LPG, as well as other parameters of the model. Claim 1 below shows that pricing above the monopoly price is suboptimal. Pricing below the marginal cost is clearly not optimal as well. Hence, if there exists \( p^G_i \) that satisfies (6) and the second-order sufficient condition \( \partial^2 \Pi^G / \partial p^2_i < 0 \), then \( p^G_i \in [c_i, p^m_i] \). To make sure that there is a unique optimum price for firm \( i \), I will assume that \( \Pi^G \) is strictly concave on \([c_i, p^m_i]\).\(^{13}\)

\(^{13}\)Since the profit \( \Pi(p) \) is assumed to be strictly concave, the concavity of \( \Pi^G(p) \) holds when the pro-
3.2. Price Effects Of LPGs

This subsection looks at the effects of LPG adoption on the price of the LPG firm (the self-effect) and on the prices of its rivals (the external effect). Consider pricing in markets with exogenous LPG adoption patterns consistent with the assumption that LPGs are associated with lower prices: for markets with partial LPG adoption, prices at LPG stores are on average lower than at other stores. LPG firms set prices according to (6) and non-LPG firms charge monopoly prices. Claims 1 through 3 compare a firm’s price when the firm adopts and does not adopt LPG, keeping all else equal. Claim 1 states that when a firm adopts LPG, it does not increase its price above the monopoly level.

**Claim 1.** Firms set weakly lower prices when they adopt LPGs.

**Proof.** All non-LPG firms charge monopoly prices. The lowest-price firm has profits
\[
\Pi_G(p) = \left( \frac{(1-\alpha)}{n} + \frac{\alpha}{m} \right) \Pi(p)
\]
when it adopts LPG and charges \( p \). Hence, the lowest-price firm also sets the monopoly price. The reason why pricing above the monopoly level is suboptimal for all other firms can be seen by examining equation (4). At \( p > p^m_i > p_0 \), firm \( i \) loses profits on the consumers who buy at the full price since \( \Pi(p) < \Pi(p^m_i) \) and it incurs losses from the consumers who claim refunds since \( \Pi(p-R(p)) < \Pi(p^m_i-R(p^m_i)) \). If the proportion of claimants were fixed, this would complete the proof. However, the proportion of sales at portion of the informed is low and the probability of claiming refunds, \( G(p) \), is not too concave. To see this, note that the second-order derivative of \( \Pi_G(p) \) is negative for \( p \in [c_i, p^m_i] \) whenever \( (\Pi_G(p))'' = \frac{\sigma}{m} (\lambda(\Pi''(p) - (G'' \Delta + 2G' \Delta' + G \Delta'')) < 0 \), where \( \lambda = \frac{(1-\alpha)}{n} + 1 > 1 \) and prime denotes a derivative with respect to \( p \). Since \( \Delta'' = \Pi''(p) - \Pi''(p^m_i) (\beta - 1)^2 \), the second-order derivative can be written as \( (\Pi_G(p))'' = \frac{\sigma}{m} (\lambda - G) \Pi''(p) - \frac{\sigma}{m} \left( G'' \Delta + 2G' \Delta' - G \Pi''(p) (\beta - 1)^2 \right) < 0 \). Since \( \lambda - G > 0, G \geq 0 \), and \( \Pi(p) \) is concave, \( \Pi_G(p) \) is concave if \( G'' \Delta + 2G' \Delta' \geq 0 \). Note that for \( p \in [c_i, p^m_i], G' \geq 0; \Delta \geq 0; \) and \( \Delta' = \Pi'(p) + \Pi'(p^m_i) (\beta - 1) \geq 0 \). Hence, \( G(p) \) that satisfies inequality \( (G''/G') \geq -2\Delta'/\Delta \) guarantees concavity of \( \Pi_G(p) \). The inequality holds if \( G(p) \) is not too concave. For instance, when consumers have a unit demand and search costs are distributed uniformly, then \( G'' = 0 \) and \( \Pi_G(p) \) is concave.
the discount price can be higher at \( p \) than at \( p_i^m; \) \( G(p) > G(p_i^m). \) If sales at the full price are more profitable than at the discount price, then we have established that charging above the monopoly price is worse than charging the monopoly price. If sales at the full price are not as profitable as at the discount price, then \( \Pi(p_i^m) > \Pi(p_i^m - R(p)) > \Pi(p - R(p)) \geq \Pi(p) \) and all sales yield lower profits at price \( p \) than at the monopoly price. Hence, in this case as well the optimal price is below the monopoly level, \( p \leq p_i^m. \) Q.E.D.

Next, I show that the lowest-cost firm (firm 1) sets the lowest price in the market, and since the lowest price is the monopoly price, \( p_0 = p_1^m. \)

**Claim 2.** The lowest-cost firm sets the lowest price in the market, which is its monopoly price.

**Proof.** Suppose that the lowest price is not set by firm 1 but by some other firm \( j \neq 1. \) Being the lowest-price firm, firm \( j \) has profits \( \Pi^G(p) = \left( \frac{(1-\alpha)}{n} + \frac{\alpha}{m} \right) \Pi(p) \) and \( \Pi^N(p) = \frac{(1-\alpha)}{n} \Pi(p) \) when it adopts and does not adopt LPG. Hence, the lowest-price firm always sets the monopoly price. It is not possible for firm 1 to have a lower cost \( (c_1 < c_j) \) but a higher price than firm \( j \) \( (p_1 > p_j) \) because \( p_j = p_i^m > p_i^m \geq p_1. \) The inequalities are due to the fact that the monopoly price is increasing in costs, \( p_j^m > p_i^m \) and firm 1’s price is the monopoly price at most, \( p_1 \leq p_i^m, \) according to Claim 1. Q.E.D.

It will be shown in Claim 3 that under certain conditions an LPG firm has a marginal incentive to reduce its price below the monopoly level.

**Claim 3.** Firm \( i \) sets a strictly lower price iff i) it offers a price-beating guarantee and some consumers claim refunds at the monopoly price, \( s < \Delta S(p_i^m); \) or ii) it offers a price-matching guarantee and some but not all consumers claim refunds at the monopoly price,
\( \mathfrak{s} < \Delta S(p^m_i) < \overline{s}. \)

The proof is in the Appendix. Claim 3 shows that firms with price-matching and price-beating guarantees have different incentives in setting prices. Intuitively, an LPG firm would lower a price below the monopoly level for two reasons. First, by lowering its price, the firm can reduce the number of the informed who claim refunds. Second, at a lower price, the loss in profits per claimant (the difference in the profit level at the full and discount prices) can be lower. For price-matching guarantees, the latter effect is absent at the monopoly price since the marginal profit at the full price is zero and the discount price is independent of the firm’s price. Hence, for price-matching guarantees to have a pro-competitive self-effect, the number of claimants should be sensitive to the firm’s price – some but not all consumers should claim refunds at the monopoly price. The conditions insuring the pro-competitive effect for price-beating firms are more lax. As long as some consumers claim refunds, price-beating firms lower their prices.\(^{14}\) In what follows it will be often assumed that condition (C1) holds.

**Condition C1.** *Some but not all the informed consumers would claim refunds at the firm’s monopoly price.*

Condition C1 is equivalent to \( \mathfrak{s} < \Delta S(p^m_i) < \overline{s}: \) search costs are sufficiently dispersed that at the monopoly price of firm \( i \) some but not all the informed consumers would ask for refunds. For (C1) to hold for all but the lowest-price firm, it suffices to assume that \( \mathfrak{s} = 0 \)

\(^{14}\)Some sellers promise to send refunds to all consumers when a lower price is discovered elsewhere. In the case of automatic LPGs, all the informed receive refunds. For a price-matching firm, the profits are \( \Pi^G(p_i) = \left(1 - \alpha \right) \Pi(p_i) + \frac{m}{n} \Pi(p_0). \) Therefore, the firm sets the monopoly price \( p^m_i. \) A price-beating firm charges less than its monopoly price since its profits are \( \Pi^G(p_i) = \left(1 - \alpha \right) \Pi(p_i) - \frac{m}{n} \Delta(p_i) \) and \( \left( \frac{\partial \Delta(p_i)}{\partial p_i} \right)|_{p^m_i} < 0. \) Hence, automatic price-matching does not have a pro-competitive self-effect but automatic price-beating still has it.
and \( \pi > \Delta S(p^m_n) \). The following corollary follows directly from Claim 3.

**Corollary.** Under (C1), an LPG promotes competitive pricing by the firm adopting it.

How does the price of an LPG firm depend on the number of other firms offering LPGs?

**Claim 4.** The pro-competitive effect of LPG adoption diminishes as a larger number (percentage) of firms adopt LPGs.

The proof is in the Appendix. The external effect of LPG adoption is anti-competitive in that the incentives of an LPG firm to reduce its price below the monopoly level diminish when more firms offer LPGs. This occurs because the relative weight of the losses due to refunds is lower when more firms share the pool of the informed consumers. The studies on LPGs that show their anti-competitive potential focus on the external effects of LPGs — their ability to change the incentives of the rival firms. In this model, the external effect only reduces the magnitude of the pro-competitive self-effect. As more firms adopt LPGs, more firms lower their prices below the monopoly level but the amount by which the price of an LPG firm is lowered is negatively related to the number (or percentage) of firms adopting LPGs.

Claim 4 has established that the prices of LPG firms move closer to the monopoly prices as LPGs become more widespread. Similar comparative statics exercises can be done for other parameters of the model. Claim 5 shows that the prices are lower when there are more consumers who are informed about LPGs and when LPG firms are more generous in their refunds.

**Claim 5.** Under (C1), the price of an LPG firm decreases in the percentage of the informed consumers, \( \alpha \), and the extent of price-beating, \( \beta \).
The proof is in the Appendix. The first result is very intuitive since the losses due to refunds have a relatively higher weight when a larger proportion of consumers are informed about LPGs. The second results implies that, all else equal, the prices of price-beating firms are lower than the prices of price-matching firms. Intuitively, LPG firms could reduce their losses due to refunds by increasing their discount (effective) price. While price-matching firms cannot change the effective price (which is equal to the lowest market price), price-beating firms can increase their discount price by decreasing the full price since $\frac{\partial \tilde{p}_i}{\partial p_i} = -(\beta - 1) \leq 0$. At higher refund percentages, the effective price is more sensitive to changes in the full price and hence the value of refund as well as the proportion of the claimants can be reduced more effectively.

Finally, I want to examine how prices of LPG firms change with their marginal costs.

Claim 6. The price of an LPG firm increases in its marginal cost when the proportion of claimants is low and not very sensitive to changes in prices.

The proof and the exact condition under which the standard comparative statics result holds are in the Appendix. Although, in general, the prices need not increase in marginal costs, for a low and relatively inflexible propensity to claim refunds this is the case.

3.3. LPG Adoption Decision

So far I have looked at the firms’ pricing when their LPG decisions are given. Next, I explore the incentives to adopt LPGs. The benefit to firm $i$ from adopting an LPG, given that $(m - 1)$ other firms adopt $(m - 1 \geq 0)$ is $B \equiv B(c_i, m) = \Pi^G(p_i^G) - \Pi^N(p_i^N)$, where $\Pi^G(p_i)$ and $\Pi^N(p_i)$ are defined in (5) and (2), and $p_i^G$ and $p_i^N = p_i^m$ are the prices set by firm $i$ when it adopts and does not adopt LPG, respectively.
If firm $i$ were to keep prices at the monopoly level, the firm’s adoption of LPG would imply extra profits from the informed who do not ask for refunds and possibly losses from the informed who claim refunds. As Claim 3 and its corollary indicate, the firm usually has incentives to cut its price in order to reduce losses due to refunds under its LPG. Hence, LPG adoption implies a trade-off between lower profits from the uniformed and a gain in profits from the informed. How does the benefit of adoption depend on the firm’s marginal cost?

**Claim 7.** The benefit of LPG adoption is strictly decreasing with the marginal cost. There exists a cut-off level for the marginal costs such that only firms with lower marginal costs adopt LPGs. The lowest-cost firm always adopts LPG.

The proof is in the Appendix. Only firms with relatively low costs adopt LPGs. This is due to the fact that it is less costly for a low-cost firm than to a high-cost firm to offer refunds to consumers in case they find a lower price elsewhere. The next claim states that the benefit of LPG adoption is also decreasing with the number of firms adopting LPGs.

**Claim 8.** LPG adoption is less attractive for firm $i$ when more firms adopt LPGs, assuming that (C1) holds if $i \neq 1$.

The proof is in the Appendix. The more widespread LPG adoption is, the lower is the benefit from the adoption. Intuitively, LPGs lose their attractiveness when more firms adopt them since the benefit of adoption – the extra demand from the informed – is shared among all LPG firms. The findings of Claim 7 and 8 explain why in reality not all firms promise to meet or beat lower competitors’ prices.
3.4. The Equilibrium In The LPG Game

The following proposition characterizes the properties of an equilibrium in the LPG game between firms who simultaneously choose their LPG policies and then set prices.

**Proposition 1.** There exists a unique equilibrium to the LPG game in which LPGs are offered. In the equilibrium, only \( m^* \geq 1 \) firms with the lowest marginal costs adopt LPGs, \( m^* \equiv \max \{ m : B(c_m, m) \geq 0 \} \). Under (C1), all LPG firms, except for the lowest-cost firm, charge below their monopoly prices and other firms charge the monopoly prices. LPG firms have uniformly lower prices than non-LPG firms, and the informed consumers choose to visit them. The informed claim refunds from LPG firms if their search costs are sufficiently low.

The proof is in the Appendix. In the equilibrium described in Proposition 1, at least one firm adopts LPG – the firm with the lowest cost. Adoption can be universal or partial, depending on the industry cost structure and LPG features. Importantly, each LPG firm (except for the lowest-cost firm) lowers its price when adopting LPG as long as condition (C1) holds, i.e. some but not all the informed consumers would claim refunds were the LPG firm to charge the monopoly price. The equilibrium price of any LPG firm is below the price of a non-LPG firm, which is consistent with the informed consumer’s beliefs and their decision to search among LPG firms.

In the equilibrium with the universal adoption, LPGs do not signal low prices in a sense that the informed consumers cannot use them to infer any information about firms’ costs. At the same time, LPGs still change firms’ incentives, pressuring them into setting lower prices. Next, compare the equilibrium prices in markets with no LPGs and in LPG markets with an equilibrium adoption pattern. The following Corollary follows immediately from
Proposition 1.

Corollary. A ban on LPG policies results in weakly higher prices for all firms. When more than one firm adopts LPG in the equilibrium, some firms would increase their prices were LPGs banned. When LPG adoption is universal in the equilibrium, all but the lowest-cost firm would set higher prices were LPGs banned.

A prohibition of LPGs increases the prices of all LPG firms in the market, except for the lowest-cost firm. The prices of non-LPG firms are unchanged. This implies that all consumers benefit from the existence of LPGs. The uninformed consumers choose a firm at random and they benefit from LPG policies as long as LPGs reduce the average price in the market. The presence of the informed conveys a positive externality on the uninformed and the externality is larger the higher is the percentage of the informed. Even if LPGs were not lowering the average price, they can still benefit the informed consumers by coordinating their demand towards low-price firms. While the uninformed consumers care about the average price, the informed only care about the prices (and effective prices) of LPG firms. The informed would usually be best-served when only one firm offered LPG. Although in that case the prices are the monopoly levels, the informed all buy at the lowest-price firm.\(^\text{15}\)

\(^{15}\)Notice what happens to the average price of LPG and non-LPG firms when a larger number of firms adopt LPGs, all else equal. Suppose there exists a fixed cost of administering LPG which can include the potential costs of litigation related to the adoption of LPG. Consider an external shock that reduces this cost (for example, there is a shift in the stand of the antitrust agencies towards LPGs). Then, a marginal firm (or firms) adopts LPG and lowers its price. Perhaps surprisingly, the average prices of non-LPG firms and LPG firms increase as a result of the LPG adoption. The average price of LPG firms increases for two reasons. First, the marginal firm has higher costs than other LPG firms, and hence its price pools up the average price of LPG firms. Second, all LPG firms increase their prices due to the external effect established in Claim 4. Although non-LPG firms continue to set monopoly prices, the average price of non-LPG firms increases as it is the lowest-cost firm among them that adopts LPG after the shock. Hence, it would appear that a more wide-spread adoption of LPGs leads to higher average prices. However, this is not the case as long as the self-effect dominates the total external effect of LPG adoption. (See the example in Section 4.) And, it surely is wrong to use this observation to suggest that LPGs should be banned. If LPGs were banned, all consumers would be worse-off.
In the equilibrium, LPG policies lead to higher levels of concentration, as measured by the Herfindahl-Hirschman index, since low-cost firms serve a larger share of the market when LPGs direct the informed towards them. At the same time, LPGs lower prices. Hence, the common measure of concentration does not capture the intensity of competition in markets where firms can adopt LPGs. The number of firms is also not a good predictor of market performance. Prices in the equilibrium do not depend on the number of firms in the market, keeping the percentage of LPG firms constant.

Finally, an interesting feature of the equilibrium is the reduced price differentials in LPG markets. The average gap between an LPG firm’s price and the lowest market price is lower since LPG firms set their prices closer to the lowest price in the market. In the presence of LPGs, prices may look more coordinated. In Hess and Gerstner (1991), the evidence of a more coordinated price behavior in the presence of LPGs is interpreted as indicating that LPGs are collusive. The current paper shows that the dispersion in prices can be lower when firms adopt LPGs. However, the prices need not be higher.

4. An Example and Alternative Formulations

An Illustrative Example

As an example, consider a market with three firms which have marginal costs $c_1 = 0$, $c_2 = 1$, and $c_3 = 2$. Assume there are an equal share of the informed and the uninformed consumers in the market, $\alpha = 0.5$. The individual consumer demand is $d(p_i) = 4 - p_i$. Search cost of the informed consumers are distributed uniformly on $[0, 1]$. Firms simultaneously choose whether to offer a price-matching policy, and then set prices. Were LPGs prohibited, the equilibrium prices would be $p_1^0 = 2$, $p_2^0 = 2.5$ and $p_3^0 = 3$. Firms would obtain profits
\( \Pi_1^0 = \frac{4}{3}, \Pi_2^0 = 0.75, \) and \( \Pi_3^0 = \frac{1}{3} \). In the equilibrium to the LPG game, firms 1 and 2 adopt price-matching, while firm 3 does not. The equilibrium prices are \( p_1^G = 2, p_2^G \approx 2.33 \) and \( p_3^N = 3 \), and the profits are \( \Pi_1^G = \frac{5}{3}, \Pi_2^G \approx 0.89, \) and \( \Pi_3^N = \frac{1}{6} \).\(^{16}\)

The prohibition of LPGs would lead to a higher price for firm 2 and the same prices for firms 1 and 3. The lowest-cost firm obtains higher profits in the presence of LPGs while other firms would rather have LPGs banned. Adoption of price-matching guarantees by firms 1 and 2 promotes competitive pricing and at the same time leads to a greater conformity in the firms’ prices.

Note that although firm 2’s LPG lowers the average market price, the average prices of LPG and non-LPG firms increase when firm 2 (in addition to firm 1) decides to offer LPG. The informed consumers prefer only one firm to adopt LPG while the uninformed prefer the adoption to be as wide-spread as possible since the average price is lower when all firms adopt LPGs.

**Alternative Demand Assumptions**

The assumption that the uninformed and the informed have the same demand can be relaxed. When the two consumer groups have different demands, additional forces come into play. Since in the equilibrium LPGs attract the informed, LPG firms get a larger share of the informed than when LPGs are absent. Suppose that the demand of the informed is more price elastic than the demand of the uninformed. Then there appears an additional rationale for an LPG firm to lower its price – it faces an overall more elastic pool of consumers. At the

\(^{16}\)This is an equilibrium because prices are optimally chosen for the equilibrium LPG adoption pattern and changes in the adoption decisions are not profitable. Without a price-matching policy, firm 2 would set a higher price, \( p_2^N = 2.5, \) and obtain lower profits \( \Pi_2^N = \frac{2}{5} \leq \Pi_2^G \). If firm 3 were to adopt price-matching, it would set a lower price \( p_3^G = 2.5 \) and earn lower profits, \( \Pi_3^G \approx 0.14 \leq \Pi_3^N \).
same time, non-LPG firms face a larger proportion of the uninformed in their consumer pool, and they would set higher prices when LPGs are present in the market. Hence, when the informed have a more elastic demand, LPG firms set lower prices while non-LPG firms set higher prices than if the demand is the same for all consumers. The opposite results would hold when the uninformed have a more elastic demand. The Appendix analyzes the case where the demand for the uninformed is linear and it is perfectly inelastic for the informed.

Price-setting incentives of an LPG firm can be expressed in terms of elasticities. Denote by \( d^U(p) \) and \( d(p) \) the demand for the uninformed and the informed consumers respectively. Let \( \varepsilon_U \) and \( \varepsilon \) be the corresponding elasticity of the demand functions. Denote by \( p^m \) the monopoly price a firm sets when it caters only to the uninformed. The pro-competitive effect of LPGs can be seen by re-writing the profits of an LPG firm as
\[
\Pi^G(p) = (p - c_i) \left(\frac{(1-n)\cdot d^U(p)}{n} + \frac{\alpha}{m}d(p)f(p)\right)\]
where \( f(p) \equiv [(1 - G(p)) + (\Pi(p - R(p))/\Pi(p_i)) G(p)] \). The price-setting incentives of the LPG firm depend on the relationship between the elasticity of demand for the uninformed and the effective elasticity of demand for the informed, which can be written as \( \varepsilon_f = \varepsilon + \varepsilon_f \), where \( \varepsilon_f \equiv p \left(\partial f(p)/\partial p\right)/f(p) \). Note that \( f(p) \geq 0 \) since otherwise the firm would have negative profits from the informed and it would not be offering LPG in the equilibrium. The sign of \( \varepsilon_f \) is negative as long as \( \partial f(p)/\partial p < 0 \), which is the case when the firm sets a price below \( p^m \). The option of getting refunds can render the effective demand for the informed more elastic.
Consumers Have A Perfectly Inelastic Demand

Next I will show that when all consumers have a perfectly inelastic demand, there is no scope for the informative role of LPGs, and the same monopoly prices prevail in markets with LPGs as in markets where LPGs are absent. The result resembles the Diamond Paradox of monopoly pricing in markets with costly search. Suppose that consumers have a perfectly inelastic demand for one unit of the product, at prices below the choke price of $r$. When LPGs are banned, there is a unique equilibrium in which all firms charge the monopoly price $r$; there is no price dispersion and no search.

In markets where LPGs are allowed, the same outcome prevails. Note that non-LPG firms always charge the monopoly price $r$. In an equilibrium where the informed prefer to buy from a non-LPG store, no firm would offer LPG, and the outcome is the same as when LPGs are banned. Next, suppose that in an equilibrium, LPGs attract the informed consumers. The profit function of an LPG firm is 
\[ \Pi^G(p) = \left( \frac{(1-\alpha)}{n} + \frac{\alpha}{m} \right)(p - c_i) - \frac{\alpha}{m} F(R(p)) R(p). \]
Let $p_0$ be the lowest price in the market. Denote by $L(p) \equiv F(R(p)) R(p)$ the loss due to refunds under the guarantee; $L(p_0) = 0$. Then, the firm’s objective can be written as 
\[ \lambda (p - c_i) - L(p), \]
where $\lambda \equiv \left( \frac{(1-\alpha)}{n} + \frac{\alpha}{m} \right) > 1$. The first-order condition is $\lambda - \partial L(p)/\partial p = 0$ and it suffices to assume that $L(p)$ is strictly convex to satisfy the second-order sufficient condition. Since $\partial^2 \Pi^G(p)/\partial p \partial c_i = 0$, the optimal price $p^G$ does not depend on $c_i$. At $p_0$, $\partial L(p)/\partial p = \beta \left( \left( \partial F/\partial R \right) R + F \right) = 0$, and it is increasing. Hence, the optimal $p^G > p_0$. This can only be the case when all LPG firms charge $p^G > r$, but charging above $r$ is suboptimal.

Finally, if in an equilibrium the informed consumers are indifferent whether to visit and buy from an LPG or non-LPG store and hence randomly choose among all stores, then the situation is as in the previous case with $m = n$. There is no equilibrium in which firms
charge prices other than the monopoly price $r$.

**Post-Purchase Search Technology**

Although the all-or-nothing post-purchase search technology may be realistic, it would be interesting to examine other search technologies. The difficulty in the analysis comes from the fact that the lowest price that the informed find after purchase is uncertain. With the uncertain price, the calculation of consumer demand is more difficult. Additionally, the profits of an LPG firm would depend on the prices of other firms (not only on the lowest price in the market). Note that each firm can then influence rival’s pricing and due to strategic complementarity of prices should have an incentive to increase its price to provoke a positive response from other LPG firms. The formal analysis of this case is rather complex and it is not undertaken here.

**Pre-Purchase Search**

In this paper, I focus on the post-purchase search and ignore the possible effects of LPGs on pre-purchase search. In contrast, the only paper that looks at the effects of LPGs on the equilibrium prices through their impact on the extent of search – Lin (1988) – shows that price-matching guarantees can be anti-competitive. The reason is that price-matching policies offer consumers a free recall of previously searched prices, and this encourages consumers to search longer, which under the assumption of increasing cost of search can imply higher prices. Low-price guarantees with the post-purchase search, in contrast, tend to discourage pre-purchase search and encourage post-purchase search.

It would be interesting to look at the effects of LPGs while allowing for both pre-purchase and post-purchase search. However, the analysis becomes complex. Moreover, it is possible
that even if one were to allow pre-purchase search, no such search would take place in an equilibrium. To see why this may be the case, consider first markets with no LPGs. The setting is then similar to that in Reinganum (1979), who models price competition between firms in markets where consumers are imperfectly informed and can engage in sequential search. Although there can exist an equilibrium with price dispersion in such a market (given asymmetric production costs and elastic demand), there is no search in the equilibrium. Comparing markets with and without LPGs, consumers seem to have even less incentive to search before purchase in the presence of LPGs for the following reasons. First, in the equilibrium, there is more conformity in prices of LPGs firms (among which the informed could search before purchase). Second, the price protection that LPGs provide renders search less attractive since drawing a high price may imply higher refunds and lower effective price. Finally, when search is much cheaper (or is free) after purchase, consumers would be likely to substitute the post-purchase search for pre-purchase search.

5. Conclusion

The paper considers an important feature of low-price guarantees that has not been previously formally examined in the literature – the option of claiming refunds for an extended period after purchase. By promising to match or beat a lower competitor’s price found by a consumer after making a purchase at a store, the store encourages the consumer to buy immediately, start utilizing the product earlier, and search after purchase. Under this scenario, LPGs are associated with lower prices because low-cost firms adopt LPGs and LPG firms lower their prices to keep consumers from searching for a better price after purchase. Universal adoption does not usually arise in the equilibrium. For sufficiently asymmetric
markets, it is too costly for some firms to match or beat other firms’ prices, even if only a fraction of consumers ask for refunds. Furthermore, the attractiveness of LPGs diminishes as they become widely adopted. Pricing of LPG firms may look more coordinated and, at the same time, be pro-competitive.

The advancement of Internet technology allows consumers to conduct a web-wide search at a fraction of the cost of visiting traditional stores. This did not discourage firms from offering LPGs. Price-matching and price-beating policies are commonly observed in online markets for consumer electronics, books, travel and mortgage loans. One difference in the features of LPGs offered online is that e-tailers limit their scope by putting tighter restrictions on the timing of post-purchase search. Whereas in conventional markets a 30-day search limit is commonly imposed, online vendors often restrict the period to 1-3 days. Still, consumers could become fully informed due to the existence of search engines, directories, and shopping bots. Such intermediaries can greatly improve the ability of buyers to find the best deals by providing them with lists of price quotes on a particular product. Although low-price guarantees differ in their characteristics, the important message they send is still, “Buy now, search later.” The post-purchase search feature of low-price guarantees can drive their competitive potential in a variety of market environments. Hence, the major result of the paper that low-price guarantees can direct consumer search to firms offering low prices is expected to carry on to other models. In light of the pro-competitive nature of LPGs with post-purchase search, an anti-trust policy against such LPGs is likely to have undesirable effects.
References


Appendix A: Proofs

Claim 3. Firm i sets a strictly lower price iff i) it offers a price-beating guarantee and some consumers claim refunds at the monopoly price, \( s < \Delta S (p_i^m) \); or ii) it offers a price-matching guarantee and some but not all consumers claim refunds at the monopoly price, \( s < \Delta S (p_i^m) < \bar{s} \).

Proof. From Claim 2, the lowest-cost (lowest-price) firm sets the monopoly price. Consider firm \( i \neq 1 \) which offers LPG. According to Claim 1, \( p_i \leq p_i^m \). From the first-order condition (6), the sign of \( \partial \Pi^G / \partial p_i \) at \( p_i^m \) is opposite to the sign of expression \( ((\partial G / \partial p_i) \Delta + G (\partial \Delta p_i)) \) since \( (\partial \Pi / \partial p) \big|_{p_i^m} = 0 \). For \( p_i \leq p_i^m \), the expression is nonnegative as all the terms in it are nonnegative. At \( p_i^m \), \( \Delta(p_i) = \Pi(p_i) - \Pi(p_i - R(p_i)) > 0 \) for \( i \neq 1 \). Hence, when a firm adopts an LPG, it has a marginal incentive to reduce its price below \( p_i^m \) iff either of two conditions hold: i) \( (\partial G(p_i) / \partial p_i) \big|_{p_i^m} > 0 \) or ii) \( G(p_i^m) > 0 \) and \( (\partial \Delta(p_i) / \partial p_i) \big|_{p_i^m} > 0 \).

Using \( \partial R(p_i) / \partial p_i = \beta \), I obtain \( (\partial \Delta(p_i) / \partial p_i) \big|_{p_i^m} = (\beta - 1) (\partial \Pi / \partial p_i) \big|_{p_i^m - R(p_i^m)} \). For price-matching guarantees, \( \beta = 1 \) and \( (\partial \Delta(p_i) / \partial p_i) \big|_{p_i^m} = 0 \); for price-beating guarantees, \( \beta > 1 \) and \( (\partial \Delta(p_i) / \partial p_i) \big|_{p_i^m} > 0 \) since \( (\partial \Pi / \partial p_i) \big|_{p_i^m - R(p_i^m)} > 0 \). For price-beating guarantees, ii) is satisfied iff some of the informed ask for refunds at firm \( i \) when it sets the monopoly price, \( G(p_i) \equiv F (\Delta S (p_i)) > 0 \) at \( p_i^m \) which is equivalent to assuming \( s < \Delta S (p_i^m) \). For price-matching guarantees, i) is satisfied iff some but not all consumers would ask for refunds at an LPG firm \( (s \leq \Delta S (p_i^m) \leq \bar{s}) \) since then \( \partial F / \partial (\Delta S) > 0 \) and \( \partial G(p_i) / \partial p_i > 0 \) because \( \partial G(p_i) / \partial p_i = (\partial F / \partial (\Delta S)) \cdot (\partial (\Delta S) / \partial p_i) \) and \( \partial (\Delta S) / \partial p_i = d(p_i) + (\beta - 1) d(p_i - R(p_i)) > 0 \) at \( p_i^m \). Assuming strict concavity of the profit function \( \Pi^G \) on \([c_i, p_i^m] \), the firm then sets a price below the monopoly price when it adopts LPG. Q.E.D.
Claim 4. The pro-competitive effect of LPG adoption diminishes as a larger number (percentage) of firms adopt LPGs.

Proof. An LPG firm maximizes the profits \( \Pi^G(p) = \frac{\lambda}{m} (\lambda \Pi(p) - G(p) \Delta(p)) \), where \( \lambda = \lambda(\alpha, m, n) = \frac{(1-\alpha) m}{\alpha n} + 1 > 1 \). Equivalently, the firm chooses \( p \) to maximize the normalized profits \( \pi(p) = \lambda \Pi(p) - G(p) \Delta(p) \). The optimal price can be defined as a solution to the first-order condition \( \frac{\partial \pi(p_i^G(m))}{\partial p} \equiv 0 \). Ignoring the integer problem, differentiate the equation with respect to parameter \( m \) and solve for \( \frac{\partial p_i^G(m)}{\partial m} \). This yields \( \frac{\partial p_i^G(m)}{\partial m} = -(\frac{\partial^2 \pi(p)/\partial p \partial m}{\partial^2 \pi(p)/\partial p^2})|_{p_i^G} \). The second-order condition – the concavity of \( \pi(p) \) – implies that the denominator is negative. Hence, the sign of the comparative statics effect of \( m \) is the same as the sign of the second-order partial derivative of the profit: \( \text{sign}(\partial p_i^G/\partial m) = \text{sign}(\partial^2 \pi/\partial p \partial m)|_{p_i^G} \). Differentiating \( \pi(p) \) with respect to \( m \) and then \( p \) yields \( (\partial^2 \pi/\partial p \partial m) = (\partial \lambda/\partial m) (\partial \Pi/\partial p) \). Since \( \partial \lambda/\partial m > 0 \) and \( \partial \Pi/\partial p > 0 \) for \( p < p_i^m \), it follows that \( \partial^2 \pi/\partial p \partial m > 0 \) and hence \( \partial p_i^G/\partial m > 0 \) at \( p_i^G < p_i^m \). Finally, note that for fixed \( n \), a larger number of LPG firms is equivalent to a higher percentage of firms adopting LPGs. Q.E.D.

Claim 5. Under (C1), the price of an LPG firm decreases in the proportion of the informed consumers, \( \alpha \), and the extent of price-beating, \( \beta \).

Proof. Consider an LPG firm that chooses \( p \) to maximize the normalized profits \( \pi(p) = \lambda \Pi(p) - G(p) \Delta(p) \), where \( \lambda = \frac{(1-\alpha) m}{\alpha n} + 1 > 1 \), \( G(p) = F(\Delta S(p)) \), \( \Delta = \Pi(p) - \Pi(\bar{p}) \), and \( \bar{p} = p - \beta (p - p_0) \). The proof of the first statement, \( \partial p_i^G/\partial \alpha < 0 \), is similar to that of Claim 4. Under (C1), \( p_i^G < p_i^m \) and \( \partial \Pi/\partial p > 0 \). From \( \partial \lambda/\partial \alpha < 0 \), it follows that \( (\partial^2 \pi/\partial p \partial \alpha) = (\partial \lambda/\partial \alpha) (\partial \Pi/\partial p) < 0 \) and hence \( \partial p_i^G/\partial \alpha < 0 \). Next, the sign of the comparative statics effect of \( \beta \) is the same as the sign of the second-order partial derivative
of profit: \( \text{sign}(\partial p^G_i/\partial \beta) = \text{sign}(\partial^2 \pi/\partial p \partial \beta) \). Differentiating the profit function \( \pi(p) \) with respect to \( \beta \) and then \( p \) yields \( (\partial^2 \pi/\partial p \partial \beta) = -\partial^2 (G \Delta)/\partial p \partial \beta \) where \( \partial^2 (G \Delta)/\partial p \partial \beta = (\partial^2 G/\partial p \partial \beta) \Delta + (\partial G/\partial \beta)(\partial \Delta/\partial p) + (\partial G/\partial p)(\partial \Delta/\partial \beta) + G (\partial^2 \Delta/\partial p \partial \beta) \). From Claim 1, \( p^G_i \leq p^m_i \). For such prices all the terms in the expression for \( (\partial^2 (G \Delta)/\partial p \partial \beta) \) are nonnegative, as straight-forward derivations can show. Hence, \( (\partial^2 \pi/\partial p \partial \beta) \leq 0 \) and \( \partial p^G_i/\partial \beta \leq 0 \). Finally, I show that the inequalities are strict under (C1) since then \( p^G_i < p^m_i \). Using prime to denote differentiation by \( p \), the first-order condition is \( \pi' = \lambda \Pi' - (G \Delta)' = 0 \). It implies that \( (G \Delta)' > 0 \) at \( p^G_i < p^m_i \) because \( \Pi' > 0 \) for \( p^G_i < p^m_i \). It follows that if \( p^G_i < p^m_i \), either \( G \) or \( G' \) are positive at \( p^G_i \). The firm that sets a strictly lower than the monopoly price is not the lowest-price firm, according to Claim 2. When \( p_0 < p^G_i \leq p^m_i \), \( (\partial \Delta/\partial \beta) = \Pi'(\tilde{p}) (p - p_0) > 0 \) and \( \partial^2 \Delta/\partial p \partial \beta = -\Pi''(\tilde{p}) (\beta - 1) (p - p_0) + \Pi'(\tilde{p}) > 0 \). Therefore, \( (\partial^2 \pi/\partial p \partial \beta) < 0 \) and \( \partial p^G_i/\partial \beta < 0 \) when \( p^G_i < p^m_i \). \textbf{Q.E.D.}

**Claim 6.** The price of an LPG firm increases in its marginal cost when the proportion of claimants is low and not very sensitive to changes in prices.

**Proof.** An LPG firm maximizes the normalized profits \( \pi(p) = (\lambda \Pi(p) - G(p)\Delta(p)) \) where \( \lambda = \frac{(1-\alpha)}{\alpha} m_0 + 1 \). Differentiating the profits by \( c_i \) yields \( \partial \Pi^G(p)/\partial c_i = (-\lambda d(p) - G(p) (d(\tilde{p}) - d(p))) \) since \( \partial \Pi/\partial c_i = -d(p) \) and \( \partial \Delta/\partial c_i = d(\tilde{p}) - d(p) \geq 0 \). Differentiate again with respect to \( p \) to obtain \( \partial^2 \Pi^G(p)/\partial p \partial c_i = \lambda \lambda d''(p) - G'(p) (d(\tilde{p}) - d(p)) - G(p) (-d'(p) - d'(\tilde{p}) (\beta - 1)) \) using \( \partial^2 \Delta(p)/\partial p \partial c_i = -d'(p) - d'(\tilde{p}) (\beta - 1) \). It follows that \( \partial^2 \pi/\partial p \partial c_i = (\lambda - G(p))(-d'(p) - G'(p) (d(\tilde{p}) - d(p)) - G(p) (-d'(p) - d(\tilde{p}))) (\beta - 1) \). The first term in the expression is positive while the second and the third are nonpositive. In general, the sign of \( \partial^2 \pi/\partial p \partial c_i \) (and hence the sign of \( \partial p^G_i/\partial c_i \)) is ambiguous but tends to be positive when the propensity to claim refunds
is relatively low and very sensitive to price changes. Q.E.D.

Claim 7. The benefit of LPG adoption is strictly decreasing with the production costs. There exists a cut-off level of the marginal costs such that only firms with lower marginal costs adopt LPGs. The lowest-cost firm always adopts LPG.

Proof. The lowest-cost is the lowest-price firm, according to Claim 2. It does not have to give any refunds, and it always benefits from offering LPG, $B > 0$. The benefit of offering an LPG decreases with the firm’s marginal cost, keeping the number of LPG firms fixed. From $B \equiv B(c, m) = \Pi^G(p_i^G) - \Pi^N(p_i^N)$, $(dB/dc_i)_{|p_i^G} = (\partial B/\partial c_i)_{|p_i^G} = (\partial \Pi^G/\partial c_i)_{|p_i^G} - (\partial \Pi^N/\partial c_i)_{|p_i^N}$.

Note that $\partial \Pi^G/\partial c_i = -d(p_i) \left( \frac{(1-\alpha)}{n} + \frac{\alpha}{m} \right) + \frac{\alpha}{m} G(p_i) (d(p_i) - d(p_i - R(p_i)))$ and $\partial \Pi^N/\partial c_i = -\frac{(1-\alpha)}{n} d(p_i)$. Hence, $(dB/dc_i)_{|p_i^G} = -d(p_i^G) \left( \frac{(1-\alpha)}{n} + \frac{\alpha}{m} \right) - \frac{\alpha}{m} G(p_i) (d(p_i^G) - R(p_i^G)) - d(p_i^G) + \frac{(1-\alpha)}{n} d(p_i^N)$. From $p_i^G \leq p_i^N$, $d(p_i^G) \geq d(p_i^N)$ and it follows that $(dB/dc_i)_{|p_i^G} < 0$ since $\alpha > 0$.

There exists a unique cut-off level in marginal cost, $\hat{c}$, such that for $c < \hat{c}$, $B(c, m) > 0$ and for $c > \hat{c}$, $B(c, m) < 0$. Firms with marginal costs $c_i < \hat{c}$ will adopt LPGs and other firms do not. Q.E.D.

Claim 8. LPG adoption is less attractive for firm $i$ when more firms adopt LPGs, assuming that (C1) holds if $i \neq 1$.

Proof. It needs to be shown that $B \equiv B(c_i, m)$ decreases in $m$; $dB/dm < 0$. Differentiation yields $dB/dm = - (\alpha/m^2) (\Pi(p_i) - G(p_i) \Delta(p_i))$, which is negative as long as the profits from the informed are positive at the optimal price $p_i^G$. The latter is true for firm $i \neq 1$ since under (C1), $p_i^G < p_i^m$. It is also true for firm 1. Q.E.D.

Proposition 1. There exists a unique equilibrium to the LPG game in which LPGs are offered. In the equilibrium, only $m^* \geq 1$ firms with the lowest marginal costs adopt LPGs,
\( m^* \equiv \max \{ m : B(c_m, m) \geq 0 \}. \) Under (C1), all LPG firms, except for the lowest-cost firm, charge below their monopoly prices and other firms charge the monopoly prices. LPG firms have uniformly lower prices than non-LPG firms, and the informed consumers choose to visit them. The informed claim refunds from LPG firms if their search costs are sufficiently low.

**Proof.** Suppose the informed consumers’ beliefs about prices of LPG and non-LPG firms are such that the informed are attracted to non-LPG firms. Then no firm offers LPG and the outcome is the same as when LPGs are banned. Suppose the informed are attracted to LPG firms. From Claims 7 and 8, there exists a unique \( m^* = \max \{ m : B(c_m, m) \geq 0 \} \) such that only firms 1 through \( m^* \) adopt LPGs.\(^\text{17}\) Under (C1), firm \( i \in \{2, ..., m^*\} \) sets a lower than monopoly price and other firms charge monopoly prices, according to Claims 2 and 3. Since LPG firms have lower costs and charge less than their monopoly prices, LPG firms set uniformly lower prices than non-LPG firms. The informed correctly believe LPG firms to have lower prices, and choose to visit them. Q.E.D.

\(^\text{17}\)Due to the integer problem, it is possible to have an equilibrium in which firm \((m^* + 1)\) adopts LPG and firm \(m^*\) does not, but allowing for such equilibria would not significantly affect the results.
Appendix B: The Informed Have A Perfectly Inelastic Demand

Appendix B deals with the case where demand functions of the informed and the uninformed are not the same: the informed have a perfectly inelastic demand for a unit of the product for prices below a choke price of $r$ and the demand for the uninformed is linear.

Denoting the demand of the uninformed by $d(p)$, the profits of firm $i$ are

$$\Pi_i^G(p_i) = (p_i - c_i) \left( \frac{1 - \alpha}{n} d(p_i) + \frac{\alpha}{m} \right) - \frac{\alpha}{m} R(p_i) F(R(p_i))$$  \tag{A1}$$

when it adopts LPG and sets $p_i \geq p_0$, where $p_0$ is the lowest market price. An LPG firm has to honor its guarantee on a fraction $F(R(p_i))$ of purchases by the informed, giving each of them a refund $R(p_i)$. As before, assume that $\Pi = \Pi(p_i) \equiv (p_i - c_i) d(p_i)$ is strictly concave.

The optimal price, $p_i^G$, satisfies the first-order condition,

$$\frac{\partial \Pi_i^G}{\partial p_i} \left( \frac{1 - \alpha}{n} \frac{\partial \Pi}{\partial p_i} + \frac{\alpha}{m} \left( 1 - \beta \left( F + R \frac{\partial F}{\partial R} \right) \right) \right) = 0$$  \tag{A2}$$

where $\beta$ is the refund percentage parameter; the arguments of functions are omitted to simplify the notations. The second-order condition is satisfied if the distribution of the informed consumers’ propensity to claim refunds is not too concave, $F''(R)/F'(R) > -2/R$. Assume that $F(\cdot)$ is such that the condition holds, which in fact is the case for many distribution functions.

In markets with no LPGs, firms have profits $\Pi_i^0(p_i) = \frac{1}{n} \Pi(p_i)$ and set monopoly prices $p_i^m$. A non-LPG firm operating in a market with at least one LPG firm has profits $\Pi_i^N(p_i) =$
\[ \frac{1 - \alpha}{n} \Pi(p_i) \] and it also sets its monopoly price \( p_i^N = p_i^m \). From equation (A2), an LPG firm sets a lower price when it has an LPG if \( \beta (F + R (\partial F \partial R)) > 1 \), i.e. when the refund parameter, \( \beta \), and the refund value, \( R \), are large, and the fraction of the informed asking for refunds, \( F \), is large and sensitive to changes in refund value. In what follows, assume that the distribution of search costs is uniform on \([0, s]\) and the demand for the uninformed is linear, \( d(p) = a - p_i \). For the uniform distribution of search costs, inequality \( \beta (F + R \frac{\partial F}{\partial R}) > 1 \) can be written as \( R > s / (2\beta) \). For a price-matching guarantee, a refund in excess of the average search cost leads to a lower price at the LPG firm. For a price-beating guarantee even a smaller refund would suffice. Denote \( \theta \equiv \frac{\alpha}{(1 - \alpha)^m} \) and \( \delta \equiv s / \beta^2 + \theta \). Claim A1 reveals that a sufficiently inefficient firm charges a lower price when it has an LPG.

**Claim A1 (Self-Effect).** *The equilibrium price of a firm is lower when it adopts an LPG than when it does not iff its marginal cost exceeds the marginal cost of the most efficient firm in the market by at least \( \delta \).*

The most efficient firm in the market always increases its price when it adopts an LPG. The opposite is true for relatively inefficient firms. The next claim provides a condition under which a larger number of LPGs reduces a price posted by an LPG firm.

**Claim A2 (External Effect).** *The equilibrium price of an LPG firm is lower when more firms adopt LPGs iff its marginal cost does not exceed the marginal cost of the most efficient firm in the market by more than \( \delta_1 \equiv \delta^2 / \beta^2 / s \).*

The comparison of \( \delta \) and \( \delta_1 \) reveals that \( \delta < \delta_1 \). Thus, there is a region of marginal costs for firm \( i \) such that both effects of LPGs are negative. An LPG firm with such costs would set a lower price when it adopts an LPG and when more of its rivals adopt LPGs. Even
when the effects work in the opposite directions, the overall effect may be pro-competitive.

A firm will adopt an LPG when its profit under an LPG is higher than that without an LPG. The difference in profits is monotonically decreasing in $c_i$. Claim A3 determines the critical level of marginal cost such that only firms with lower costs adopt LPGs.

**Claim A3. (Adoption).** A firm adopts an LPG iff its marginal cost does not exceed the marginal cost of the most efficient firm by more than $\delta_2$, $c_i - c_1 < \delta_2$, where $\delta_2 = \sqrt{\delta (\delta + (a - c_1 + \theta))} - \delta > 0$.

The comparison of Claims A1 and A2 with Claims 3 and 4 reveals that whereas in the benchmark model all but the lowest-cost LPG firms increase their prices, in the case of the perfectly inelastic demand for the informed and linear demand for the uninformed, only firms with sufficiently high costs reduce their prices. When a firm offers LPGs, it receives a larger share of the informed, and if their demand is more inelastic than the demand of the uninformed, the LPG adoption may impel the firm to increase its price unless its cost (and hence the cost of refunds) is high. As in the benchmark model, the attractiveness of LPG policy to a firm diminishes as the practice becomes more widespread. The equilibrium similar to the one outlined in Proposition 1 exists when the firms’ marginal costs are sufficiently asymmetric.