Common Sources of Variation in Natural Gas Futures Prices

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Abstract

We identify the common sources of variation in the term structure of natural gas futures prices and the term structure of convenience yields using principal component analysis. In the term structure of futures prices we require at least 5 components to explain 90% of the variation: the first component is a level component that affects futures prices equally across maturity and the remaining components are seasonal. For the term structure of convenience yields, a level, slope and curvature component explain 93%, 4% and 1% of the variation, respectively.

1 Introduction

Natural Gas prices are notoriously volatile. During a cold spell in February 1996, the spot price spiked to $12 after averaging $2.90 in January and the implied volatility of the traded options contracts was over 150% (Simons (1997)). Fitzgerald and Pokalski (1995) attribute this volatility to an interplay of the inelasticity of storage and transportation facilities and the extreme weather conditions that arise unexpectedly. Following the recent deregulation of the natural gas markets, distribution companies are no
longer guaranteed a "fair return" by the Federal Power Commission; instead their returns are likely to fluctuate with the natural gas price. One method of reducing the risk and uncertainty of their returns is hedging. The two main objectives of hedging are a reduction in earnings volatility and a lower probability of "financial distress. Additional benefits accrue in the form of enhanced creditworthiness, higher market capitalization and a lower after-tax cost of capital. There are also benefits to shareholders in the form of higher and less risky returns to shareholders' equity.

To hedge the risk inherent in natural gas prices we first need to determine the sources of variation in natural gas prices. For example, one source of variation is the seasonal demand for gas; the price responds sharply to unexpected Winter cold spells and Summer heat waves. Although the identification of the sources of variation is an empirical problem, we need to be guided by theory. The natural starting point is the commodity options pricing model introduced by Black (1976). Like the Black and Scholes (1973) stock option pricing model, it has only one source of stochastic variation. However, one source of variation is inadequate for pricing commodity options. This begs the question: if one source of variation is not enough, how many do we need to adequately model the variation of commodity prices?

The theoretical guide to answering this question is a model of commodity pricing developed by Amin, Ng and Pirrong (1995); it can accommodate more than one source of variation. The difference between the Black (1976) model and the model proposed by Amin, Ng and Pirrong (1995) is analogous to the difference between the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT). Whereas the CAPM only has one source of systematic risk or variation, the APT can accommodate more than one source, but is silent on the number of sources of systematic risk.

The statistical tool that we use to identify the number of sources of variation is principal components analysis (PCA). PCA is a method for identifying the number of sources of variation within a group of variables and the relative contribution of each source to the total variation. Litterman and Scheinkman (1991) introduced the method of PCA in the context of hedging "xed income portfolios. Whereas stocks are known to have a lot of unsystematic risk that can be measured by the volatility of their returns, "xed income securities have a lot more systematic risk that is common across maturities. Litterman and Scheinkman nd that at least two sources of variation are needed to adequately explain the variation in the term structure of interest rates. More recently, Cortazar and Schwartz (1994) applied PCA to the term structure
of copper prices. They identify three sources of variation that explain 98% of the variation.

The objective of this paper is to replicate the Cortazar and Schwartz (1994) analysis for natural gas prices and extend the analysis to convenience yields. Based on the theory of asset pricing underlying futures pricing, PCA should identify one less source of variation. In section 2 we discuss the theory of futures pricing and the role of a second state variable. In section 3 we discuss the data, provide further background to the method of PCA and discuss the results of our analysis. We conclude with section 4.

2 The Theory of Futures Pricing

The natural starting point for the pricing of commodity futures contracts is the cost-of-carry model that relates the futures price to the cost of purchasing the commodity on the spot market and carrying or storing it until maturity. The model consists of two components: the first component is a stochastic differential equation that characterizes the dynamics of the spot price,

$$\frac{dS(t)}{S(t)} = \mu(t) dt + \sigma(t) dW_1(t);$$

where $S(t)$ is the spot price, $\mu(t)$ is the expected instantaneous price change, $\sigma(t)$ is the instantaneous standard deviation, $W_1(t)$ is a Wiener process and is the source of variance of the spot price. The second component is the theory of storage relation,

$$F(t; T) = S(t) \exp \int_t^T (r(u) - c(t; u)) du$$

where $F(t; T)$ is the futures price at time $t$ for delivery at $T$, $r(t)$ is the riskless rate of interest and $c(t; u)$ is the deterministic convenience yield, in excess of the cost of storage, that the commodity holder earns at time $u$ based on information at time $t$. The convenience yield can be thought of as the benefit of having the commodity on hand to smooth production in case a shortage arises (Kaldor (1939)). The storage relation is an arbitrage condition that equates the futures price to the opportunity cost of buying

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1Amin, Ng and Pirrong (1995) have a very detailed exposition of the different futures pricing models in the context of pricing options on futures.
the commodity on the spot market, i.e. the opportunity cost includes the cost of buying the commodity on the spot market, the interest foregone and the cost of storage less the convenience yield of holding the commodity from \( t \) to \( T \).

By substituting (1) into (2), we obtain the risk-adjusted dynamics for the futures price,

\[
\frac{dF(t;T)}{F(t;T)} = r(t)dt + \frac{\sigma(t)}{2}dW^1_1(t); \quad (3)
\]

where \( W^1_1(t) = W_1(t) \), \( \sigma(t) \) and \( r(t) \) is the price of risk associated with the spot price. The dynamics of the forward price suggest another interpretation of the convenience yield: it is a "dividend" stream that accrues to the holder of the commodity, but not to the holder of the futures contract. The drawback of this model is that futures prices are perfectly correlated across maturities because there is only one source of variation common to all futures prices, the Wiener process, \( W_1(t) \).

One method of reducing the correlation between the futures prices of different maturities is to allow the convenience yield to be stochastic: the introduction of a second source of variation that is imperfectly correlated with the first, breaks the perfect co-movement between the futures prices of different maturities. Assume that the dynamics of the convenience yield are determined by the stochastic differential equation,

\[
dc(t;T) = \mu(t;T)dt + \sigma(t;T)dW_2(t); \quad (4)
\]

where \( \mu(t;T) \) is the expected instantaneous change in the convenience yield, \( \sigma(t;T) \) is the instantaneous standard deviation, \( W_2(t) \) is a Wiener process that is correlated with \( W_1(t) \) and we denote the correlation by \( \rho \). The Wiener process, \( W_2(t) \), is the second source of variation in the model and is imperfectly correlated with \( W_1(t) \), thus it can reduce the correlation of futures prices across the term structure.

In order to obtain the risk-adjusted dynamics of futures prices, we turn to more recent developments in the theory of asset pricing. Harrison and Kreps (1979) and Harrison and Pliska (1981) have shown that in the absence of arbitrage opportunities the risk-adjusted asset price is a martingale and the associated probability measure is referred to as the equivalent martingale measure. Under the equivalent martingale measure, \( F(t;T) \) is a martingale.
and its dynamics are characterized by the following stochastic differential equation:

$$\frac{dF(t; T)}{F(t; T)} = \frac{1}{2} \frac{dW^2(t)}{W^2(t)} \cdot Z_T \cdot \int \limits_{t}^{T} \pm(t; u) du \cdot dW^2(t);$$

Under the equivalent martingale measure the drift of the convenience yield is restricted by the correlation with the spot price and the requirement that the convenience yield equal zero at the maturity of the contract. The risk-adjusted dynamics of the convenience yield are given as follows,

$$dc(t; T) = \pm(t; T) \cdot Z_T \cdot \int \limits_{t}^{T} \pm(t; u) du \cdot dW^2(t);$$

where $W^2(t)$, $W^2(t)$, and $W^2(t)$ is the price of risk associated with the stochastic convenience yield. The immediate payoff to the introduction of a stochastic convenience yield is that futures prices are no longer perfectly correlated, i.e. the additional source of variation relaxes the degree of comovement between futures prices along the term structure.

Equations (1), (2) and (4), and the theory of asset pricing provide a complete model for the pricing of futures and options on futures contracts. Before implementing this model in a hedging program, we need to determine if two sources of variation are sufficient to model the variation that is observed in the term structure of futures prices. If the model is inadequate in some dimension, a significant source of risk will not be priced and not be hedged. For example, in the fixed income literature, Canabarro (1995) shows that one-factor interest rate models have poor hedging characteristics especially in a two-factor economy.

We can determine the number of sources of variation using principal components analysis (PCA). PCA is a purely statistical method for determining the number of stochastic terms that generate the variation in futures prices. Within the asset pricing framework that we have used above, there are two

\[1\] Amin, Ng and Pirrong (1995) develop a model of futures pricing using the Heath, Jarrow and Morton (1992) framework which leads to the same stochastic differential for futures pricing with multiple stochastic state variables.

\[2\] The requirement that the futures price be a martingale imposes a constraint on the drift of the dynamics of the convenience yield under the equivalent martingale measure, similar in spirit to the Heath, Jarrow and Morton (1992) restrictions on the drift of the term structure of forward rates of interest.
observationally equivalent ways of determining the number of sources of variation. First, if we regard the spot price as containing the first source of variation, PCA of the convenience yields can determine the number of additional sources of variation. Second, PCA of the futures prices, excluding the spot price, should identify the same number of sources of variation as the first method (Cortazar and Schwartz (1994)). In the next section we discuss the method of PCA and its application to natural gas futures prices.

3 Application

3.1 Data

Daily spot and futures price data for the Henry Hub contract are available from Datastream International. Futures prices are available from the inception of the market on April 3, 1990. Spot price series are only available from November 5, 1993, coinciding with the date that the price was first published in the Wall Street Journal. We construct a series of constant maturity futures prices with 1 through 12 months to maturity by choosing the Friday closing price of the contract with expiration date closest to within two weeks of the required maturity. This method is similar to Schwartz (1997).

The natural gas futures contract has the following contract specifications. Each trading unit is 10,000 million British thermal units (MMBtu). The contract is traded from 10:00 A.M. - 3:10 P.M. (E.S.T.), for the open outcry session. After-hours trading is conducted via the NYMEX ACCESS® electronic trading system from 4 P.M. to 7 P.M., Monday through Thursday. The contract trades for 36 consecutive months commencing with the next calendar month (for example, on October 3, 1998, trading occurs in all months from November 1998 through October 2001) although trading is light in the first 24 months. Trading terminates three business days prior to the first calendar day of the delivery month and delivery is made at the Sabine Pipe Line Co.'s Henry Hub in Louisiana. If contracts have not been settled prior to expiry, delivery takes place no earlier than the first calendar day of

4 The Datastream codes for the spot price and futures prices are NNG and NNGMMYY, respectively, where MM denotes the month and YY the year of maturity of the contract. For example, the futures contract maturing in December 1999 is NNG1299.

5 Herbert (1995) has documented the maturity and volume effects of contracts close to maturity so we do not choose a 'one-month' contract that has less than three weeks to maturity.
the delivery month and is completed no later than the last calendar day of the delivery month. All deliveries are to be made at an hourly and daily rate of flow as uniform as possible over the course of the delivery month.

The convenience yield is calculated by inverting the theory of storage relation, i.e.

\[ c(t; T) = \ln(F(t; T)) e^{-\int_t^T \alpha(u) \, du} \]

where \( c(t; T) \) is the convenience yield earned from \( t \) to \( T \). As an approximation to the continuous time differential, we analyse the discrete, weekly changes in the futures prices and convenience yields, i.e. from equation (5) we have,

\[ \frac{dF(t; T)}{F(t; T)} = \frac{F(t; T) - F(t-1; T)}{F(t-1; T)} \]

and from (6) we have,

\[ dc(t; T) = \alpha(t; T) - dc(t-1; T) \]

where \( \alpha \) denotes the discrete change in the convenience yield that is used to approximate the differential, \( dc(t; T) \).

In the next section we provide a discussion of the PCA method.

### 3.2 Principal Components Analysis

PCA is a statistical technique for determining the number of stochastic components that adequately summarize the systematic variation in a group of variables. PCA is based on the assumption that there is a finite and unknown number of stochastic components that generate the variation within a group of variables. The financial economic analog to this assumption is the distinction between the number of sources of systematic variation of a stock returns that are embodied in the CAPM and the APT. Whereas the CAPM only has one source of systematic risk, the APT can accommodate more than one source of systematic risk, but is silent on the number of sources of risk. In this section we discuss the PCA of convenience yields, but the method also applies to the term structure of futures prices.
We conjecture that the convenience yields are a linear combination of $k$ unknown stochastic state variables or factors, i.e.

$$
\epsilon_t^c = L \epsilon_t^Z + \epsilon_t^m;
$$

where $\epsilon_t^c$ is the $m \times 1$ vector of changes in the convenience yields in period $t$, $\epsilon_t^Z$ is the $m \times 1$ vector of expected values of the changes in the convenience yields, $L$ is the $m \times k$ matrix of factor loadings, $Z_t$ is the $k \times 1$ vector of factors at period $t$ with expectation zero and $\epsilon_t^m$ is the $m \times 1$ vector of random shocks. By construction, the covariance matrix of $Z_t$ is the identity matrix, and $Z_t$ is uncorrelated with $\epsilon_t^m$.

Based on the assumed factor structure in (8), the covariance matrix of the convenience yields, $\Sigma$, can be decomposed as follows:

$$
\Sigma = LL^0 + \sigma^2;
$$

where $\text{cov}(\epsilon_t^m) = \sigma^2$. Given an estimate of $\Sigma$, PCA provides an estimate of $L$. However, PCA is a decomposition of the co-variance of the $m$ convenience yields, thus the estimate of $L$ is affected by the relative size of the individual variances. Instead of decomposing the covariance matrix we prefer to use the correlation matrix, $\rho$, because all the variables are scaled to have the same variance. We proceed as follows: first, we obtain the eigendecomposition of the correlation matrix,

$$
\rho = U \Sigma U^T;
$$

where $U$ is an orthonormal matrix containing the eigenvectors and $\Sigma$ is a diagonal matrix with the eigenvalues, $\lambda_i$, in descending order down the diagonal. The sum of the eigenvalues is a measure of the total variation within the group of variables. The proportion of variation explained by each principal component can be measured by the ratio of each eigenvalue to the sum. One rule-of-thumb for determining $k$, the number of stochastic components that adequately summarizes the variation, is the ratio of the sum of the $k$ eigenvalues and the sum of all the eigenvalues. Like the coefficient of determination or $R^2$ in regression analysis, this ratio is not a statistical test, but a measure of the variation that is explained by the $k$ components. For example, Litterman and Scheinkman (1991) and Cortazar and Schwartz (1994) find that for $k = 3$; more than 95% of the variation is explained. Although the principal components are purely a statistical construction, we
can analyze each column of $U$ to determine the weighting that is attached to each of the original $m$ variables. The analysis of the weights helps to add some economic content to each component. For example, in Litterman and Scheinkman's (1991) analysis of the term structure they identify the first factor as a level factor that has the same effect across maturities, i.e. a positive shock to the first factor increases all interest rates by the same increment. The second and third factors are related to the slope and curvature of the term structure, respectively.

The principal components are linear combinations of the original variables:

$$Z = X U;$$

where $X$ is the $n \times m$ data matrix such that $\text{corr}(X) = -$. The first principal component is the linear combination of the convenience yields that explains the greatest proportion of their variation. The second component is constructed to be orthogonal to the first component and explains the greatest proportion of the remaining variation. This procedure is repeated $m$ times to produce $m$ orthogonal principal components. In essence, we have constructed a set of $m$ orthogonal principal components from $m$ correlated convenience yields.

In summary, although we require $m$ stochastic components to explain all the variation in the $m$ convenience yields, PCA provides a method of reducing the dimensionality of the problem when there are $k < m$ stochastic components or sources of variation that explain a large proportion of the variation of a group of variables. Returning to the financial economic analogy at the beginning of this subsection, the $k$ stochastic components are analogous to the $k$ sources of systematic risk in the APT and the remaining $m - k$ components reflect unsystematic risk or noise.

4 Results

4.1 Forward Rates

Table 1 contains the correlation matrix of changes in the futures prices for the period January 31, 1992 to November 20, 1998. The decrease in the correlation coefficients as the time interval between maturities increases, indicates
that we need more than one source of systematic variation to model the term structure of futures prices. Table 3 contains the percentage of variation explained by the principal components of the correlation matrix in Table 1. It is immediately clear that we need 5 principal components to explain at least 90% of the variation in the forward rates.

Turning to Figure 1, we see that the first component is a "level" effect, i.e. it affects the term structure of futures prices uniformly across maturity and accounts for 52% of the variation in the futures prices. The second, third and fourth components account for 15%, 13% and 8% of the variation, respectively, and capture the seasonality in futures prices. The second component captures the dominant annual cycle of the natural gas market, namely, the demand for gas in the Winter Heating period. The third component also reflects a seasonal component and the fourth component captures both the Winter heating and Summer cooling. In summary, the PCA of the term structure of futures prices identifies the dominant level effect across maturities and the strong seasonal cycle in demand.

4.2 Convenience Yields

Table 2 contains the correlation matrix of changes in the convenience yields for the period November 5, 1993 until November 20, 1998. We omit four weekly observations from February 2 to February 23 when a Winter cold spell caused the spot price to spike to $12. Except for the convenience yield of the one-month contract, the correlations are all greater than 0.84. It is well-known that the one-month contract has more idiosyncratic or unsystematic variation (Herbert (1995)) so the large positive correlation across maturities suggests that there is one dominant stochastic component in convenience yields. Table 3 contains the percentage of variation explained by the principal components of the correlation matrix in Table 2. At most three factors are needed to explain 98% of the variation in the convenience yields. The first component accounts for 93% of the variation while the second and third only account for 4% and 1% of the variation, respectively. An interesting feature of the first three factor loadings is that they are identical to those obtained by Phao (1998) in the principal component analysis of U.S. Treasury yields.

The first component is a "level" effect that affects the futures prices uniformly across the term structure. The second component has been called a "slope" effect: a shock to the second component causes convenience yields
at the short end to decrease and those at the long end to increase, thus increasing the slope of the term structure. The third factor is referred to as a "curvature" or "twist" effect: a shock to the third factor causes the term structure of convenience yields to increase at the short and long end, while decreasing at the intermediate maturities.

None of the first three factors have any seasonal component to them. Although the fourth factor does have a seasonal component, it explains less than 1% of the variation so it plays a very small role. Thus the results suggest that if we are going to model the seasonality in the term structure, we would have to include it in the spot rate (Pilipovic (1998)) and not the convenience yields.

5 Conclusion

Adequate modelling of the systematic risk in the term structure of futures prices, excluding the spot price, requires at least 5 sources of variation and they are all seasonal. Adequate modelling of the systematic risk in the term structure of convenience yields requires one dominant source of variation and at most 2 other sources. The absence of a dominant seasonal component in the convenience yields suggests that a simple model of the spot price and the term structure of convenience yields can be constructed by including a seasonal component in the drift of the spot prices (Pilipovic (1998)) and the dominant level effect of the convenience yields. The empirical performance of this model is left for future research (Beelders (1999)).

References


Table 1: The Correlation Matrix of the percentage changes of the Futures Prices

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Table 2: The Correlation Matrix of the Changes of the Convenience Yields

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Table 3. Proportion of Variation Explained by each Principal Component

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Figure 1: