Tests of the Random Walk Hypothesis and Long-Run Dependence on the Johannesburg Stock Exchange

Asst. Prof. Owen Beelders*  
Department of Economics  
Emory University  
Atlanta Ga 30322-2240  
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Abstract

We test for predictability and long-run dependence in four broad South African Stock Indexes using the variance ratio test, two tests for fractional differencing and the modified rescaled range statistic. For the All Share Index and the Gold Index we fail to reject the null of the random walk model. For the Industrial Index, we can reject the null hypothesis of a random walk using the variance ratio test at the 4 and 8-week horizon, but we find no evidence of long-run dependence using the test for fractional differencing or the rescaled range test. For the Financial Index, we find evidence of long-run dependence using a test developed by Newbold and Agiakloglou (1994), but this may be due to the thin trading of these stocks. In conclusion, there is little evidence to suggest that there is long-run dependence on the Johannesburg Stock Exchange.

JEL Classification Number: G14, G15  
Key words: Random Walk, Variance Ratio, Predictability

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1 Introduction

Two competing schools of thought have arisen in the debate about market efficiency. One posits that markets are efficient and returns are unpredictable. The other draws on work by Fama and French (1988) and Poterba and Summers (1988) who find that returns are predictable at longer time horizons. In fact, the evidence seems to point to the fact that in short-run there is a lot of noise or volatility in stock returns, but at longer horizons returns this washes out and returns are predictable.\(^1\) Two competing statistical models are used to promote these two schools of thought. The random walk model, as popularized by Fama (1965) and Malkiel (1973), is the main work horse of the supporters of efficient markets. The random walk model of stock prices implies that the best forecast of any future price is the current price, price changes and returns are unpredictable at any horizon and forecast standard errors grow linearly with the horizon. One drawback of the random walk model is the assumption that price changes are independent; this is untestable. Since the publication of Harrison and Kreps (1979), the random walk hypotheses has been superceded by the martingale hypothesis which maintains the three implications above, but replaces the independence of price changes with that of uncorrelated price changes. The martingale hypothesis introduces an additional testable hypothesis of no short-run dependence, i.e. stock price changes or returns are not serially correlated, and permits the modelling of higher order dependence such as conditional heteroskedasticity.

Researchers who support the notion of long-run dependence have adopted the model of fractional differencing for its purposes. Fractional ARIMA models display long-run dependence that is consistent with predictability at the longer horizons that Fama and French (1988) and others have found. The basic motivation for fractional integration is aggregation within the error-duration model where the observed process is an aggregate of shocks with a stochastic magnitude and stochastic duration (Parke (1999)). This motivation is consistent with the notion that the long horizon asset returns are an aggregate of information shocks of different magnitudes and duration. The magnitude can be determined by the relative importance of the shock to the company or economy and the duration can be determined by the protracted nature of the information revelation. For example, the litigation against Microsoft has a material impact on the how the company will operate in the future and the duration of the shock is determined by the length of the proceedings and negotiation. Recently, Diebold and Inoue (1999) have shown that a small amount of regime switching may be observationally equivalent to the observation of fractional differencing in economic data. Thus we are likely to find evidence of fractional integration in stock markets that have experienced regime switching or structural changes.

Both short- and long-run dependence have consequences in financial modelling. First, the underlying models for valuing derivatives exclude the possibil-

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\(^1\) For example, Harrison and Zhang (1999) find that the risk-return relationship only appears at two-year holding intervals because noise tends to dominate returns at shorter horizons.
ity of any form of dependence in returns. Second, statistical tests of market efficiency and other hypotheses derived from financial theory are constructed under the assumption that the null is correct. If these assumptions are incorrect, the usual forms of statistical inference are no longer valid and the joint hypothesis problem rears its head.

Because of the noise in asset returns and the presence of serial correlation due to thin trading, greater focus has been placed on testing for long-run dependence. The objective of this paper is to test the two competing theories in the context of broad stock indexes on the Johannesburg Stock Exchange (JSE). Section 2 contains a brief literature survey of results for other countries. In section 3 we describe our data and discuss the descriptive statistics. In Section 4, 5 and 6 we test the two competing hypotheses by applying the variance ratio test of Lo and MacKinley, two tests for fractional differencing developed independently by Geweke and Porter-Hudak (1983) and Newbold and Akiaikloglou (1994) and the modified rescaled-range statistic of Lo (1991), respectively. We conclude with section 7.

2 Literature Review

To date there has been little research on market efficiency on the JSE. A\textsuperscript{2} exception is Lo and Wang (1995) who consider the implementation of option pricing models when returns are predictable.

An exception in the literature is Lo and Wang (1995) who consider the implementation of option pricing models when returns are predictable.
Urrutia (1995) examined the South and Central American markets of Argentina, Brazil, Chile and Mexico and found evidence against the random walk model. However, he did not use the Chow and Denning (1993) critical values that control the level of significance when there are multiple comparisons. Using the correct critical values, Ojah and Karemera (1999) fail to reject the random walk hypothesis for these markets and nd no evidence of fractional differencing. A runs test detects a lack of weak form efficiency for the Chilean market, but this is only at short horizons. Frennberg and Hannson (1993) reject the random walk model for Swedish stock prices because the variance ratios are statistically greater than 1 at lags of less than 24 months, however, a caveat to their analysis is that they do not use the Chow and Denning critical values. Ayadi and Pyun (1994) test the random walk model using variance ratios in the Korean stock market. Although this market is also characterized by thin trading they do not nd evidence to reject the random walk model. In conclusions, there is not much evidence against the random walk hypothesis when the Chow and Denning (1993) critical values are used.

3 Data and Descriptive Statistics

We focus on the broad, market index and three broad sector indexes that represent the largest percentage of market capitalization, i.e. the All Share Index (ALSI), the Industrial Index (INDI), the Gold Index (GLDI) and the Financial Index (FINI). The weekly data for the ALSI, INDI, GLDI and FINI indexes were obtained from Datastream for the period January 1988 to December 1999. The indexes are value-weighted and are adjusted for mergers, de-listings, but not dividends.

Returns are computed as the natural log of the price relative, i.e. \( R_t = 100 \cdot \ln(\frac{P_t}{P_{t-1}}) \) where \( P_t \) is the price index at time \( t \).

The purpose of this section is to compare the descriptive statistics of the indexes, to look for deviations from normality in the unconditional distribution and to determine the amount of dependence in the rst and second conditional moments. The descriptive statistics are reported in Table 1 and include tests of skewness and excess kurtosis based on the GMM estimator and the traditional estimator that is derived under the assumption of iid returns. We include tests based on the GMM estimator because the traditional estimator suffers from the drawback that its variance is underestimated when returns are non-normal and conditionally heteroskedastic (Pagan (1996)).

Except for the Gold Index, the mean returns of all the broad indexes are statistically different from zero. There are a number of possible reasons for this: rst, we are analyzing returns de ned as log price relatives and not excess returns so the non-zero mean may reflect the non-zero riskless rate of return under a no-arbitrage equilibrium. Second, during the sample period, the rate of ination and the risk-free rate was in excess of 10% per annum and this may be

\footnote{Detailed information about the index construction is available at the website of the JSE, http://www.jse.co.za/ .}

\footnote{Someone said that dividend adjustment does not matter - what is the reference?}
reflected in the raw returns of the indexes. Finally, the statistical significance may be as a result of the large sample size of 2993 observations.

Turning to the second moment, the Gold Index is approximately four times more volatile than the other three indexes. The difference in volatility is reflected in Figure 1 where we plot the time series of returns for the four broad indexes on the same axes. The Gold Index also displays its uniqueness in the third moment: in contrast to the other indexes, the Gold Index has a positive coefficient of skewness although it is not statistically significant. The importance of testing for skewness and kurtosis using the GMM estimator is apparent from the huge difference in magnitude of the test statistics. For example, the Gold Index is negatively skewed and has a t-statistic of 4.283 under the assumption of iid returns whereas the test statistic is only 0.002 when the iid assumption is relaxed. Only the All Share and Industrial Indexes display statistically significant negative skewness, but all four indexes display excess kurtosis relative to the normal distribution that is significant at the 1% level.

The first five autocorrelations are reported in Table 1. The JSE is known for its low turnover ratio and thin trading so we expect to see some short-run dependence. Except for the Gold Index, the first autocorrelation is statistically significant although this may be due to conditional heteroskedasticity (Diebold (1986)). The first autocorrelation of the Financial Index is the largest of all the indexes because it contains stocks that suffer most from thin trading. The Gold Index is least likely to suffer from thin trading because the gold stocks are highly traded by domestic and foreign investors. The Ljung-Box Q-statistics confirm the presence of serial correlation in all the indexes and are statistically significant at the 1% level at both 5 and 10 lags. The Q-statistics of the squared returns are also statistically significant at the 1% level at both 5 and 10 lags. The presence of conditional heteroskedasticity is consistent with the excess kurtosis in returns.

In conclusion, the gold sector is different to the other sectors in that it is more volatile and has a symmetric distribution. All the indexes display excess kurtosis that may be due to the presence of conditional heteroskedasticity or leptokurtosis in the underlying distribution. There is significant time dependence in the returns of the indexes especially at the first lag and the serial correlation appears to be related to thin trading.

4 The Variance Ratio Test

Under the null hypothesis that a stock price, $P_t$, follows a random walk, the variance of the k-horizon return is proportional to k, i.e. if we define the k-th horizon return as $R_t(k) = 100 \Delta \ln(P_{t+k} = P_t)$, then its variance satisfies the relationship,

$$\text{Var}(R_t(k)) = k \cdot \text{Var}(R_t(1))$$

These results are broadly consistent with the descriptive statistics of daily returns in Beelders (2000) although the deviations from normality for the weekly returns are not as extreme. The idiosyncrasy of the gold index is still present in the weekly returns.
We define the k-horizon variance ratio as
\[
VR(k) = \frac{\text{Var}(R_t(k))}{k \cdot \text{Var}(R_t(1))}.
\] (2)

Under the null hypothesis, the variance ratio equals 1 at all horizons for \( k > 1 \). Several studies have found variance ratios greater than one at horizons of less than one year and variance ratios less than one at horizons greater than one year. This is consistent with investor over-reaction (positive serial correlation) in the short-run and mean-reversion (negative serial correlation) in the long run.

Following Campbell, Lo and McKinlay (1997), we define the q-horizon variance ratio as
\[
VR(q) = \frac{\hat{\sigma}_q^2}{\hat{\sigma}_1^2}.
\] (3)

where the unbiased estimators \( \hat{\sigma}_q^2 \) and \( \hat{\sigma}_1^2 \) are defined as
\[
\hat{\sigma}_q^2 = \frac{1}{nq} \sum_{k=1}^{nq} (R_t(1) - \mu)^2
\] (4)
\[
\hat{\sigma}_1^2 = \frac{1}{nq} \sum_{k=1}^{nq} (R_t(q) - \mu q)^2.
\]

where \( n \) is the number of non-overlapping sub-samples of length \( q \). Under the assumption that the returns are serially uncorrelated and homoskedastic,
\[
Z_1(q) \sim \text{P} \left( \frac{\hat{\sigma}_q^2(1)}{\hat{\sigma}_1^2} \right) \sim \text{N}(0, 1).
\] (5)

Under the assumption that the returns are serially uncorrelated and heteroskedastic, the heteroskedasticity consistent variance estimator of \( VR(q) \) is
\[
\mu(q) \sim \text{P} \left( \frac{1}{4} \sum_{j=1}^{nq} \frac{k^2}{q} b_k \right)
\] (6)

where
\[
b_k = \frac{nq \sum_{j=k+1}^{nq} (R_t(j) - \hat{\mu}_j)^2}{\sum_{j=1}^{nq} (R_t(j) - \hat{\mu}_j)^2}.
\] (7)

The test statistic for a null of a random walk is
\[
Z_2(q) \sim \text{P} \left( \frac{\hat{\sigma}_q^2(1)}{\mu(q)} \right) \sim \text{N}(0, 1).
\] (8)
By recognizing that the application of the variance ratio of Lo and McKinley (1988) involves multiple comparisons, i.e. at each horizon we conduct a hypothesis test, Chow and Denning (1993) obtain a bound for the overall size of the test. For a sequence of $m$ horizons we can obtain the test statistics for the variance ratios,

$$Z(q_1); \ldots; Z(q_n);$$

where the $q_i$ are the horizons that satisfy the condition $q_1(=2) < q_2 < \ldots < q_n \cdot N=2$. We define $Z^\nu(q)$ as the largest of the absolute values of the test statistics, i.e.

$$Z^\nu(q) = \max_j Z(q_j); \ldots; jZ(q_n);$$

The $(1 - \alpha)\%$ confidence interval for $Z^\nu(q)$ is defined as

$$Z^\nu(q) \pm SM_M(\alpha; m; 1)$$

where $SM_M(\alpha; m; 1)$ is the asymptotic critical value of the Studentized maximum modulus (SMM) distribution with parameter $m$ and degrees of freedom 1. The critical value is obtained from the normal distribution by the equality, $SM_M(\alpha; m; 1) = Z^{\nu=2}$, where $Z^{\nu=1} = 1 \cdot (1 \cdot \alpha)^{1-m}$. Under the assumption that the returns are uncorrelated and homoskedastic, the asymptotic confidence interval for each variance ratio is given by

$$\frac{P}{\bar{P}}\{VR(q) \mid 1\} \pm (2(2q \mid 1) = \exists q)^{1-m} \cdot SM_M(\alpha; m; 1) \quad i = 1; \ldots; m$$

and under the assumption that the returns are uncorrelated and heteroskedastic the asymptotic confidence interval for each variance ratio is given by

$$\frac{P}{\bar{P}}\{VR(q) \mid 1\} \pm \mu^{1-m} \cdot SM_M(\alpha; m; 1) \quad i = 1; \ldots; m$$

The results of the variance ratio tests for both the homoskedastic and heteroskedastic consistent standard errors are reported in Table 2. We focus on the tests based on the heteroskedastic consistent standard errors. It is interesting to note that only the Gold Index has variance ratios that are less than 1. In fact we expect to find mean reversion in the Gold Index because the main driving force behind this index is a commodity price, the gold price, which is typically modelled as a mean reverting process. We reject the null of a random walk for the All Share Index at 4 and 8 weeks at the 10% level of significance, but none of the test statistics are significant when we use the Chow and Denning critical values. Contrary to our expectations, we fail to reject the null of a random walk for the Gold Index at all lags. We reject the null of a random walk for the Industrial Index at 4 and 8 week lags at the 1% level of significance and for 2 and 16 weeks at the 10% level using the Chow and Denning critical values. Finally, we reject the null of a random walk for the Industrial and Financial Indexes at 2 week lag using the Chow and Denning critical
values at the 1% level of significance under the assumption of homoskedastic
returns, but we cannot reject the null when we use the heteroskedastic errors. 
This extreme rejection at 2 weeks is consistent with evidence of a high degree 
of positive serial correlation and conditional heteroskedasticity in the returns of 
these two indexes and underscores the importance of using the heteroskedastic 
consistent standard errors. We reject the null of a random walk at a lag of 4 
weeks when using the heteroskedastic consistent standard errors. In conclusion, 
there is little evidence to reject the null hypothesis of a random walk except for 
the Industrial Index.

5 Fractional Differencing

Fractionally integrated processes display long-run dependence and are a feasible 
alternative to difference stationary models because they display the typical 
spectral shape characterized by Granger (1966). An autoregressive fractionally 
integrated and moving average process (ARFIMA(p; δ; q)) is denoted by

\[ \hat{A}(L) (1 - L)^d P_t = \mu(L)^" \]

where \( j d < 1; L \) is the lag operator, \( \hat{A}(L) \) is AR polynomial and \( \mu(L) \) is the 
MA polynomial. Asset prices are weakly stationary if \( d < 0.5 \), invertible for 
\( d > -0.5 \) and nonstationary for \( d > 0.5 \). The intuition behind the long-run 
dependence of the process is derived from the behavior of the spectral density 
at frequency zero; it behaves like \( \lambda^d \) as \( \lambda \) (the frequency) converges to zero. 
Thus the typical shape of the spectral density can be captured by \( 0 < d < 1 \). 
In practice, unit root tests have very low power against fractional alternatives 
so we may be differencing too much and not taking into account long-range 
dependence.

Geweke and Porter-Hudak (1983) have developed a test for fractional differencing 
using the spectral regression,

\[ \ln(I(\lambda_j)) = \ln(\lambda_j) + \ln(\lambda_j) - 2 + \lambda_j^2 \] 

where \( \lambda_j = 2\frac{j}{T} \) for \( j = 1; 2; \cdots; n \), is the harmonic ordinate, the sample size or 
the number of periodogram ordinates \( n = g(T) \) \( T \); \( I(\lambda_j) \) is the periodogram 
of returns at frequency \( \lambda_j \) and \( \lambda_j^2 \) is an error term. Geweke and Porter-Hudak 
suggest that the best choice of \( n \) is given by \( g(T) = T^{0.5} \). The null hypothesis 
is \( d = 0 \), i.e. the process is a random walk. Under the null, the standard error 
of \( d \) is \( \frac{1}{2} T^{0.5} \) and we can compute a t-statistic to test the null hypothesis against 
the alternative of fractional differencing.

The estimated values of \( d \) for the four indexes are reported in Table 3a for 
\( @ = 0.45; 0.5; 0.55 \) and \( n = T^{0.5} \). We fail to reject the null hypothesis of no 
long-run dependence for each of the indexes at every value of \( @ \) i.e. we have no 
evidence to reject the null hypothesis of a random walk.

To test the robustness of the Geweke and Porter-Hudak test, we also implement 
a test for fractional differencing developed by Akaike and Newbold
For the fitted ARIMA \((p; q)\) model of returns, \(\hat{R}(L)R_t = \hat{B}(L)b\), Akiakloglou and Newbold have developed an LM test for the hypothesis that \(d = 0\) as the t-test of \(\pm\) in the regression,

\[
\hat{b}_t = \sum_{i=1}^{p} \phi_i W_{t-i} + \sum_{j=1}^{q} \theta_j Z_{t-j} + \dfrac{1}{2}K_t(m) + u_t
\]

where

\[
\hat{R}(L)W_t = R_t, \quad \hat{R}(L)Z_t = \hat{b}_t; \quad K_t(m) = \sum_{j=1}^{m} \hat{b}_{t-j}.
\]

Newbold and Akiakloglou nd that the test has empirical size close to nominal size, but low power when \(p\) and \(q\) are not zero and the process has a non-zero mean. They also nd that the sample autocorrelation function is severely biased in small samples so it is very difficult to detect fractional differencing in small samples.

We estimate AR(5) models for the returns of the four indexes thus setting \(p = 5\) and \(q = 0\) and saving the residuals as \(\hat{b}_t\). We construct \(K_t(m)\) for \(m = 3; 4; 5\) and report the results of (16) in Table 3b. We cannot reject the null of no long-run dependence for the All Share and Industrial indexes although the estimates of \(\pm\) fluctuate wildly across \(m\). The estimates of \(\pm\) for the Gold Index are quite uniform across \(m\) and range between 0.311 and 0.400, but we also fail to reject the null hypothesis of no long-run dependence. We reject the null of no long-run dependence for the Financial Index for all values of \(m\). The rejection of the null hypothesis may be due to the thin trading of stocks in this index and it is consistent with the high level of serial correlation of the returns of the index.

The overwhelming support for the random walk hypothesis is surprising for two reasons: rst, Diebold and Inoue (1999) nd that evidence in support of fractional differencing may be due to a small amount of regime changes in a series. Second, South Africa was immersed in economic and socio economic turmoil during the sample period and there were a number of regime changes during the latter part of the sample period (Beelders (2000)). Despite the economic milieu, we only nd evidence of fractional differencing in the Financial Index and this may be due to the thin trading of these stocks.

6 The Modified Rescaled Range Statistic

The rescaled range (R/S) statistic was developed for the purpose of detecting long-range dependence and has certain robustness properties that make it better suited than autocorrelations, variance ratios or spectral analysis. Mandelbrot and Wallis (1969) show that the R/S statistic can detect long-range dependence in time series that are non-Gaussian and have skewness and excess kurtosis. This is clearly an advantage for financial data where the distributions tend
be leptokurtic. Second, Mandelbrot (1972, 1975) shows that the R/S statistic converges almost surely even if the process has infinite variance; this does not hold for the autocorrelations or variance ratios. Finally, Mandelbrot (1972) argues that the R/S statistic can detect non-periodic cycles with periods greater than the sample size; this is a distinct advantage over spectral analysis.

Lo (1991) develops the modified R/S statistic that is robust to short-run dependence by using an estimate of the long-run standard deviation to standardize the statistic. Lo finds that the modified R/S statistic has less bias and has better size. For example, for the alternative of a fractionally differenced process, the probability that the modified R/S statistic rejects the null of no long-range dependence converges to 1. Using the CRSP equally- and value-weighted indexes Lo finds that there is no evidence of long-run dependence, but the data are consistent with a short-memory process.

The modified R/S statistic is calculated as follows:

\[ Q_n = \frac{1}{n} \sum_{j=1}^{j} \left( R_j - R_n \right) \]

where

\[ b^2_{(q)} = b^2 + 2 \sum_{j=1}^{q} \left( b^2_{(q)j} \right) \]

and \( n \) is the sample size. The key modification of the R/S is the use of a consistent estimator of the variance of the partial sums under the assumption that there is short-run dependence in the data, i.e. use \( b^2_{(q)} \) instead of \( b^2 \) in computing the R/S test statistic. The estimated values of the R/S and modified R/S test statistics are reported in Table 4. We fail to reject the null hypothesis of no long-run dependence for each of the four indexes. This result is consistent with the variance ratio tests where we found little long-run dependence. We accept our results with a note of caution because Pagan (1996) has noted that the choice of \( q \) is an unresolved issue, i.e. how do we distinguish between the short- and long-run (Mills (1999), p.118). There is also evidence that the distribution of the test statistic is sensitive to fat-tailed distributions, thus Lo suggest that the R/S test should be regarded as a portmanteau test before conducting a more thorough analysis of long-run dependence. Keeping these caveats in mind, we conclude that there is no evidence to suggest the presence of long-run dependence in the JSE Indexes.
7 Conclusion

We have used the variance ratio test, two tests for fractional differencing and the modified rescaled range statistic to test for long-run dependence in the returns of four broad South African indexes. For the All Share Index and the Gold Index we fail to reject the null of the random walk model. This is particularly surprising for the Gold Index where we expect to find evidence of mean reversion. For the Industrial Index, we can reject the null hypothesis of a random walk using the variance ratio test at the 4 and 8-week horizon, but we find no evidence of long-run dependence using the test for fractional differencing or the rescaled range test. For the Financial Index, we find evidence of long-run dependence using the Newbold and Agiakloglou (1994) test, but this may be due to the thin trading of the stocks in this index. In conclusion, there is little evidence to suggest that the JSE is not efficient despite the economic turmoil surrounding South Africa during the sample period and the only rejection of the hypothesis of efficiency may be due to thin trading.

References


Table 1: Descriptive Statistics of the Weekly Returns of the JSE Indexes

A superscript a, b and c denotes the statistical significance at the 10%, 5% and 1% levels of significance, respectively. $Q_R (k)$ denotes the Box-Ljung Q-statistic that is computed for the returns using k lags and $Q_{R^2}(k)$ denotes the Box-Ljung Q-statistic that is computed for the squared returns using k lags.

<table>
<thead>
<tr>
<th></th>
<th>All Share</th>
<th>Gold</th>
<th>Industrial</th>
<th>Financial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Size</td>
<td>606</td>
<td>606</td>
<td>606</td>
<td>606</td>
</tr>
<tr>
<td>Mean</td>
<td>0.109(^b)</td>
<td>-0.030</td>
<td>0.127(^b)</td>
<td>0.159(^b)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.272</td>
<td>5.570</td>
<td>1.497</td>
<td>1.494</td>
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<tr>
<td>Skewness</td>
<td>-0.797</td>
<td>0.427</td>
<td>-0.525</td>
<td>-1.678</td>
</tr>
<tr>
<td>iid t-statistic</td>
<td>(-7.987)^c</td>
<td>(4.283)^c</td>
<td>(-5.272)^c</td>
<td>(-16.825)^c</td>
</tr>
<tr>
<td>GMM t-statistic</td>
<td>(-3.322)^c</td>
<td>(0.002)^c</td>
<td>(-1.658)^a</td>
<td>(-0.784)^c</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.828</td>
<td>1.187</td>
<td>3.803</td>
<td>14.486</td>
</tr>
<tr>
<td>iid t-statistic</td>
<td>(19.123)^c</td>
<td>(5.898)^c</td>
<td>(18.999)^c</td>
<td>(72.503)^c</td>
</tr>
<tr>
<td>GMM t-statistic</td>
<td>(2.676)^c</td>
<td>(2.660)^c</td>
<td>(3.054)^c</td>
<td>(2.657)^c</td>
</tr>
<tr>
<td>Lags</td>
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</tr>
<tr>
<td>1</td>
<td>0.089(^b)</td>
<td>0.063</td>
<td>0.132(^c)</td>
<td>0.157(^c)</td>
</tr>
<tr>
<td>2</td>
<td>0.077(^a)</td>
<td>-0.114(^c)</td>
<td>0.105(^c)</td>
<td>0.071(^a)</td>
</tr>
<tr>
<td>3</td>
<td>0.011</td>
<td>-0.006</td>
<td>0.007</td>
<td>0.013</td>
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<tr>
<td>4</td>
<td>0.071(^a)</td>
<td>0.074</td>
<td>0.068(^a)</td>
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<tr>
<td>5</td>
<td>-0.0261</td>
<td>-0.114(^c)</td>
<td>0.020</td>
<td>0.039</td>
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<tr>
<td>$Q_R (5)$</td>
<td>12.045(^c)</td>
<td>21.718(^c)</td>
<td>20.498(^c)</td>
<td>19.246(^c)</td>
</tr>
<tr>
<td>$Q_R (10)$</td>
<td>13.846(^c)</td>
<td>29.799(^c)</td>
<td>29.651(^c)</td>
<td>36.428(^c)</td>
</tr>
<tr>
<td>$Q_{R^2}(5)$</td>
<td>30.98(^c)</td>
<td>30.04(^c)</td>
<td>28.45(^c)</td>
<td>110.54(^c)</td>
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<tr>
<td>$Q_{R^2}(10)$</td>
<td>42.77(^c)</td>
<td>40.82(^c)</td>
<td>63.01(^c)</td>
<td>170.00(^c)</td>
</tr>
</tbody>
</table>
Table 2: Variance Ratios for the Weekly Returns of the JSE Indexes

The standard errors under the assumption of homoskedastic returns are reported in parentheses and the standard errors under the assumption of heteroskedastic returns are reported in brackets. One, two and three asterisks denote that the test statistic is statistically significant using the Chow and Denning (1993) critical values at the 10%, 5% and 1% level of significance, respectively. A superscript a, b and c denotes the statistical significance at the 10%, 5% and 1% levels of significance, respectively.

<table>
<thead>
<tr>
<th>Index</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Share</td>
<td>1.088</td>
<td>1.203</td>
<td>1.265</td>
<td>1.261</td>
</tr>
<tr>
<td></td>
<td>(2.175)(^b)</td>
<td>(2.682)(^c)</td>
<td>(2.204)(^b)</td>
<td>(1.460)</td>
</tr>
<tr>
<td></td>
<td>[1.455]</td>
<td>[1.931]</td>
<td>[1.753]</td>
<td>[1.238]</td>
</tr>
<tr>
<td>All Gold</td>
<td>1.063</td>
<td>0.973</td>
<td>0.928</td>
<td>0.925</td>
</tr>
<tr>
<td></td>
<td>(1.561)</td>
<td>(-0.358)</td>
<td>(-0.595)</td>
<td>(-0.421)</td>
</tr>
<tr>
<td></td>
<td>[1.427]</td>
<td>[-0.326]</td>
<td>[-0.533]</td>
<td>[-0.379]</td>
</tr>
<tr>
<td>Industrial</td>
<td>1.133</td>
<td>1.306</td>
<td>1.447</td>
<td>1.453</td>
</tr>
<tr>
<td></td>
<td>(3.287)(^c)</td>
<td>(4.029)(^c)</td>
<td>(3.721)(^c)</td>
<td>(2.534)(^b)</td>
</tr>
<tr>
<td></td>
<td>[2.229]</td>
<td>[2.905]</td>
<td>[2.923]</td>
<td>[2.069]</td>
</tr>
<tr>
<td>Financial</td>
<td>1.160</td>
<td>1.314</td>
<td>1.370</td>
<td>1.224</td>
</tr>
<tr>
<td></td>
<td>(3.945)(^c)</td>
<td>(4.127)(^c)</td>
<td>(3.078)(^c)</td>
<td>(1.256)</td>
</tr>
<tr>
<td></td>
<td>[1.457]</td>
<td>[1.749]</td>
<td>[1.506]</td>
<td>[0.677]</td>
</tr>
</tbody>
</table>
We report the estimate of $d$ and its standard error in parenthesis where the number of weekly observations is 606. We obtain the same conclusion when using daily data, i.e. there is no evidence of long-run predictability.

### Table 3 a: Estimates of $d$ using the Geweke and Porter-Hudak Method

We report the estimate of $d$ and its standard error in parenthesis where the number of weekly observations is 606. We obtain the same conclusion when using daily data, i.e. there is no evidence of long-run predictability.

<table>
<thead>
<tr>
<th>Index</th>
<th>0.45</th>
<th>0.5</th>
<th>0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Share</td>
<td>0.197</td>
<td>0.063</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.634)</td>
<td>(0.244)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>All Gold</td>
<td>0.133</td>
<td>-0.040</td>
<td>-0.065</td>
</tr>
<tr>
<td></td>
<td>(0.427)</td>
<td>(-0.156)</td>
<td>(-0.291)</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.097</td>
<td>-0.003</td>
<td>-0.056</td>
</tr>
<tr>
<td></td>
<td>(0.312)</td>
<td>(-0.013)</td>
<td>(-0.251)</td>
</tr>
<tr>
<td>Financial</td>
<td>0.258</td>
<td>0.0628</td>
<td>-0.046</td>
</tr>
<tr>
<td></td>
<td>(0.829)</td>
<td>(0.244)</td>
<td>(-0.207)</td>
</tr>
</tbody>
</table>
Table 3 b: Estimates of ± using the Akikiologlou and Newbold Method

We fit AR(5) models to the returns and use the residuals ($\hat{b}_t$) to compute the regressors in (16). The estimates of ± and the t-statistic are reported in the table above. A superscript a, b and c denote statistical significance at the 10%, 5% and 1% levels of significance respectively.

<table>
<thead>
<tr>
<th>Index</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Share</td>
<td>0.139</td>
<td>1.088</td>
<td>0.622</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.408)</td>
<td>(0.478)</td>
</tr>
<tr>
<td>All Gold</td>
<td>0.383</td>
<td>0.311</td>
<td>0.398</td>
</tr>
<tr>
<td></td>
<td>(1.049)</td>
<td>(1.038)</td>
<td>(1.323)</td>
</tr>
<tr>
<td>Industrial</td>
<td>1.402</td>
<td>0.257</td>
<td>0.236</td>
</tr>
<tr>
<td></td>
<td>(1.223)</td>
<td>(0.331)</td>
<td>(0.449)</td>
</tr>
<tr>
<td>Financial</td>
<td>2.256</td>
<td>1.427</td>
<td>0.879</td>
</tr>
<tr>
<td></td>
<td>(2.449)\textsuperscript{a}</td>
<td>(1.918)\textsuperscript{a}</td>
<td>(1.847)\textsuperscript{a}</td>
</tr>
</tbody>
</table>
Table 4: Rescaled Range Statistic

The critical values for the R/S statistic are obtained from Table II of Lo (1991).

<table>
<thead>
<tr>
<th>Index</th>
<th>R/S Statistic</th>
<th>Modified R/S Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Share</td>
<td>1.122</td>
<td>0.995</td>
</tr>
<tr>
<td>All Gold</td>
<td>0.942</td>
<td>0.947</td>
</tr>
<tr>
<td>Industrial</td>
<td>1.386</td>
<td>1.185</td>
</tr>
<tr>
<td>Financial</td>
<td>1.228</td>
<td>1.057</td>
</tr>
</tbody>
</table>