ABSTRACT

Sensitivity to scope has become an acid test for the validity of responses to non-market valuation scenarios. We examine the theoretical relationship between whether a subject’s responses exhibit sensitivity to scope and whether consistent preferences underlie those responses. We find that sensitivity to scope is neither necessary nor sufficient for preference consistency. Moreover, while continuous, strongly monotonic and total preferences yield scope, the converse fails to hold. Our results suggest caution in scope’s application as a survey validation tool.

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The Scope Test Revisited

I. INTRODUCTION

In its assessment of survey-based methods such as contingent valuation (CV) as a tool for natural resource damage assessment, the National Oceanic and Atmospheric Administration (NOAA) Blue-Ribbon Panel discusses the nature of preferences presumed to underlie survey responses. These range from the “...weakest form [requiring] certain kinds of consistency among choices made by individuals”, which entails underlying preferences are regular (i.e., reflexive, total and transitive),\(^1\) to more restrictive preferences which satisfy continuity and strict monotonicity in addition, as can be inferred from the following quote:

> [u]sually, though not always, it is reasonable to suppose that more of something regarded as good is better so long as an individual is not satiated. This is in general translated into a willingness to pay somewhat more for more of a good, as judged by the individual (both quotes from Arrow et al., 1993, p. 4604).

Based in part on this more restrictive presumption, the Panel recommended a number of conditions as burden-of-proof requirements, i.e., necessary conditions for survey validity, including “responsiveness to the scope of the environmental insult” (Arrow et al., 1993, p. 4614). Consequently, scope—the willingness to pay somewhat more for more of a good—has come to be “regarded by many as an acid test” of survey-derived values (Carson et al., 1996, p. 3).

The consumer preferences underlying the NOAA Panel’s deliberations were probably presumed to be regular, continuous, strongly monotonic and strictly convex, much as one might find in standard texts (e.g., Mas-Colell, Whinston and Green, 1995, pp. 41-45). Subsequent

\(^1\)Formal definitions are given in Section III.
advances in behavioral economics and recent experimental studies suggest that this standardized, restrictive presumption concerning a population’s underlying preferences may be unrealistic (see the discussion in Section II). A fuller understanding of scope’s relationship to the properties of preferences would be useful under such circumstances and is the focus of this note.

We analyze scope’s theoretical properties and find that scope is neither necessary nor sufficient for regular preferences (the NOAA Panel’s less restrictive notion of preferences). Moreover, preferences need be only continuous, strictly monotonic and total to yield scope; the converse, however, does not hold. Consequently, the importance of scope for testing empirical methods under either of the preference scenarios is called into question.

The remainder of this note is laid out as follows. In Sections II and III we provide background and formal definitions. Sections IV and V contain our analysis and a discussion of our findings.

II. BACKGROUND

The NOAA Panel convened in the midst of a vigorous debate focused in no small part on scope’s application. Some interpreted the failure of scope in empirical studies (e.g., Kahneman and Knetsch, 1992; Diamond et al., 1993; and Boyle et al., 1994) as evidence of generic failure of CV as a measurement tool (Kahneman and Knetsch, 1992) or of the inability of subjects to construct preferences for public goods (Diamond and Hausman, 1994; Kahneman, Ritov and Schkade, 1999).² Others (Harrison, 1992; V. K. Smith, 1992; Carson and Mitchell, 1993, 1995)

²For empirical purposes, responses are deemed to satisfy scope if values differ across different quantitative or qualitative levels of a good. For instance, the split sample scope test asks separate population groups to value differently-sized changes of a good. Imposing the
contended that insensitivity to scope arises from study-specific methodological shortcomings rather than any widespread lack of the more restrictive notion of preference or general methodological failure. Rollins and Lyke (1998) reconciled regular, continuous, strongly monotonic and strictly convex preferences with empirical violations of scope by considering the effects of initial endowment and sample size. However, their empirical findings also suggest some subjects would not willingly pay for any increment, raising the question of whether (1) the empirical study fails to capture preferences or (2) the preferences underlying these responses are regular, but not continuous or strictly monotonic, or (3) underlying preferences are inconsistent, i.e., either non-reflexive, non-total or non-transitive.

Recent experimental evidence suggests (2) may be the case. Andreoni and Miller (2002) tested 34 subjects’ responses for preference consistency (as measured by the Generalized Axiom of Revealed Preference) and also tested them for a form of monotonicity. Their findings suggest considerable heterogeneity among preferences, including cases in which subjects were consistent (i.e., regular), but not monotonic. Furthermore, statistical analysis suggests monotonicity does not lead to a greater likelihood of consistency. These findings bring into question the

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representative consumer assumption, the analyst pools subjects’ responses and estimates a willingness-to-pay (WTP) function. The test is considered satisfied if the coefficient of the quantity (or quality) variable is positive and significant (Carson, 1995).

3Assuming away measurement issues, such preferences yield responses which empirically fail scope when the initial level of the good in question is large (and consequently, the marginal utility of an increase is small) and the sample size is inadequate.

4“Respondents who check no...indicat[e] that they would vote no even if it cost their household nothing to create the new park....” (Rollins and Lyke, 1998, p. 341)

5Twenty-three subjects satisfied both monotonicity and consistency; two failed both; six satisfied consistency but not monotonicity; three satisfied monotonicity, but were inconsistent. A
presumption, implicit in the NOAA Panel’s listing of scope as a burden-of-proof requirement, that regular, continuous, and strictly monotonic preferences underlie survey responses.

III. DEFINITIONS

Let $\succeq$ denote weak preference, $\sim$ the symmetric part of $\succeq$ (i.e., indifference), and $\succ$ the asymmetric part of $\succeq$ (i.e., strict preference). Consider a world with two goods $a$ and $b$, where $a$ is the quantity or quality of a commodity and $b$ is a numéraire good. Definition 1 below is standard and follows Richter (1966):

Definition 1: A preference relation $\succeq$ on $\mathbb{R}^2$, is regular if it is (1) reflexive: for all $(a, b) \in \mathbb{R}^2_+$, $(a, b) \succeq (a, b)$; (2) total: for all $(a, b)$ and $(a', b')$ in $\mathbb{R}^2_+$ with $(a, b) \neq (a', b')$, either $(a, b) \succeq (a', b')$, or $(a', b') \succeq (a, b)$, or both, and (3) transitive: for all $(a, b), (a', b')$ and $(a'', b'')$ in $\mathbb{R}^2_+$, $(a, b) \succeq (a', b')$ and $(a', b') \succeq (a'', b'')$ imply $(a, b) \succeq (a'', b'')$.

We identify consistent preferences with regular preferences.

In addition to the properties required for regularity, additional restrictions such as continuity and various forms of monotonicity and convexity are frequently imposed on consistent preferences (e.g., Mas-Colell, Whinston and Green, 1995, pp. 41-45). Definition 2 characterizes monotonic and strongly monotonic preferences, and Definitions 3 and 4 characterize continuous one-tailed Chi-squared test fails to reject the null that subjects with monotonic choice are as or less likely to be consistent than non-monotonic ones at the 0.1 level of significance.

Consistent, non-monotonic choice was most frequently associated with the decision-maker receiving a minority of the overall gains her decisions yielded; consequently, Andreoni and Miller termed the underlying preferences as “jealous” ones. In the context of environmental valuation such sentiments could be expressed as, “Yes, I’d like to see more South American rainforest preserved, but why should I pay for it? Shouldn’t the locals pitch in?”
and strictly convex preferences, respectively.

Definition 2: A preference relation $\succeq$ on $\mathbb{R}^2_+$ is (1) monotonic if for all $(a, b) \succ (a', b') \in \mathbb{R}^2_+$, $(a, b) \succ (a', b')$, and (2) strongly monotonic if for all $(a, b) \succeq (a', b') \in \mathbb{R}^2_+$ with $(a, b) \neq (a', b')$, $(a, b) \succ (a', b')$.

Definition 3: A preference relation $\succeq$ on $\mathbb{R}^2_+$ is continuous if for any point $(a, b)$, every upper contour set $\{(a', b'): (a', b') \succeq (a, b)\}$ and every lower contour set $\{(a', b'): (a, b) \succeq (a', b')\}$ is closed in $\mathbb{R}^2_+$.

Definition 4: A preference relation $\succeq$ on $\mathbb{R}^2_+$ is strictly convex if for any point $(a, b)$ every upper contour set $\{(a', b'): (a', b') \succeq (a, b)\}$ is a strictly convex set in $\mathbb{R}^2_+$.

Finally, let an individual endowed with bundle $(a, b)$ be asked to value an increment $\delta > 0$ in $a$. We offer Definition 5 as a formal definition of scope in the spirit of the NOAA Panel’s report.

Definition 5: A preference relation $\succeq$ on $\mathbb{R}^2_+$ satisfies scope if for all $(a, b) \in \mathbb{R}^2_+$ and any $\delta > 0$, either (1) there exists a unique $\epsilon > 0$ such that $(a + \delta, b - \epsilon) \succeq (a, b)$ where $b - \epsilon \geq 0$, or (2) $(a + \delta, 0) \succ (a, b)$.

Definition 5 attempts to capture the notion of scope at the level of preferences as follows. Starting from any bundle $(a, b)$, if $a$ is increased by $\delta$, either there is an amount $\epsilon$ of the numéraire good that can be taken away to leave the consumer indifferent between the bundles $(a + \delta, b - \epsilon)$ and $(a, b)$ or the consumer never reaches indifference even when her entire endowment of the numéraire is taken away. In the first case, $\epsilon \leq b$ may be interpreted as her willingness to pay in terms of the numéraire good for the $\delta$ increment in $a$. In the second case, she must prefer the resulting bundle $(a + \delta, 0)$ to $(a, b)$.

Definition 5 implies that if a consumer is indifferent between two bundles, then one
bundle must lie to the southeast (or northwest) of the other in $\mathbb{R}^2_+$. It rules out thick indifference curves (i.e., local satiation) as well as preferences with horizontal indifference curve segments.

IV. ANALYSIS

In this section, we show that scope (as captured by Definition 5) is neither necessary nor sufficient for regular preferences. To show that scope is not necessary, consider a consumer whose preferences are represented by a utility function given by $U(a, b) = \min\{a + b, 2b\}$. The expansion path of this utility lies along a 45° line through the origin; a typical indifference curve is piecewise linear with slope -1 above the 45° line and slope zero below. Pick any bundle, $(a, b)$ where $a$ lies on or below the 45° line. Since the indifference curve through the bundle $(a, b)$ is horizontal for any increment $\delta$ of $a$, Definition 5 is violated. But because the preferences of this consumer are represented by a utility function, her preferences are regular. Hence regular preferences do not guarantee scope. More generally, any preferences with horizontal indifference curve segments in some region of the commodity space will fail scope if measured along the flat segment. Such a case could arise when a consumer valuing the survival of a bird species attaches a positive value to marginal population increments up to a survival threshold, but does not value marginal increases above the threshold (when survival is assured).

To show that scope is not sufficient for regular preferences, consider the following counterexample. Define a preference relation $\succeq$ on $\mathbb{R}^2_+$ as given by the following rules:

Rule 1: \[(a, b) \sim (a', b') \text{ if and only if } a + b = a' + b',\]

Note that this counterexample is monotonic (Definition 2.1), but not strongly monotonic (Definition 2.2).
Rule 2: \((a', 0) > (a, b)\) if and only if \(a' > a + b\), and

Rule 3: \((a, b') > (a, b)\) if and only if \(b' > b\).

Rule 1 says that two bundles are indifferent if each bundle has the same number of total units of 
\(a\) and \(b\). In this case, indifference sets are well-defined (\(a\) and \(b\) are perfect substitutes), but it is 
not possible to compare across indifference sets. Rule 2 says that if a bundle contains positive 
amounts of \(a\) but none of \(b\), then this bundle is strictly preferred to any bundle where the total 
units of \(a\) and \(b\) are fewer. One consequence of Rule 2 is that given two bundles on the 
horizontal axis, the one to the right is preferred. Rule 3 implies a consumer prefers strictly more 
of the numéraire to less.

It follows that this preference relation is reflexive: any bundle \((a, b)\) is at least as good as 
itself by Rule 1. However, this relation is not total: for example, the distinct bundles \((3, 2)\) and 
\((2, 2)\) cannot be compared using either Rules 1, 2 or 3. Neither is this preference relation 
transitive. To see this, consider the bundles \((3, 2)\), \((5, 0)\) and \((2, 2)\). It is true that \((3, 2) \succeq (5, 0)\) 
by Rule 1 and \((5, 0) \succeq (2, 2)\) by Rule 2; however, \((3, 2)\) and \((2, 2)\) are not comparable, so it is not 
possible to conclude that \((3, 2) \succeq (2, 2)\) as transitivity would require.

It remains to be shown that this preference relation satisfies scope. Suppose a bundle \((a, b)\) is such that \(a \geq 0, b > 0\), and let \(\delta > 0\). So long as \(\delta \leq b\), there is a unique \(\epsilon = b - \delta\) such that 
\((a + \delta, \epsilon) \sim (a, b)\) by Rule 1. If \(\delta > b\), then the bundle \((a + \delta, 0) > (a, b)\) by Rule 2. In either 
case, scope (as given by Definition 5) is satisfied. Thus, scope is neither necessary nor sufficient 
for preference consistency.

A question of interest is what properties yield scope. Let \(\succeq\) be continuous, strongly 
monotonic and total. Beginning with an initial endowment \((a, b)\) where \(a \geq 0\) and \(b > 0\), an
increment $\delta > 0$ yields a bundle $(a + \delta, b)$ which is strictly preferred to $(a, b)$ by strong monotonicity. This property also implies for a small enough $\epsilon$ ($0 < \epsilon < b$) that $(a + \delta, b) \succ (a + \delta, b - \epsilon)$. By continuity and totality, $\epsilon$ can be increased to yield either (1) a bundle which is as desirable as $(a, b)$, or (2) the endowment of the numéraire, $b$, is exhausted before reaching indifference, in which case $(a + \delta, 0) \succ (a, b)$. Thus, continuity, strong monotonicity and totality imply scope as defined in Definition 5. The converse, however, does not hold, as shown by the sufficiency counterexample above which satisfies scope but neither strong monotonicity nor totality (e.g., it is not possible to establish $(3, 2) \succ (2, 2)$ or otherwise order them).

V. DISCUSSION

In this note we have shown that scope is neither necessary nor sufficient for consistent preferences; moreover, while continuous, strictly monotonic and total preferences imply scope, the converse does not hold. The usefulness of these findings lies in the dichotomy between the heterogeneous nature of preferences Andreoni and Miller uncovered and the representative agent with regular, continuous, strongly monotonic and strictly convex preferences which appear to underlie the NOAA Panel’s recommendations. For scope to achieve its intended purpose as a validity test of a given measurement tool, the analyst must assume and subjects must possess preferences which are at a minimum continuous, strongly monotonic and total.

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7Note that reflexivity and transitivity, the two other properties required for regularity, are not needed.

8If the analyst’s expectations include consistency and diminishing marginal WTP, then reflexivity, transitivity and strict convexity (guaranteeing a strictly diminishing marginal rate of substitution) would also be required.
Divergence between the form of preferences the analyst assumes and subjects possess presents a confound which may limit scope’s usefulness as an acid test. For example, a subject with preferences which are consistent, but not strongly monotonic (e.g., Andreoni and Miller’s “jealous preferences”) would not report a positive marginal WTP for an increment of a commodity. Such responses would fail scope at the individual level or be classified as protest zeros (Mitchell and Carson, 1989). In either case, the analyst most likely deletes such responses from her data set. Assuming the analyst’s objective is to obtain preference-consistent data, these inappropriate deletions would upwardly bias estimates of resource damages. For another example, consider the analyst whose sample consists largely of subjects with regular but non-monotonic preferences. She regresses the quantity of the environmental good on WTP, fails to reject the hypothesis her estimated coefficient exceeds zero, and incorrectly concludes her measurement tool has failed. Both examples, while somewhat overstating the case for emphasis, highlight the dangers employing scope poses when the structure of preferences differs from the one scope presumes.

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REFERENCES


