A Hotelling Model with a Ceiling on the Stock of Pollution

Ujjayant Chakravorty, Bertrand Magné and Michel Moreaux*

Abstract

Environmental agreements such as the Kyoto Protocol aim to stabilize the amount of carbon in the atmosphere, which is mainly caused by the burning of nonrenewable resources such as coal. We characterize the solution to the textbook Hotelling model when there is a ceiling on the stock of emissions. We consider both increasing and decreasing demand for energy. We show that when the ceiling is binding, both the low-cost nonrenewable resource and the high-cost renewable resource may be used jointly. A key implication is that if energy demand were to decline in the long run, we may supplement energy supply through "clean" renewables to meet the environmental standard, but then revert back to using only "dirty" fossil fuels in the future when the ceiling has become non-binding. That is, the much heralded societal "transition" to clean energy resources may be somewhat short-lived.

Keywords: Environmental Agreements, Externalities, Fossil Fuels, Nonrenewable Resources, Nonstationary Demand

JEL codes : Q12, Q32, Q41.

*Respectively, Emory University, Atlanta, GA 30322, unc@emory.edu; Université de Toulouse I (LERNA), 21 Allée de Brienne, 31000 Toulouse, France, bertrand.magne@univ-tlse1.fr; and Université de Toulouse I (IUF, IDEI and LERNA), 21 Allée de Brienne, 31000 Toulouse, France, mmoreaux@cict.fr.
1 Introduction

Many nonrenewable resources, such as coal, oil and natural gas, pollute the atmosphere through emissions of carbon, sulphur and other compounds. Environmental agreements such as the Kyoto Protocol have tended to control such pollution by proposing a target pollution concentration, or equivalently, an upper bound on the stock of pollution.\(^1\) The Montreal Protocol, which aims to phase out the stock of chemicals that deplete the ozone layer, adopts a similar approach, with an eventual goal of a complete phase-out of the stock of these harmful chemicals.

In this paper, we ask how a ceiling on the stock of emissions affects the standard Hotelling model (Hotelling, 1931). We consider a polluting nonrenewable resource and a clean renewable resource. As in Hotelling, it is clear that there will be an eventual transition from the nonrenewable to the renewable resource. What is not clear is the dynamics of this transition, which is the focus of our analysis.

Little attention has been paid to this problem of how a limit on the stock of emissions may alter the sequence of extraction of a fossil fuel and the backstop renewable resource over time. The theoretical literature has mainly relied on models that specify damage functions caused by the use of nonrenewable resources.\(^2\) Forster (1980) first examined pollution in a model of nonrenewable resources. Pollution has a negative effect on the utility function, but the clean substitute plays no role except in the terminal phase. Other studies such as Sinclair (1994) and Ulph and Ulph (1994) have examined the time profile of the carbon tax in an infinite horizon framework but without a backstop resource.\(^3\) Hoel and Kverndokk (1996) and Tahvonen (1997) analyze the path of optimal carbon taxes in a model with a nonrenewable resource and a clean backstop.

\(^1\)The Kyoto Protocol aims at staying below an exogenous upper bound, although country commitments are formulated in terms of emission flows.

\(^2\)A ceiling on the stock of emissions may be considered a special case of an increasing and convex damage function in which damages are negligible until the stock reaches a threshold level (the ceiling) and sufficiently high beyond. In general, there is a high degree of uncertainty with respect to the precise shape of the damage function because the actual costs of climate change are difficult to estimate (McKibbin and Wilcoxen, 2002). Another justification for using a ceiling is that most operational climate change models have used Kyoto-type ceilings to examine the economic effects of global warming (e.g., see Zhang and Folmer, 1998).

\(^3\)Farzin (1996) and Gjerde et al. (1999) address the optimal timing of a carbon tax under threshold effects beyond which the damages become irreversible.
Using stock-dependent extraction costs, they show that there may be a period of simultaneous extraction of the nonrenewable and renewable resource. Toman and Withagen (2000) use a general equilibrium framework to examine the role of economic incentives in managing the stock of pollution arising from use of a polluting input but they do not have resource scarcity in their model. Fisher et al. (2004) also do not consider resource scarcity in modeling the relationship between the pollution stock and the development of a clean technology. On the other hand, most empirical work on global warming either assumes a general equilibrium framework that does not explicitly recognize the scarcity of fossil fuels or models the problem by imposing exogenous carbon taxes (e.g., see Manne and Richels, 1991, Nordhaus and Yang, 1996, and Chakravorty et al. 1997).

This paper combines two features that are important in the assessment of long-run impacts of any international agreement to limit fossil fuel emissions. The first is the scarcity of the nonrenewable resource, which drives up its price over time. The second is the ceiling placed on aggregate emissions from consumption of the resource. As we shall see below, the scarcity of the fossil fuel drives the dynamics of pollution accumulation and the ultimate transition to the cleaner backstop. However, the constraint on the stock of emissions causes the renewable resource to be used even though the cheaper nonrenewable has not yet been exhausted. Once this constraint is no longer binding, the solution reverts to the standard Hotelling case.

We first consider stationary demand for energy. We show that the nonrenewable resource is used until the stock of pollution reaches the ceiling. At the ceiling, one of two things may happen. The fossil fuel may be extracted at a constant rate until it becomes scarce and its extraction rate begins to fall. Then the ceiling ceases to be binding and nonrenewable resource use follows a Hotelling-like path until exhaustion. However, if the cost of the renewable is relatively low, we get an alternative solution in which both resources are extracted simultaneously at the ceiling until the former gets exhausted.

We then consider non-stationary demand. When demand for energy is increasing over time, such as from an increase in per capita consumption or growth in population, the sequence of resource use depends upon the relative costs of the nonrenewable and renewable resource, the abundance of the former, whether it is highly (or mildly) polluting, and whether the imposed ceiling is high or low. Only the fossil fuel may be used when the ceiling is binding, or both
resources may be used jointly. Or the ceiling may begin with the exclusive use of the nonrenewable and end with joint extraction of both resources.

In many of the cases examined, we show that a costly renewable is likely to kick in much before the nonrenewable resource is close to exhaustion. These results provide a point of departure from the standard Hotelling notion of a switch from a cheap nonrenewable to a costly renewable resource.

A sharper result is obtained when energy demand decreases over time, which may be a plausible scenario to consider in the long run. The nonrenewable resource is used at first, then both resources are used at the ceiling, followed again by exclusive use of the nonrenewable resource until it is exhausted and finally, a complete transition to the renewable. That is, the more expensive renewable resource is used at the ceiling even though the cheaper nonrenewable resource has not yet been exhausted. This is followed by a phase when only the nonrenewable resource is used.

When emissions can be abated at constant unit cost, we show that it is never optimal to abate as well as use the renewable resource simultaneously. If the unit cost of abatement is higher than the cost of the renewable resource, there will never be any abatement. In general, however cheap abatement may be, it is never done before the pollution ceiling is attained or when the ceiling ceases to be binding. Abatement takes place only during the period when the ceiling is binding. We develop conditions under which abatement may start exactly at the instant the ceiling is attained, or the abatement period may lie strictly in the interior of the ceiling period.

The analysis suggests that if an agreement such as the Kyoto Protocol were to be implemented, we may see the joint use of fossil fuels and renewable energy. For example, renewables such as solar energy may be employed in electricity generation even though they are costly relative to coal. However, even with energy demand increasing over time, if global populations and energy demand peak and then begin a decline, the pollution ceiling may not be binding any longer. We

---

4 For example, energy demand may decrease over time if there is a decline in global population and the inevitable levelling off of per capita energy consumption in the developing countries. Recent population projections have significantly downgraded earlier estimates of global population growth and the level from which world population will begin a steady decline (United Nations, 2002, Lutz et al., 2001).

5 This result violates Herfindahl’s (1967) theorem of “least cost first,” which suggests that in partial equilibrium, nonrenewable resources must be used in order of increasing extraction cost (see Amigues et al., 1998).
may then abandon the expensive solar energy and revert back to using fossil fuels exclusively. The relative abundance of coal over the other fossil fuels (oil and natural gas) may suggest that the second period with exclusive use of coal may be an extended one.

Section 2 describes the Hotelling model with a ceiling on the stock of pollution. Section 3 develops intuition by focusing on special cases including stationary and increasing (decreasing) demand. Section 4 considers pollution abatement. Section 5 concludes the paper.

2 The Hotelling Model with a Ceiling on the Stock of Pollution

2.1 Basic assumptions

We consider an economy in which the instantaneous gross surplus at time \( t \) generated by energy consumption \( q_t \) is given by \( u(q_t, t) \). We assume that:

**Assumption U.1.** \( u:R^2_+ \rightarrow R^++ \) is of class \( \mathbb{C}^3 \), strictly increasing and strictly concave in \( q_t \):

\[
    u_1(q_t, t) \equiv \delta u(q_t, t)/\delta q_t > 0 \quad \text{and} \quad u_{11}(q_t, t) \equiv \delta^2 u(q_t, t)/\delta q_t^2 < 0, \quad q_t > 0, t > 0.
\]

We are mainly interested in the most realistic case in which energy demand is increasing over time. This case is denoted \( ID \) and is compared to the case of decreasing demand \( (DD) \). Demand is increasing (decreasing) if the gross surplus and the marginal gross surplus are both increasing (decreasing) and if \( u \) is bounded from above (below) by \( \pi(u) \).

**Assumption U.ID (DD).** For any \( q_t > 0, t \geq 0 \):

\[
    u_2(q_t, t) \equiv \delta u(q_t, t)/\delta t > 0 (< 0) \quad \text{,} \quad u_{12}(q_t, t) \equiv \delta^2 u(q_t, t)/\delta q_t \delta t > 0 (< 0)
\]

and

\[
\lim_{t \rightarrow +\infty} u(q_t, t) = \pi(q_t) (u(q_t)).
\]

\footnote{The latter may happen with a decline in world population so that aggregate energy consumption falls even with the expected increases in per capita energy demand in the developing countries.}
The following regularity assumption will be useful in characterizing the optimal path in the ID case:

**Assumption U.R (Regularity).** In the ID (DD) case, for any $q_t > 0$, $t \geq 0$:

$$\frac{\delta^3 u(q_t, t)}{\delta q_t \delta t^2} = u_{122}(q_t, t) < 0 (> 0).$$

An immediate implication of the above condition is that, for any $q_t > 0$, the instantaneous marginal gross surplus at a given $q_t$ is an increasing and concave function of time in the ID case, and a decreasing and convex function in the DD case.

The economy uses two resources: a polluting fossil fuel (e.g., coal) and a clean renewable resource (e.g., solar power). They are perfect substitutes: i.e., if $x_t$ and $y_t$ are the respective extraction (consumption) rates of the nonrenewable (call it NR) and the renewable (R) resource, then aggregate energy consumption at time $t$ is given by $q_t = x_t + y_t$. The instantaneous gross surplus is equal to $u(x_t + y_t, t)$.

The NR resource is characterized by four parameters: the amount of initial reserves $X_0$, its average extraction cost to the user, $c_e$ (assumed constant), the pollution per unit of the resource $\zeta$ and the unit cost of abatement, given by $c_a$, also assumed constant. We denote by $X_t$ the amount of resource available at time $t$, so that $\dot{X} = -x_t$.

The NR resource is assumed to be scarce even in the DD case. More precisely, let $x_{ct}$ be the consumption level of the resource for which the marginal gross surplus is equal to its marginal cost, $u_1(x_{ct}, t) = c_e$. Then we assume that the stock necessary to sustain this path (infinite in the ID case) is higher than $X_0$.

**Assumption NRS (Nonrenewable resource scarcity):**

$$\lim_{t \uparrow +\infty} \int_0^t x_{ct} \, dr > X_0.$$  

---

7The extraction cost is interpreted as a delivery cost, i.e., inclusive of processing and transportation.
Let $Z_t$ be the stock of pollution. The emission of pollution $z_t$ is proportional to the consumption of the $NR$ resource, $z_t = \zeta x_t$. As is standard in the literature, we assume that the natural regeneration capacity of the atmosphere is proportional to the stock of pollution $Z_t$ (see e.g., Kolstad and Krautkraemer, 1993). Let $a_t$ be the abatement at time $t$, then:

$$Z_t = \zeta x_t - a_t - \alpha Z_t, \quad \zeta x_t - a_t \geq 0, \quad \alpha > 0, \quad t \geq 0.$$  

The pollutant stock must be less than or equal to some exogenously fixed ceiling level $\overline{Z}$, so that $\overline{Z} - Z_t \geq 0$. We denote by $\overline{x}(a_t)$ the maximum consumption rate of the $NR$ resource if $Z_t$ is at its upper bound $\overline{Z}$, and abatement is equal to $a_t$ : $\overline{x}(a_t) = (a_t + \alpha \overline{Z})/\zeta$. Let $\overline{p}_{ct}(a_t)$ be the corresponding marginal gross surplus, defined by $\overline{p}_{ct}(a_t) = u_1(\overline{x}(a_t), t)$. Since $\partial \overline{p}_{ct}(a_t)/\partial t = u_{12}(\overline{x}(a_t), t)$ and $\partial^2 \overline{p}_{ct}(a_t)/\partial t^2 = u_{122}(\overline{x}(a_t), t)$, for any given $a_t$, $\overline{x}(a_t)$ is increasing and concave in the ID case, and decreasing and convex in the DD case. When there is no active abatement, $\overline{x}(0)$ and $\overline{p}_{ct}(0)$ can be denoted more simply by $\overline{x}$ and $\overline{p}_{ct}$, respectively.

There exists a constant instantaneous natural flow of the renewable resource, $\overline{y}$. This resource is non-storable so that the part $\overline{y} - y_t$ of the natural flow which is not immediately used, is definitely lost. We denote by $c_r$ the constant average delivery cost of the $R$ resource. This cost is higher than the cost of the $NR$ resource: $c_r > c_e$. We assume that the marginal gross surplus at small consumption rates is higher than $c_r$ even in the DD case, so that the $R$ resource has to be used once the $NR$ resource is exhausted, and as we shall show, in some cases even before the exhaustion of the $NR$ resource.

**Assumption U.2**  
$\exists \varepsilon > 0: \lim_{t \uparrow + \infty} u_1(q_t, t) \geq c_r + \varepsilon, \quad t \geq 0.$

Lastly we assume that the $R$ resource is abundant even in the ID case. Define $y_{ct}$ as the consumption rate of the $R$ resource for which the marginal gross surplus is equal to the marginal cost of the resource, $u_1(y_{ct}, t) = c_r, \quad t \geq 0$. Then $y_{ct}$ is time increasing in the ID case and decreasing in the DD case. Finally, we can state:

**Assumption RA (Renewable resource abundance):**  
$y_{ct} \leq \overline{y}, \quad t \geq 0.$
2.2 The Optimization Problem

The social planner's problem (defined by $P$) is to maximize the net social welfare by choosing the quantities of the NR and the R resource as well as carbon abated at any given instant:

\[
(P) \quad \max_{\{(x_t, y_t, a_t), t \geq 0\}} \int_0^{+\infty} \{ u(x_t + y_t, t) - c_e x_t - c_a a_t - c_r y_t \} e^{-\rho_t} dt
\]  

(1a)

\[
s.t. \quad \dot{X}_t = -x_t, \quad X_0 \text{ given, } X_t \geq 0,
\]  

(1b)

\[
\zeta x_t - a_t \geq 0,
\]  

(1c)

\[
x_t \geq 0,
\]  

(1d)

\[
a_t \geq 0,
\]  

(1e)

\[
\dot{Z}_t = \zeta x_t - a_t - \alpha Z_t, \quad Z_0 < \bar{Z} \text{ given, } \bar{Z} - Z_t \geq 0, \text{ and}
\]  

(1f)

\[
y_t \geq 0.
\]  

(1g)

Let $L_t$ be the current value Lagrangian associated with the problem $(P)$:

\[
L_t = u(x_t + y_t, t) - c_e x_t - c_a a_t - c_r y_t - \lambda t x_t + \mu t \left[ \zeta x_t - a_t - \alpha Z_t \right] + \nu_t \left[ \bar{Z} - Z_t \right] + \gamma_t a_t + \gamma_r t y_t.
\]  

Then the first order conditions are:

\[
\frac{\delta L_t}{\delta x_t} = 0 \Leftrightarrow u_1 - c_e - \lambda t + \zeta \mu_t + \gamma_{et} + \zeta \gamma_{at} = 0,
\]  

(2)

\[
\frac{\delta L_t}{\delta y_t} = 0 \Leftrightarrow u_1 - c_r + \gamma_{rt} = 0, \quad \text{and}
\]  

(3)

\[
\frac{\delta L_t}{\delta a_t} = 0 \Leftrightarrow -c_a - \mu_t - \gamma_{at} + \gamma_{at} = 0,
\]  

(4)

together with the complementary slackness conditions:

\[
\nu_t \geq 0, \quad \text{and} \quad \nu_t \left[ \bar{Z} - Z_t \right] = 0,
\]  

(5)

7
\( \gamma_{et} \geq 0, \) and \( \gamma_{et}x_t = 0, \) \( \quad (6) \)

\( \gamma_{rt} \geq 0, \) and \( \gamma_{rt}y_t = 0, \) \( \quad (7) \)

\( \bar{\gamma}_{at} \geq 0, \) and \( \bar{\gamma}_{at} [\zeta x_t - a_t] = 0, \) and \( \quad (8) \)

\( \gamma_{at} \geq 0, \) and \( \gamma_{at} a_t = 0. \) \( \quad (9) \)

The dynamics of the costate variables are determined by:

\[ \dot{\lambda}_t = \rho \lambda_t - \frac{\delta L_t}{\delta X_t} \Rightarrow \dot{\lambda}_t = \rho \lambda_t \Rightarrow \lambda_t = \lambda_0 e^{\rho t}, \] \( \quad (10) \)

\[ \dot{\mu}_t = \rho \mu_t - \frac{\delta L}{\delta Z_t} \Rightarrow \dot{\mu}_t = (\rho + \alpha) \mu_t + \nu_t. \] \( \quad (11) \)

Note that costate variable \( \mu_t \) is non-positive. Furthermore, if \( Z_t < Z \) over some time interval \([t_1, t_2]\), then \( \nu_t = 0 \) over that interval and \( \mu_t = \mu_{t_1} e^{(\rho + \alpha)(t_1 - t)}, t \in [t_1, t_2]. \)

Lastly, the transversality conditions at infinity are given by:

\[ \lim_{t \to +\infty} e^{-\rho t} \lambda_t X_t = \lambda_0 \lim_{t \to +\infty} X_t = 0, \] \( \quad (12) \)

and

\[ \lim_{t \to +\infty} e^{-\rho t} \nu_t Z_t = 0. \] \( \quad (13) \)

### 2.3 Preliminary Remarks and Definitions

Before solving the planner’s problem, let us define the following functions and benchmarks which will be used repeatedly in characterizing the solution.

For any \( \lambda_0 \in (0, c_r - c_e) \), let \( x_t(\lambda_0) \) be either the solution of:

\[ u_1(x, t) = c_e + \lambda_0 e^{\rho t} \equiv \tilde{p}_t(\lambda_0), \]
if such a solution exists, or 0 if not. Then $\tilde{x}_t(\lambda_0)$ would be the optimal consumption rate of the NR resource with the ceiling non-binding at time $t$ and never tight at any time $\tau > t$, provided that $\tilde{p}_t(\lambda_0) < c_r$ and $\lambda_0$ is the right initial unitary rent.

Note that the time derivative of $\tilde{x}_t(\lambda_0)$ cannot be signed under the purely qualitative assumptions defining the ID case since
\[
\frac{\delta \tilde{x}_t(\lambda_0)}{\delta t} = -\frac{u_{12}(\tilde{x}_t(\lambda_0), t) + \rho \lambda_0 e^{\rho t}}{u_{11}(\tilde{x}_t(\lambda_0), t)},
\]
while in the DD case, $\delta \tilde{x}_t(\lambda_0)/\delta t < 0$. As a function of $\lambda_0$, $\tilde{x}_t(\lambda_0)$ is clearly decreasing:
\[
\delta \tilde{x}_t(\lambda_0)/\delta t < 0 \quad \text{and} \quad \lim_{\lambda_0 \uparrow 0} \tilde{x}_t(\lambda_0) = x_{ct}.
\]

We define $\bar{\theta}(\lambda_0)$ as the time at which $\tilde{p}_t(\lambda_0) = c_r$:
\[
\bar{\theta}(\lambda_0) = \rho^{-1} \left[ \log (c_r - c_e) - \log \lambda_0 \right].
\]

Without the ceiling constraint, the optimal value of $\lambda_0$, denoted by $\lambda_0'$, would be the solution to the cumulative demand/supply balance equation:
\[
\int_0^{\bar{\theta}(\lambda_0)} \tilde{x}_t(\lambda_0) dt = X_0.
\]

Let $\tilde{Z}_t(\lambda_0)$ be the pollution stock induced by $\tilde{x}_t(\lambda_0)$, starting from $Z_0$, that is:
\[
\frac{d\tilde{Z}_t(\lambda_0)}{dt} = \zeta \tilde{x}_t(\lambda_0) - \alpha \tilde{Z}_t(\lambda_0) \quad \text{and} \quad \tilde{Z}_t(\lambda_0) = Z_0.
\]

Then $\tilde{Z}_t(\lambda_0')$ would be the pollutant stock trajectory within the time interval $[0, \bar{\theta}(\lambda_0')]$ in the absence of the ceiling constraint. After $\bar{\theta}(\lambda_0')$ there is no more pollutant emission and $Z_t$ decreases to 0 at infinity. In what follows, we assume that the ceiling constraint is binding, that is

**Assumption PCB (Pollution constraint binding):**
\[
\max \{ \tilde{Z}_t(\lambda_0), \ t \in [0, \tilde{\theta}(\lambda_0')] \} > Z.
\]
Although $\tilde{x}_t(\lambda_0)$ is not the optimal consumption trajectory at the beginning of the planning period, it will be useful for characterizing the end of the phase during which the users are supplied, either wholly or partially, from the NR resource.

Before defining $\tilde{x}_t(\lambda_0)$, let us first note that by (4), $-\mu_t = c_\alpha + \overline{\gamma}_{at} - \gamma_{at}'$. Suppose that $Z_t = \overline{Z}$. Then the instantaneous cost of consumption at time $t$ is given by $c_e + \lambda_0 e^{\rho t}$ for $x_t < \overline{x}$ because in this case no abatement is necessary. Thus for $a_t$ to be positive, abatement must equal $\zeta(x_t - \overline{x})$ so that the constraint $\zeta x_t - a_t \geq 0$ is never tight and $\overline{\gamma}_{at} = 0$. Since $a_t > 0$, we have $\gamma_{at} = 0$.

Substituting for $-\mu_t$ from the above expression in (2), we get $u_1(x_t + y_t, t) = c_e + \zeta c_\alpha + \lambda_0 e^{\rho t} \equiv \tilde{p}_t(\lambda_0)$.

Define $\tilde{x}_t(\lambda_0)$ as either the solution of $u_1(x_t, t) = \tilde{p}_t(\lambda_0)$, if such a solution exists, or by 0, if not. Then $\tilde{x}_t(\lambda_0)$ is the optimal rate of consumption of the NR resource if at time $t$ the ceiling is binding, a portion of the emission flow has to be abated, and if $\tilde{p}_t(\lambda_0) < c_r$. The sign of the derivative of $\tilde{x}_t(\lambda_0)$ cannot be determined from the purely qualitative assumptions defining the ID case, and is negative in the DD case.

Denote by $\tilde{\theta}(\lambda_0)$ the time at which $\tilde{p}_t(\lambda_0) = c_r$. Then $\tilde{\theta}(\lambda_0) = \rho^{-1}[\log(c_r - c_e - \zeta c_\alpha) - \log \lambda_0]$.

Let $\tilde{\tau}(\lambda_0)$ be the time at which $\tilde{p}_t(\lambda_0) = \overline{p}_{et}$, if it exists and is unique over $[0, \infty)$. Let $\tilde{\tau}_1(\lambda_0)$ and $\tilde{\tau}_2(\lambda_0)$, $\tilde{\tau}_1(\lambda_0) < \tilde{\tau}_2(\lambda_0)$, be the two instants of time at which this equality is satisfied if both exist over $[0, \infty)$.

Lastly, for any $\lambda_0 \in [0, \overline{p}_{et} - c_e]$ we denote by $\tilde{\tau}(\lambda_0)$ the time at which $\tilde{p}_t(\lambda_0) = \overline{p}_{et}$. Since $\tilde{p}_t(\lambda_0)$ is increasing and concave, while $\overline{p}_{et}$ is increasing and concave in the ID case and decreasing and convex for DD, $\tilde{\tau}(\lambda_0)$ is well-defined and unique and $\tilde{p}_t(\lambda_0) <, > \overline{p}_{et}$ depending on whether $t <, > \tilde{\tau}(\lambda_0)$.

For any $\lambda_0 \in [0, c_r - c_e]$ and $\mu_0 \in (- \lfloor c_r - (c_e + \lambda_0) \rfloor / \zeta, 0)$, let $\tilde{x}_t(\lambda_0, \mu_0)$ be the solution of:

$$u_1(x, t) = c_e + \lambda_0 e^{\rho t} - \zeta \mu_0 e^{(\rho + \alpha)t} \equiv \tilde{p}_t(\lambda_0, \mu_0),$$

if such a solution exists, or 0 if not. Then $\tilde{x}_t(\lambda_0, \mu_0)$ is the optimal consumption rate of the NR resource if at time $t$ the ceiling is not binding and has never been from time 0 (so that $v_t = 0$, $\tau \in [0, t]$, and $\mu_t = \mu_0 e^{(\rho + \alpha)t}$ by (11)) but will be binding in the future (provided that $\lambda_0$ and $\mu_0$ be the right initial values of the costate variables).
As in the case of \( \bar{x}_t(\lambda_0) \), the time derivative of \( \bar{x}_t(\lambda_0, \mu_0) \) cannot be signed in the ID case but is negative in the DD case.

Denote by \( \hat{\theta}(\lambda_0, \mu_0) \) the time at which \( \hat{p}_t(\lambda_0, \mu_0) = c_r \). Then

\[
\hat{\theta}(\lambda_0, \mu_0) = \rho^{-1} \left[ \log (c_r - c_e) - \log (\lambda_0 - \zeta \mu_0) \right].
\]

Define \( \hat{Z}_t(\lambda_0, \mu_0) \) as the pollution stock generated by \( \bar{x}_t(\lambda_0, \mu_0) \), starting from \( Z_0 \), so that:

\[
\frac{\delta \hat{Z}_t(\lambda_0, \mu_0)}{\delta t} = \zeta \bar{x}_t(\lambda_0, \mu_0) - \alpha \hat{Z}_t(\lambda_0, \mu_0) \quad \text{and} \quad \hat{Z}_t(\lambda_0, \mu_0) = Z_0.
\]

For any \( \lambda_0 \in [0, p_{e0} - c_e] \) and \( \mu_0 \in (- [p_{e0} - (c_e + \lambda_0)] / \zeta, 0) \), denote by \( \hat{\tau}(\lambda_0, \mu_0) \) the time at which \( \hat{p}_t(\lambda_0, \mu_0) = \bar{p}_{et} \). As in the case of \( \hat{\tau}(\lambda_0) \), under the regularity assumptions, \( \hat{\tau}(\lambda_0, \mu_0) \) is well-defined and unique even in the ID case, and \( \hat{p}_t(\lambda_0, \mu_0) <, > \bar{p}_{et} \) if \( t <, > \hat{\tau}(\lambda_0, \mu_0) \).

Next, for any \( \lambda_0 \in [0, p_{e0} - c_e] \) and \( \mu_0 \in (- [c_r - (c_e + \lambda_0)] / \zeta, 0) \), denote by \( \bar{\delta}(\lambda_0, \mu_0) \) the time at which \( \bar{p}_t(\lambda_0, \mu_0) = \bar{p}_t(\lambda_0) \). Under the regularity assumptions, \( \bar{\delta}(\lambda_0, \mu_0) \) is well-defined and unique.

Lastly, define \( \bar{\theta} \) as the time at which \( \bar{p}_{et} = c_r \), either in the ID case when \( p_{e0} < c_r < p_{e\infty} \), or in the DD case when \( p_{e\infty} < c_r < p_{e0} \).

3 No Abatement and (Non-)Stationary Demand

In this and the following section, we characterize the solution for the special case when there is never any abatement of emissions at equilibrium. This is plausible if for instance, abatement is too costly. In section 6, we consider the case when abatement is economically feasible. We first develop intuition by examining the simplest case - when demand is stationary, then discuss the non-stationary (ID and DD) cases.

3.1 The Benchmark Case: When Demand is Stationary

With a fixed demand and no abatement, we get the standard Hotelling model with a pollution ceiling. Without abatement the pollution stock grows with emissions net of natural decay,
\[ \dot{Z}_t = \zeta x_t - \alpha Z_t. \] At the ceiling, \( \bar{x} = \alpha Z / \zeta. \) We need to consider two cases according to whether (a) \( y_c < \bar{x} \) or (b) \( y_c > \bar{x}. \)

### 3.2 Case (a) \( y_c < \bar{x} \)

Consider the case when \( y_c < \bar{x}, \) i.e., the maximum extraction rate of the \( R \) resource is lower than the extraction rate of the \( NR \) resource at the ceiling. This only happens if \( \tilde{p}_c < c_r, \) i.e., the price of the \( NR \) resource at the ceiling is lower than the cost of the \( R \) resource (see Fig. 1). When the pollution stock is at the ceiling, the nonrenewable resource can supply all of the demand. Quadrant 1 (in Fig. 1) shows the relevant price paths and quadrant 4 shows resource use over time. The sequence of resource use can be completely described by four different phases. In phase I, the true marginal cost for consumption of the \( NR \) resource is given by the curve \( \tilde{p}_t (\lambda_0, \mu_0) = c_e + \lambda_0 e^{\rho t} - \zeta \mu_0 e^{(\rho + \alpha)t}. \) Note from quadrant 4, that the pollution emitted in this phase is higher than \( \zeta \bar{x}, \) the rate at which the flow is exactly neutralized by natural decay. Thus the stock of pollution increases over time from some initial level \( Z_0 < Z. \) However, at time \( \tilde{\tau}(\lambda_0), \) the stock reaches the ceiling \( \bar{Z}. \) This is the beginning of phase II, in which the consumption of the \( NR \) resource equals \( \bar{x}, \) and the stock of pollution is binding at the ceiling \( \bar{Z}. \) The price of the \( NR \) resource equals \( \bar{p}_c \) and is constant in this phase, which ends at time \( \tilde{\tau}(\lambda_0). \) This marks the beginning of phase III, in which \( NR \) use declines from \( \bar{x} \) and the ceiling is no longer binding. This is a transition phase from the nonrenewable to the renewable resource. This phase is analogous to a Hotelling price path, since the price given by \( \tilde{p}_t (\lambda_0) \) rises at the rate of discount, i.e., \( c_e + \lambda_0 e^{\rho t}. \) There are no more externality costs since the pollution ceiling is never binding beyond \( \tilde{\tau}(\lambda_0). \) Finally at time \( \tilde{\theta}(\lambda_0) \), the price of the \( NR \) resource equals the cost of the backstop resource. At this point, the \( NR \) resource gets exhausted and the economy is supplied by the backstop \( R \) resource, \( q_t = y_c. \)

In both phases I and II, the optimal solution could be achieved through a tax on emissions equal to \(-\mu_t\) per unit of emission, or equivalently, \(-\zeta \mu_t\) per unit of the fossil fuel consumed.

---

8Since \( \tilde{p}_t, x_{ct} \) and \( y_{ct} \) are constant under stationary demand, we drop the subscript \( t \) in the present section.

9In the Appendix we check that the necessary conditions are satisfied for the more general case of non-stationary demand.
The tax grows at a constant rate $\rho + \alpha$ until the target $\bar{Z}$ is reached. Then it declines steadily to zero during the period when the ceiling is binding. The quota or number of permits for the nonrenewable resource at time $t$ equal $x_t$, with a permit price of $-\zeta \mu_t$. Equivalently, the number of pollution permits is $\zeta x_t$, with price $-\mu_t$.\(^{10}\)

**Fig 1 here**

### 3.3 Case (b) $y_c > \bar{x}$

Now consider the alternative case in which $y_c > \bar{x}$, i.e., the extraction rate of the NR resource at the ceiling is lower than the maximum extraction rate of the R resource. In this case, $\bar{p}_e > c_r$, i.e., the renewable resource becomes economical at a price that is lower than the equilibrium price when the ceiling is binding (see Fig. 2). As in the previous case, in phase I, the fossil fuel is extracted and the stock of pollution is increasing. However, since $\bar{x} < y_c$, and the price of the NR resource increases over time, it must equal the cost of the R resource $c_r$ at or before the ceiling is attained. It does not make economic sense to use the renewable before the ceiling is attained, since it is more costly than the nonrenewable. Thus the ceiling becomes tight exactly at the instant $\tilde{\theta} (\lambda_0, \mu_0)$ when $\bar{p}_e (\lambda_0, \mu_0)$ equals $c_r$, the cost of the backstop resource. In phase II, there is joint consumption of both resources with the deficit $y_c - \bar{x}$ supplied by the R resource. The equilibrium price of energy is constant and the pollution stock is binding over an interval until the NR resource is exhausted at time $\tilde{\theta} (\lambda_0)$. The key difference between this case and the previous one is that here the R resource is used at the ceiling and there is no Hotelling-type transition from the ceiling to the backstop. Emissions decline from their maximum level to zero at $\tilde{\theta} (\lambda_0)$.\(^{11}\)

**Fig 2 here**

We now consider the case of non-stationary demand. First we discuss the case of increasing demand.

---

\(^{10}\)Analogous interpretations could be made for each of the cases analyzed below.

\(^{11}\)Several other interesting cases may arise if we consider the flow of the backstop resource (say $\bar{y}$) to be limited, i.e., $\bar{y} < y_c$. These are developed elsewhere for the stationary demand case (see Chakravorty et al. 2003). The focus of the present paper is to examine resource substitution under non-stationary demand and abatement.
4 Increasing Demand

It will be convenient to develop the solution for three distinct cases, when the cost of the backstop resource is high, low and medium relative to $p_{et}$, the limit price when the emission ceiling is binding, i.e., (a) $p_{e\infty} < c_r$; (b) $c_r < p_{e0}$ and (c) $p_{e0} < c_r < p_{e\infty}$, where $p_{e\infty} = \lim_{t \to +\infty} p_{et}$.

4.1 Case (a) $p_{e\infty} < c_r$

This is illustrated in Fig. 3. The pattern of resource substitution is somewhat analogous to the stationary demand case shown in Fig. 1. If the ceiling is to be binding, the instantaneous marginal surplus must be equal to $p_{et}$ during some time interval within which the constraint is binding. Since $Z_0 < Z$ there must exist an initial period during which the constraint is slack while $Z_t$ is increasing. Thus the marginal surplus path must be given by $\hat{p}_t(\lambda_0, \mu_0)$ during an initial time period $[0, \hat{t}(\lambda_0, \mu_0)]$ when the users are supplied with the NR resource, $q_t = x_t = \hat{x}_t(\lambda_0, \mu_0)$ and the ceiling is not binding. At $\hat{t}(\lambda_0, \mu_0)$ the ceiling is attained and a second phase begins, during which the consumption of the NR resource is constrained to $\bar{p}$, so that $q_t = x_t = \bar{p}$, until date $\hat{t}(\lambda_0)$ when $p_{et} = \hat{p}_t(\lambda_0)$. Next comes the last phase during which the NR resource is consumed, until time $\hat{\theta}(\lambda_0)$ when $p_{et} = c_r$. Note that during this phase since $\hat{p}_t(\lambda_0) > p_{et}$ we have $\hat{x}_t(\lambda_0) < \bar{p}$. Since $Z_{\hat{\theta}(\lambda_0)} = Z$ at the beginning of this phase, the ceiling is no longer binding suggesting that $\hat{p}_t(\lambda_0)$ is the correct value of the marginal surplus. At $\hat{\theta}(\lambda_0)$ the NR resource is exhausted and the economy switches to the costly $R$ resource.

![Fig. 3 here](image-url)

The co-state variables $\lambda_0$ and $\mu_0$ are determined as the solution to the system of the two following equations (14)-(15):

---

12 The case of increasing demand is reasonable to explore since global energy demand is expected to increase at least for the next several decades both from increases in population as well as in energy consumption per capita, especially in the developing countries.

13 Recall from section 2.3 that $\hat{x}_t(\lambda_0)$ is not necessarily monotonic but if $\hat{p}_t(\lambda_0, \mu_0) < p_{et} \iff \hat{x}_t(\lambda_0, \mu_0) > \bar{p}$ and $Z_0 < Z$ then the equation defining $\hat{Z}_t$ implies that $\hat{Z}_t(\lambda_0, \mu_0)$ is monotonically increasing up to the time when $\hat{Z}_t(\lambda_0, \mu_0) = Z$. 

---
i. the supply of the NR resource must equal cumulative consumption:
\[
\int_0^\tau(\lambda_0,\mu_0) \tilde{x}_t(\lambda_0,\mu_0) dt + \pi [\tau(\lambda_0) - \tilde{\tau}(\lambda_0,\mu_0)] + \int_{\tilde{\tau}(\lambda_0)}^{\tau(\lambda_0)} \tilde{x}_t(\lambda_0) dt = X_0; \tag{14}
\]

ii. the ceiling is attained at date \( \hat{\tau}(\lambda_0,\mu_0) \):
\[
\hat{Z}_{\hat{\tau}(\lambda_0,\mu_0)}(\lambda_0,\mu_0) = \mathcal{Z}. \tag{15}
\]

Note that if (14)-(15) are satisfied then the consumption path defined above is continuous.

In the Appendix we show that there exists values of the other dual variables, \( v_t, \gamma_{ct}, \) and \( \gamma_{rt} \) such that all the F.O.Cs. (2)-(12) are satisfied. We summarize as follows:

**Proposition 1** In the increasing demand case with \( \underline{p}_{\text{exc}} < c_r \), if \( \lambda_0 \) and \( \mu_0 \) satisfy (14)-(15), then the optimal consumption plan is given by:
\[
q_t = x_t = \tilde{x}_t(\lambda_0,\mu_0) \quad \text{for } t \in [0, \hat{\tau}(\lambda_0,\mu_0)];
\]
\[
q_t = \pi \quad \text{for } t \in [\hat{\tau}(\lambda_0,\mu_0), \tilde{\tau}(\lambda_0)];
\]
\[
q_t = x_t = \tilde{x}_t(\lambda_0) \quad \text{for } t \in [\tilde{\tau}(\lambda_0), \tilde{\theta}(\lambda_0)];
\]
\[
q_t = y_t = y_{ct} \quad \text{for } t \in [\tilde{\theta}(\lambda_0), +\infty).
\]

### 4.2 Case (b) \( c_r < \underline{p}_{ct0} \)

This case is illustrated in Fig. 4 and the solution is somewhat analogous to Fig. 2. Since \( \underline{p}_{ct} > c_r \), \( t \geq 0 \), then during the time interval within which the ceiling is tight, the marginal surplus of consumption must equal \( c_r \) and both resources are used. The optimal plan then has the following structure. The first phase during which the consumption of the NR resource is not constrained, we have \( q_t = x_t = \tilde{x}_t(\lambda_0,\mu_0) \). This phase ends at time \( \tilde{\theta}(\lambda_0,\mu_0) \) when \( \tilde{\theta}(\lambda_0,\mu_0) = c_r \). Since \( c_r < \underline{p}_{\tilde{\theta}(\lambda_0,\mu_0)} \), at time \( \tilde{\theta}(\lambda_0,\mu_0) \) the consumption of the NR resource must fall, from \( \tilde{x}_{\tilde{\theta}(\lambda_0,\mu_0)}(\lambda_0,\mu_0) \) to \( \pi \) while the consumption of the R resource jumps from 0 to
\[
y_{\tilde{\theta}(\lambda_0,\mu_0)} - \pi,
\]
so that aggregate energy consumption is continuous. During the second phase
\[
[\tilde{\theta}(\lambda_0,\mu_0), \tilde{\theta}(\lambda_0)]
\]
both resources are simultaneously used, \( q_t = y_{ct}, x_t = \pi \) and \( y_t = y_{ct} - \pi \), and at \( \tilde{\theta}(\lambda_0) \) the NR resource is exhausted. At \( \tilde{\theta}(\lambda_0) \) the consumption rate of the NR resource falls from \( \pi \) to 0 and the consumption of the R resource jumps from \( y_{\tilde{\theta}(\lambda_0)} - \pi \) to \( y_{\tilde{\theta}(\lambda_0)} \), keeping aggregate consumption continuous. Beginning from \( \tilde{\theta}(\lambda_0) \), only the R resource is exploited.

**Fig. 4 here**
The two conditions determining \( \lambda_0 \) and \( \mu_0 \) now take the following form:

\[
\int_0^{\hat{\theta}(\lambda_0, \mu_0)} \tilde{x}_t(\lambda_0, \mu_0) dt + \varpi \left[ \tilde{\theta}(\lambda_0) - \hat{\theta}(\lambda_0, \mu_0) \right] = X_0, \quad (16)
\]

\[
\hat{Z}_{\hat{\theta}(\lambda_0, \mu_0)}(\lambda_0, \mu_0) = \bar{Z}. \quad (17)
\]

In the Appendix, we show that if (16)-(17) are satisfied, then all the F.O.C.s. (2)-(12) are also satisfied.

**Proposition 2** In the increasing needs case with \( c_r < \overline{p}_e \), if \( \lambda_0 \) and \( \mu_0 \) satisfy (16)-(17), then the optimal consumption plan is given by:

\[
x_t = \begin{cases} 
\tilde{x}_t(\lambda_0, \mu_0), & t \in \left[ 0, \hat{\theta}(\lambda_0, \mu_0) \right] \\
\varpi, & t \in \left[ \hat{\theta}(\lambda_0, \mu_0), \tilde{\theta}(\lambda_0) \right] \\
0, & t \in \left[ \tilde{\theta}(\lambda_0), +\infty \right)
\end{cases} \quad \text{and} \quad
y_t = \begin{cases} 
0, & t \in \left[ 0, \hat{\theta}(\lambda_0, \mu_0) \right] \\
y_c t - \varpi, & t \in \left[ \hat{\theta}(\lambda_0, \mu_0), \tilde{\theta}(\lambda_0) \right) \\
y_c t, & t \in \left[ \tilde{\theta}(\lambda_0), +\infty \right)
\end{cases}
\]

### 4.3 Case (c) \( \overline{p}_{e_0} < c_r < \overline{p}_{e_\infty} \)

There may exist three types of optimal consumption plans in this case. The first two are like the optimal plans of the extreme cases \( \overline{p}_{e_\infty} < c_r \) and \( c_r < \overline{p}_{e_0} \), while the third one is specific to the present case.

In order to explain why we may have an optimal plan of the type characterized in Proposition 1, let us assume that \( \tilde{\theta} \left( \lambda_0^e \right) < \bar{\theta} \) and that \( \bar{\theta} - \tilde{\theta} \left( \lambda_0^e \right) \) is large, that the overshooting of the upper bound \( \bar{Z} \), \( \max \left\{ \tilde{Z}_t \left( \lambda_0^e \right), t \in \left[ 0, \tilde{\theta} \left( \lambda_0^e \right) \right] \right\} - \bar{Z} \), is not too large and the time interval during which the pollutant stock \( \tilde{Z}_t \left( \lambda_0^e \right) \) is higher than the ceiling \( \bar{Z} \), is not too long. Then the period during which the consumption of the resource is constrained by \( \varpi \) when the ceiling is binding is likely to be short and ending before \( \bar{\theta} \) (because \( \bar{\theta} - \tilde{\theta} \left( \lambda_0^e \right) \) is large and \( \lambda_0^e - \lambda_0 \) is small) as illustrated in Fig. 5. Several factors may lead to this outcome, such as a sufficiently high stock of the NR resource, a large cost differential between the two resources, or a relatively "clean" quality nonrenewable resource (small \( \zeta \)).

**Fig. 5 here**
Symmetrically, let us assume that $\overline{\theta} < \tilde{\theta} \left( \lambda_0' \right)$ and that $\tilde{\theta} \left( \lambda_0' \right) - \overline{\theta}$ is large, that $\max \left\{ \tilde{Z}_t (\lambda_0, \mu_0), t \in [0, \tilde{\theta} \left( \lambda_0' \right)] \right\} - \underline{Z}$, is not too large and that the time interval during which $\tilde{Z}_t (\lambda_0, \mu_0) > \underline{Z}$ is not too long, as shown in Fig. 6. Then the time period during which $x_t = \overline{x}$, $[\tilde{\theta} (\lambda_0, \mu_0), \tilde{\theta} (\lambda_0)]$, when the constraint binds, is short, and the optimal path is like the one characterized in Proposition 2. This is likely to happen if the stock of the NR resource is small (leading to a higher scarcity rent), the cost of the R resource is relatively low or the NR resource is highly polluting.

**Fig. 6 here**

Now consider again the case illustrated in Fig. 7. If $X_0$ is sufficiently high, the time period during which the consumption of the NR resource is constrained by $\overline{x}$, is long enough to end after time $\overline{\theta}$. Once $\overline{p}_{ct} > c_r$, then for $t > \overline{\theta}$, it is optimal to use the least costly NR resource at its maximal rate $\overline{x}$ and to supplement consumption with the R resource. Thus the optimal path is like the one illustrated in Fig. 5 below. During a first phase $[0, \tilde{\tau} \left( \lambda_0' \right)]$, $q_t = x_t = \tilde{x}_t (\lambda_0, \mu_0)$ and at $\tilde{\tau} (\lambda_0, \mu_0)$, the ceiling is attained. During a second phase, from $\tilde{\tau} (\lambda_0, \mu_0)$ to $\overline{\theta}$, the consumption of the NR resource is given by $q_t = x_t = \overline{x}$ while the R resource is not competitive. At $\overline{\theta}$ the R resource becomes competitive and during the third phase $[\overline{\theta}, \tilde{\theta} (\lambda_0)]$ both resources are used, the least costly NR resource at its maximal rate $x_t = \overline{x}$ and the R resource providing the complement $y_t = y_{ct} - \overline{x}$. At $\tilde{\theta} (\lambda_0)$ the NR resource is exhausted and only the R resource is used subsequently.

**Fig. 7 here**

$\lambda_0$ and $\mu_0$ are solutions to the system of two equations (15)-(18):

$$\int_0^{\tilde{\tau} (\lambda_0, \mu_0)} \tilde{x}_t (\lambda_0, \mu_0) dt + \overline{x} \left[ \tilde{\theta} (\lambda_0) - \tilde{\tau} (\lambda_0, \mu_0) \right] = X_0.$$  

(18)

In the Appendix, we show that if (15)-(18) are satisfied then all the F.O.Cs (2)-(12) are also satisfied.

**Proposition 3** In the increasing demand case with $\overline{p}_{e0} < c_r < \overline{p}_{e\infty}$:

14In this case, it must occur at the end of the time interval $[0, \tilde{\theta} \left( \lambda_0' \right)]$.
- either there exists values of \( \lambda_0 \) and \( \mu_0 \) satisfying (14) and (15), then the optimal path is the same as in Proposition 1;

- either there exists values of \( \lambda_0 \) and \( \mu_0 \) satisfying (16) and (17), then the optimal path is the same as in Proposition 2;

- or, lastly, there exists values of \( \lambda_0 \) and \( \mu_0 \) satisfying (15) and (18), in which case the optimal path is given by:

\[
\begin{align*}
x_t &= \begin{cases} 
\hat{x}_t(\lambda_0, \mu_0) & , t \in [0, \hat{\tau}(\lambda_0, \mu_0)] \\
\overline{x} & , t \in [\hat{\tau}(\lambda_0, \mu_0), \hat{\theta}(\lambda_0)] \\
0 & , t \in [\hat{\theta}(\lambda_0, \mu_0), +\infty)
\end{cases} \\
y_t &= \begin{cases} 
0 & , t \in [0, \overline{\theta}] \\
y_{ct} - \overline{x} & , t \in [\overline{\theta}, \hat{\theta}(\lambda_0)] \\
y_{ct} & , t \in [\hat{\theta}(\lambda_0), +\infty)
\end{cases}
\end{align*}
\]

In general, we get three distinct cases: (i) only the nonrenewable resource is used at the ceiling followed by a Hotelling transition to the renewable resource; (ii) both nonrenewable and renewable resources are used at the ceiling until the former is exhausted and emissions decline, and (iii) a combination of the two: the ceiling stage itself has two phases: initial exclusive use of the nonrenewable resource, followed by the use of both resources until the nonrenewable resource gets exhausted.

## 5 Decreasing Demand

The case of decreasing demand may be important in the long-run, if for instance, global fertility rates and population decline faster than expected, as has been predicted by several recent projections (Lutz et al., 2001).\(^{15}\) With decreasing demand, \( \overline{p}_{ct} \) decreases with time, hence \( \overline{p}_{e0} > \overline{p}_{e\infty} \). Again, it is convenient to classify the solution according to the cost of the NR resource relative to the marginal utility at the ceiling., i.e., (a) \( \overline{p}_{e0} < c_r \), (b) \( c_r < \overline{p}_{e\infty} \) and (c) \( \overline{p}_{e\infty} < c_r < \overline{p}_{e0} \).

\[^{15}\text{World population is expected to rise from its present level of 6 billion to about 9 billion in 2070, then decline to 8.4 billion in 2100. Fertility rates are falling below replacement levels not only in the developed countries but also in some 74 intermediate-fertility developing countries. With the general aging of these societies, energy demand will likely follow this declining trend, albeit with a time lag that accounts for an increase in energy consumption per capita for residents of developing countries.}\]
5.1 Case (a) $\overline{p}_{e0} < c_r$

The case is illustrated in Fig. 8. The optimal consumption path is similar to the case $\overline{p}_{e\infty} < c_r$ under ID, except that here the marginal surplus $\overline{p}_{et}$ is decreasing during the phase $[\overline{\tau}(\lambda_0, \mu_0), \overline{\tau}(\lambda_0)]$ when consumption of the NR resource is constrained by $\overline{\tau}$. It follows that the price path first increases, then decreases, and increases again before equaling $c_r$ once the NR resource is exhausted. However energy consumption always decreases except during the phase $[\overline{\tau}(\lambda_0, \mu_0), \overline{\tau}(\lambda_0)]$.

Fig. 8 here

The values of $\lambda_0$ and $\mu_0$ are determined as the solution to (14)-(15), as in the case $\overline{p}_{e\infty} < c_r$ under ID.

**Proposition 4** In the decreasing needs case with $\overline{p}_{e0} < c_r$, if $\lambda_0$ and $\mu_0$ satisfy (14)-(15) then the optimal consumption path is the same as in Proposition 1.

5.2 Case (b) $c_r < \overline{p}_{e\infty}$

This case is illustrated in Fig. 9. The marginal surplus path is the same as in the case $c_r < \overline{p}_{e0}$ under ID, because in both cases $\overline{p}_{et} > c_r, t \geq 0$. The consumption path is permanently decreasing.

Fig. 9 here

The values of $\lambda_0$ and $\mu_0$ are determined as the solution of (16)-(17), as in the case $c_r < \overline{p}_{e0}$ under ID.

**Proposition 5** In the decreasing needs case with $c_r < \overline{p}_{e\infty}$, if $\lambda_0$ and $\mu_0$ satisfy (16)-(17) then the optimal consumption path is the same as in Proposition 2.
5.3 Case (c) $\bar{p}_{e\infty} < c_r < \bar{p}_e$

As in case $\bar{p}_e < c_r < \bar{p}_{e\infty}$ under ID, we again have three solutions, two of which are similar to the above two cases and are not discussed separately. We only describe the third path, illustrated in Fig. 10. This case is unique because both the low cost NR resource and the high cost R resource are exploited simultaneously for a period followed by exclusive use of the NR resource and then the R resource. The backstop R resource is used during two disjoint time periods.

Consider the case illustrated in Fig. 8. If $X_0$ is higher than $\overline{X}_0$ and sufficiently large, then the period during which $x_t = \overline{x}$ has to be so long so that $\hat{\theta}(\lambda_0, \mu_0) < \bar{\tau}(\tau, \mu_0) < \overline{\theta}$ as illustrated in Fig. 10, and the optimal marginal surplus path has five phases. During the first phase $[0, \hat{\theta}(\lambda_0, \mu_0)]$ the marginal surplus is given by $\hat{\psi}_t(\lambda_0, \mu_0)$ and $q_t = x_t = \hat{x}_t(\lambda_0, \mu_0)$, and at the end of the phase the ceiling is attained. At $t = \hat{\theta}(\lambda_0, \mu_0)$, we have $c_r < \bar{p}_e$ and $\hat{x}_t(\lambda_0, \mu_0) = y_{ct} > \overline{x}$ so that consumption of the NR resource must fall from $\hat{x}_t(\lambda_0, \mu_0)$ to $\overline{x}$ while consumption of the R resource jumps from 0 to $y_{ct} - \overline{x}$. During the second phase $[\hat{\theta}(\lambda_0, \mu_0), \overline{\theta}]$, both resources are used, the NR resource being constrained by $\overline{x}$: $q_t = y_{ct}$, $x_t = \overline{x}$ and $y_t = y_{ct} - \overline{x}$. In the third phase $[\overline{\theta}, \bar{\tau}(\lambda_0)]$, the R resource is no longer exploited and the consumption of the NR resource is constrained by $\overline{x}$: $q_t = x_t = \overline{x}$. During the fourth phase $[\bar{\tau}(\lambda_0), \hat{\theta}(\lambda_0)]$ consumption of the NR resource decreases, $q_t = x_t = \bar{x}_t(\lambda_0)$ to $y_{ct}\hat{\theta}(\lambda_0)$. At $t = \bar{\tau}(\lambda_0)$ the NR resource is exhausted and during the last phase $[\hat{\theta}(\lambda_0), +\infty)$ the R resource is the sole supplier, $q_t = y_t = y_{ct}$. In summary, the expensive R resource is used along with the cheaper NR resource so that the stock of emissions is kept under the ceiling, and the cheaper NR resource is used exclusively (first at the maximal rate, then lower) when the ceiling is no longer binding. Finally, exhaustion of the NR resource leads to the terminal phase with the R resource.

Fig. 10 here

The two conditions determining $\lambda_0$ and $\mu_0$ are (17) and at date $\hat{\theta}(\lambda_0, \mu_0)$ when the ceiling is reached, the following cumulative consumption/supply balance must hold:

$$\int_0^{\hat{\theta}(\lambda_0, \mu_0)} \hat{x}_t(\lambda_0, \mu_0) dt + \overline{x} [\bar{\tau}(\lambda_0) - \hat{\theta}(\lambda_0, \mu_0)] + \int_{\hat{\tau}(\lambda_0)}^{\overline{\tau}(\lambda_0)} \bar{x}_t(\lambda_0) dt = X_0. \quad (19)$$
Proposition 6 Under decreasing demand with \( \overline{p}_{e0} < c_r < \overline{p}_{e0} \):

- either there exists values of \( \lambda_0 \) and \( \mu_0 \) satisfying (14) and (15), then the optimal path is the same as in Proposition 4 (or 1);

- either there exists values of \( \lambda_0 \) and \( \mu_0 \) satisfying (16) and (17), then the optimal path is the same as in Proposition 5 (or 2);

- or, lastly, there exists values of \( \lambda_0 \) and \( \mu_0 \) satisfying (17) and (19), in which case the optimal path is given by:

\[
x_t = \begin{cases} 
\tilde{x}_t(\lambda_0, \mu_0), & t \in [0, \tilde{\theta}(\lambda_0, \mu_0)] \\
\bar{x}, & t \in [\tilde{\theta}(\lambda_0, \mu_0), \tilde{\tau}(\lambda_0)] \\
\tilde{x}_t(\lambda_0), & t \in [\tilde{\tau}(\lambda_0), \tilde{\theta}(\lambda_0)] \\
0, & t \in [\tilde{\theta}(\lambda_0), +\infty) 
\end{cases}
\]

\[
y_t = \begin{cases} 
0, & t \in [0, \tilde{\theta}(\lambda_0, \mu_0)] \cup [\bar{\theta}, \tilde{\theta}(\lambda_0)] \\
y_{ct} - \bar{x}, & t \in [\tilde{\theta}(\lambda_0, \mu_0), \bar{\theta}] \\
y_{ct}, & t \in [\tilde{\theta}(\lambda_0), +\infty) 
\end{cases}
\]

6 Abatement with Non-stationary Demand

All the cases examined above can be viewed as special cases of the general model with abatement when the unit cost of pollution abatement is high enough for it not to be economically feasible. For instance, as shown in Fig. 3, a sufficiently high abatement cost \( c_a \) would imply that the price path \( \tilde{p}_t(\lambda_0) \) is higher than the path \( \hat{p}_t(\lambda_0, \mu_0) \) until the former intersects the cost of the \( R \) resource at time \( \tilde{\theta}(\lambda_0) \). Beyond this point, it is cheaper to "reduce" emissions by replacing a unit of the \( NR \) resource with a unit of the \( R \) resource.¹⁶

In what follows, we avoid the complete characterization of the solution with abatement (which is similar to the no abatement cases and is available from the authors), but only describe the different types of solutions for the timing of emission reductions by focusing on two typical cases: a medium and a low cost of abatement, \( c_a \).¹⁷

¹⁶By definition, \( \tilde{p}_t(\lambda_0) \) is parallel to the price path \( \hat{p}_t(\lambda_0) \) because \( \tilde{p}_t(\lambda_0) = \hat{p}_t(\lambda_0) + \zeta c_a \).

¹⁷The high cost case has been subsumed in the cases described in the previous section.
6.1 Increasing Demand

6.1.1 (a) $\pi_{\infty} < c_r$ and low cost of abatement, $c_a$.

Consider the case when the cost of abatement $c_a$ is low. Here abatement starts the instant the ceiling is reached. Note that $\tilde{p}_t(\lambda_0) < \pi_{st}$ at $t = \tilde{\tau}(\lambda_0, \mu_0)$ as shown in Fig. 11, so that $\tilde{\tau}(\lambda_0, \mu_0) \in [\delta(\lambda_0, \mu_0), \tilde{\tau}(\lambda_0))$. Once the ceiling is attained at time $\delta(\lambda_0, \mu_0)$, abatement becomes immediately economical and continues until time $\tilde{\tau}(\lambda_0)$. The amount of pollution abated at any instant is given by $a_t = \zeta(x_t - \bar{x})$. In the next phase $(\tau_2(\lambda_0), \tilde{\tau}(\lambda_0))$, $p_t(\lambda_0) > \pi_{st}$, the pollution stock is still at the ceiling but there is no abatement. The ceiling becomes nonbinding at time $\tilde{\tau}(\lambda_0)$ and the subsequent phases which are shown in the figure are the same as discussed previously.

Fig. 11 here

6.1.2 (b) $\pi_{\infty} < c_r$ and medium cost of abatement, $c_a$.

If the cost of abatement is relatively high, then the period of abatement $[\tilde{\tau}_1(\lambda_0), \tilde{\tau}_2(\lambda_0)]$, shown in Fig. 12, is a strict subset of the time period during which the ceiling is binding, $[\tilde{\tau}(\lambda_0, \mu_0), \tilde{\tau}(\lambda_0)]$. At the ceiling the NR resource is the sole supplier of energy until time $\tilde{\tau}_1(\lambda_0)$ when abatement becomes optimal. Abatement stops at time $\tilde{\tau}_2(\lambda_0)$ and the NR resource supplies exclusively both at the ceiling and beyond, until it is exhausted. The remaining path mimics cases described earlier.

Fig. 12 here

6.1.3 (c) $c_r < \pi_{e0}$ and low cost of abatement, $c_a$.

When the cost of the $R$ resource is low, and abatement is cheap, these two act as substitutes for the polluting NR resource at the ceiling, as shown in Fig. 13. Active abatement starts exactly when the ceiling is attained, but the $R$ resource becomes economical at time $\tilde{\theta}(\lambda_0)$. Because the cost of the $R$ resource is constant and the shadow price of abatement rises with time, the two cannot be used jointly. The two resources are used jointly at the ceiling until the NR resource
is exhausted.

Fig. 13 here

6.2 Decreasing Demand

Assuming \( c_r < \bar{p}_{e0} \) and a low cost of abatement \( c_a \), active abatement may also occur under decreasing demand, as shown in Fig. 14. When the cost of abatement is low, active abatement may occur immediately upon hitting the ceiling. If abatement is feasible, then it must be done at the beginning of the ceiling period, since demand declines over time. Under decreasing demand, we do not get the case shown in Figure 12 in which the abatement interval is strictly in the interior of the ceiling.

Fig 14 here

We do not characterize all the possible solutions under abatement but focus here on the more interesting cases. Several of the cases described earlier may occur when \( \bar{p}_{e0} < c_r < \bar{p}_{e\infty} \) (ID) and \( \bar{p}_{e\infty} < c_r < \bar{p}_{e0} \) (DD). However, we can make some general observations on the timing of pollution abatement. Abatement emerges as an alternative to the use of the clean \( R \) resource. There is never a period with active abatement and use of the \( R \) resource. Since the real cost of abatement rises at an exponential rate, it may be cheaper than the \( R \) resource in the beginning but will eventually become more expensive. Pollution will only be abated during a period when the stock is at its ceiling. It will never be done before the ceiling becomes binding because it is always beneficial to postpone it to the future, given the positive discount rate.

7 Concluding Remarks

This paper is a first attempt at extending Hotelling theory to the case when the stock of pollution from a nonrenewable resource is constrained. We consider both increasing and decreasing demand, as well as the possibility of abatement. One general result is that in all cases, the stock of pollution builds up over time followed by an interval in which the ceiling is binding. Beyond this interval, the emission stock declines to zero as energy supply shifts from
the exclusive use of the nonrenewable to that of the renewable resource. However, the details of this transition differ from case to case. If the renewable backstop is cheap or the nonrenewable resource is highly polluting, or the imposed ceiling is low, the renewable may be used along with the nonrenewable resource exactly at the instant the ceiling is attained. This pattern is maintained until the nonrenewable resource is exhausted. In another case when the nonrenewable resource is abundant or mildly polluting, or the backstop is expensive, the supply of energy at the ceiling is provided only by the nonrenewable resource, followed by a transition phase when extraction declines until the nonrenewable resource is exhausted and is replaced by the renewable resource.

One particular case of decreasing demand is unique because both resources may be used at the ceiling, followed by the exclusive use of the nonrenewable, and finally the renewable resource is used again in the terminal period. It suggests two disjoint periods of time when the renewable resource may be used.

Modeling a nonrenewable resource with a pollution ceiling is a first step towards developing theory that can examine substitution across multiple energy resources (oil, coal, natural gas and renewables) under agreements like the Kyoto Protocol. Empirical trends such as the recent transition from coal burning to the cleaner natural gas in power generation can be better examined in a model with multiple nonrenewable resources each with different emission characteristics. For example, what is the optimal extraction sequence when we have two scarce resources - a cheap dirty resource (e.g., coal) and a costly clean resource (e.g., natural gas)?

However, useful policy insights can be obtained even from the simple model developed here. One implication is that the standard Hotelling solution of a transition from a polluting fossil fuel to a clean renewable resource may be overly simplistic when there is a ceiling on the stock of emissions. There may be a strong case for use of the renewable resource during the period when the ceiling is tight, even though the cost of the backstop is higher than that of the fossil fuel and the latter has not been exhausted. Thus, solar or other renewable technologies may need to be used to supplement the use of fossil fuel resources, even if they are not economical in terms of the unit cost of energy. Conventional wisdom, which suggests that renewables cannot be used because they are currently costlier than fossil fuels, may be somewhat misplaced.
and it is realistic to expect global energy demand to peak and then decline over the long-term, the joint extraction of fossil fuels and solar energy may be feasible, if an international agreement were in place. For instance, we may use expensive solar energy for a time when the ceiling is binding (e.g., meet limits set by the Kyoto Protocol) then revert back to a "Hotelling" world with coal as the primary source of energy. Empirical work needs to be done using the Hotelling framework to see which of the cases considered here are likely given plausible parameter values.

Reducing emissions through abatement, such as through carbon sequestration by forests, emerges as a clear alternative to renewable energy. If the cost of abatement is sufficiently high, it will never be used. The renewable resource will be employed jointly with the nonrenewable resource at the ceiling. When abatement is economically feasible, it is used only during the period when the emissions ceiling is binding. Over time, the true cost of abatement increases exponentially, and eventually the renewable resource becomes cheaper. Thus, abatement technologies compete with the renewable resource and only one or the other will be employed at any given time. However, in future work, one could specify nonlinear abatement (e.g., the unit cost of bringing land under forests may rise with volume abated) and renewable energy cost functions, in which case both these options may be used simultaneously at the ceiling.

Yet another possible extension could include the endogenous choice of a ceiling and its welfare and political economy implications. For example, it may be realistic to expect the ceiling to become more stringent over time as societies become more sensitive to environmental externalities. In a model with multiple countries, the choice of a ceiling may be the outcome of a bargaining process.
Appendix

For the various cases discussed in sections 4 and 5, it is easy to check that all the first order conditions (2)-(12) are satisfied by the values of $\gamma_{et}, \gamma_{rt}, \mu_t$ and $v_t$. In all cases, the NR resource is exhausted in finite time and $\lim_{t \to +\infty} Z_t = 0$, so that the transversality conditions (12) are satisfied. We only consider the following cases:

- **ID with $\overline{p}_{e\infty} < c_r$:**
  \[
  \gamma_{et} = \begin{cases} 
  0, & t \in \left[0, \tilde{\theta}(\lambda_0)\right] \\
  \tilde{p}_t(\lambda_0) - c_r, & t \in \left[\tilde{\theta}(\lambda_0), +\infty\right),
  \end{cases}
  \]
  \[
  \gamma_{rt} = \begin{cases} 
  c_r - \tilde{p}_t(\lambda_0), & t \in \left[0, \tilde{\tau}(\lambda_0, \mu_0)\right] \\
  c_r - \overline{p}_{et}, & t \in \left[\tilde{\tau}(\lambda_0, \mu_0), \tilde{\tau}(\lambda_0)\right) \\
  c_r - \tilde{p}_t(\lambda_0), & t \in \left[\tilde{\tau}(\lambda_0, \tilde{\theta}(\lambda_0)\right) \\
  0, & t \in \left[\tilde{\theta}(\lambda_0), +\infty\right),
  \end{cases}
  \]
  \[
  \mu_t = \begin{cases} 
  \mu_0 e^{(\rho + \alpha)t}, & t \in \left[0, \tilde{\tau}(\lambda_0, \mu_0)\right] \\
  -\zeta^{-1} \left[\overline{p}_{et} - \tilde{p}_t(\lambda_0)\right], & t \in \left[\tilde{\tau}(\lambda_0, \mu_0), \tilde{\tau}(\lambda_0)\right) \text{ and} \\
  0, & t \in \left[\tilde{\tau}(\lambda_0), +\infty\right),
  \end{cases}
  \]
  \[
  v_t = \begin{cases} 
  0, & t \notin \left[\tilde{\tau}(\lambda_0, \tilde{\theta}(\lambda_0)\right) \\
  -\zeta^{-1} \left[(\rho + \alpha) \left(\overline{p}_{et} - \tilde{p}_t(\lambda_0)\right) - \left(\overline{p}_t - \tilde{p}_t(\lambda_0)\right)\right], & t \in \left[\tilde{\tau}(\lambda_0, \tilde{\theta}(\lambda_0)\right).
  \end{cases}
  \]

Since $\overline{p}_{et} - \tilde{p}_t(\lambda_0)$ is decreasing in the time interval $[\tilde{\tau}(\lambda_0, \mu_0), \tilde{\tau}(\lambda_0))$, $v_t > 0$.

- **ID with $c_r < \overline{p}_{e0}$:**
  \[
  \gamma_{et} = \begin{cases} 
  0, & t \in \left[0, \tilde{\theta}(\lambda_0)\right] \\
  \tilde{p}_t(\lambda_0) - c_r, & t \in \left[\tilde{\theta}(\lambda_0), +\infty\right),
  \end{cases}
  \]
  \[
  \gamma_{rt} = \begin{cases} 
  c_r - \tilde{p}_t(\lambda_0), & t \in \left[0, \tilde{\theta}(\lambda_0, \mu_0)\right] \\
  0, & t \in \left[\tilde{\theta}(\lambda_0, \mu_0), +\infty\right),
  \end{cases}
  \]
\[
\mu_t = \begin{cases} 
\mu_0 e^{(\rho+\alpha)t}, & t \in \left[0, \tilde{\theta} (\lambda_0, \mu_0)\right] \\
-\zeta^{-1} [c_r - \tilde{p}_t (\lambda_0)], & t \in \left[\tilde{\theta} (\lambda_0, \mu_0), \tilde{\theta} (\lambda_0)\right] \text{ and} \\
0, & t \in \left[\tilde{\theta} (\lambda_0), +\infty\right), 
\end{cases}
\]

\[
v_t = \begin{cases} 
0, & t \in \left[0, \tilde{\theta} (\lambda_0)\right] \\
-\zeta^{-1} \left[ (\rho + \alpha) (c_r - \tilde{p}_t (\lambda_0)) + \tilde{p}_t (\lambda_0) \right], & t \in \left[\tilde{\theta} (\lambda_0, \mu_0), \tilde{\theta} (\lambda_0)\right) \text{ and} \\
0, & t \in \left[\tilde{\theta} (\lambda_0), +\infty\right).
\end{cases}
\]

- **ID with** \( p_{e0} < c_r < p_{\infty} \), illustrated in Fig. 5:

\[
\gamma_{et} = \begin{cases} 
0, & t \in \left[0, \tilde{\theta} (\lambda_0)\right] \\
\tilde{p}_t (\lambda_0) - c_r, & t \in \left[\tilde{\theta} (\lambda_0), +\infty\right), 
\end{cases}
\]

\[
\gamma_{rt} = \begin{cases} 
0, & t \in \left[0, \tilde{\theta} (\lambda_0, \mu_0)\right) \\
c_r - \tilde{p}_t (\lambda_0), & t \in \left[\tilde{\theta} (\lambda_0, \mu_0), \tilde{\theta} (\lambda_0)\right) \text{ and} \\
c_r - p_{et}, & t \in \left[\tilde{\theta} (\lambda_0, \mu_0), \tilde{\theta} (\lambda_0)\right), 
\end{cases}
\]

- **DD with** \( p_{\infty} < c_r < p_{e0} \), illustrated in Fig. 10:

\[
\gamma_{et} = \begin{cases} 
0, & t \in \left[0, \tilde{\theta} (\lambda_0)\right) \\
\tilde{p}_t (\lambda_0) - c_r, & t \in \left[\tilde{\theta} (\lambda_0), +\infty\right), 
\end{cases}
\]
\[ \gamma_{rt} = \begin{cases} 
& c_r - \hat{p}_t (\lambda_0, \mu_0), t \in [0, \hat{\theta} (\lambda_0, \mu_0)) \\
& 0, t \in \left(\hat{\theta} (\lambda_0, \mu_0), \hat{\theta} \right) \cup \left(\hat{\theta} (\lambda_0), +\infty\right) \\
& c_r - \bar{p}_{et}, t \in \left[\hat{\theta}, \bar{\tau} (\lambda_0)\right) \\
& c_r - \tilde{p}_t (\lambda_0), t \in \left[\bar{\tau} (\lambda_0), \hat{\theta} (\lambda_0)\right), 
\end{cases} \]

\[ \mu_t = \begin{cases} 
& \mu_0 e^{(\rho + \alpha)t}, t \in [0, \hat{\theta} (\lambda_0, \mu_0)) \\
& -\zeta^{-1} [c_r - \hat{p}_t (\lambda_0)], t \in \left(\hat{\theta} (\lambda_0, \mu_0), \hat{\theta} \right) \text{ and} \\
& -\zeta^{-1} [\bar{p}_{et} - \tilde{p}_t (\lambda_0)], t \in \left[\hat{\theta}, \bar{\tau} (\lambda_0)\right) \\
& 0, t \in \left[\bar{\tau} (\lambda_0), +\infty\right), 
\end{cases} \]

\[ v_t = \begin{cases} 
& 0, t \notin [0, \hat{\theta} (\lambda_0, \mu_0)) \cup [\bar{\tau} (\lambda_0), +\infty) \\
& -\zeta^{-1} \left[(\rho + \alpha) (c_r - \hat{p}_t (\lambda_0)) + \hat{p}_t (\lambda_0)\right], t \in [\hat{\theta} (\lambda_0, \mu_0), \hat{\theta}] \\
& -\zeta^{-1} \left[(\rho + \alpha) (\bar{p}_{et} - \tilde{p}_t (\lambda_0)) - \left(\bar{p}_{et} - \tilde{p}_t (\lambda_0)\right)\right], t \in [\hat{\theta}, \bar{\tau} (\lambda_0)). 
\end{cases} \]
References


Figure 1: Stationary Demand ($\bar{p}_e < c_r$) - only the nonrenewable resource is used at the ceiling. 
NB: The dashed curve $c_e + \lambda_0 e^{p_t}$ is the standard Hotelling price path when the ceiling is never binding.
Figure 2: Stationary Demand ($\bar{p}_e > c_r$) - both resources are used at the ceiling
Figure 3: Increasing Demand ($\overline{p}_e < c_r$) - only the nonrenewable resource is used at the ceiling
Figure 4: Increasing Demand ($c_r < \bar{p}_{e0}$) - both resources are used at the ceiling
Figure 5: Increasing Demand ($p_{e0} < c_r < p_{e\infty}$) - only the nonrenewable resource is used at the ceiling
Figure 6: Increasing Demand ($p_{e0} < c_r < p_{e\infty}$) - both resources are used at the ceiling
Figure 7: Increasing Demand ($\bar{p}_{e0} < c_r < \bar{p}_{e\infty}$) - the ceiling has two phases - the nonrenewable is used first, followed by both resources
Figure 8: Decreasing Demand ($\overline{p}_e < c_r$) - only the nonrenewable resource is used at the ceiling
Pollution stock at the ceiling

Figure 9: Decreasing Demand ($\bar{p}_{\infty} > c_r$) - both resources are used at the ceiling
Figure 10: Decreasing Demand ($p_{e0} > c_r > p_{e\infty}$) - the renewable resource is used during two disjoint time periods
Figure 11: Abatement starts exactly at the instant the ceiling becomes binding
Figure 12: Abatement occurs strictly within the period when the ceiling is binding
Figure 13: Abatement starts when the ceiling becomes binding but is replaced by the renewable resource.
Figure 14: Abatement starts when the ceiling becomes binding but is replaced by the renewable