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Business Fixed Investment and "Bubbles":
The Japanese Case

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In recent years, there has been a lively debate on whether stock market prices always equal the expected present value of future dividends. A variety of theoretical work has called the simple present value model of stock prices into question. Empirical studies have provided evidence that stock prices may vary too much relative to dividends, that investors may overreact, that there may be fads in stock market prices, and that there may be a tendency to be overly optimistic about the future performance of stocks that have done well in the recent past. Much of this theoretical and empirical work has been vigorously challenged, so the academic debate is far from over.

Among policy-makers, there is a long-standing concern that extreme movements in asset markets may adversely affect the real economy. For example, Friedman and Schwartz's Monetary History of the United States shows that the main concern of the Federal Reserve in the late 1920s was how to curb stock market speculation (see especially pages 253-270). The almost universal reaction of central bankers to the 1987 stock market crash was to inject liquidity into the system. More recently, the high price of U.S. equities led to Federal Reserve Board Chairman Greenspan's concern about "irrational exuberance" and the mid-1998 worldwide decline in equity markets raised fears about subsequent effects on real economic activity.

In this paper, we refer to deviations of actual stock market prices from the expected present value of future dividends as "bubbles." This could include fads, noise trading, and other phenomena as well as the more restricted class of deviations sometimes referred to as "rational bubbles". The two key questions which motivate our work are: Do bubbles exist? If bubbles exist, do they have an effect on the real economy?

We focus in particular on the possible effects of bubbles on business fixed investment. It is widely understood that business fixed investment is relatively volatile and may play an important role in economic fluctuations. Previous research on this issue has focused primarily on the U.S. stock market, which tends to be less volatile than many other major stock markets. As a result, tests for the effects of bubbles on investment may be less powerful.

The recent experience in Japan provides a particularly interesting case. The rise of the Japanese stock market in the late 1980s and its decline in the early 1990s dwarf the 1987 stock market crash in the U.S. From 1980 to the peak in 1989, in real terms, the Japanese stock market rose 373%, and fell by 50% in the subsequent three years. (Comparable figures for the U.S.)
indicate a run-up of 73% from 1980 to 1987; the 1987 decline was restored in one year.) This phenomenon was widely perceived as a bubble not only by the Japanese public (for whom the phrase "the bubble economy" has come to characterize this period), but also for many economists and policy-makers. For example, Chairman Greenspan described the plunge in the Japanese stock market as "a correction of the bubble in asset prices" (Washington Post, April 18, 1992, p. A13). In Japan, there was a strong perception that the bubble was an important issue for monetary policy. In fact, Yasushi Mieno, who took over as governor of the Bank of Japan in 1989, is reported to pride himself on being the man who made ending speculative excesses a primary objective of monetary policy (The Economist, February 15, 1992, p. 91).

Higher stock prices provide firms with a potentially cheap source of finance, so we might expect firms to react by issuing equity. Economic theory, however, leaves open the question of whether business fixed investment will respond to a bubble. On the one hand, firms may use the earnings-price ratio as a measure of the cost of capital so that a stock price bubble will lower the discount rate applied to future cash flows and stimulate investment. We refer to this as an active financing mechanism. On the other hand, firms might view stock prices as unreasonably high and issue new shares but put the proceeds into cash and securities, rather than fixed investment. We refer to this as an inactive financing mechanism. These contrasting theoretical possibilities are reflected in differing interpretations of the importance of the putative bubble on real variables. Some prominent academics have concluded that the fall in equity prices had little effect on the Japanese economy (Malkiel, 1995, p.37) or that stock prices generally have little impact on Japanese firms (Porter, 1992, p.9). Alternatively, others have claimed that the bursting of the bubble has been a primary culprit in the prolonged Japanese recession (Aliber and Solomon, 1993, p.2; The Economist, July 9, 1994, p.14).

In Section I, we examine informal evidence on whether bubbles existed in Japan and, if so, whether they affected business fixed investment. In Section II, we show how the theory of fixed investment can be used to derive an innovative set of tests for whether bubbles exist and whether they affect fixed investment and present the main empirical results using these tests. Like West (1987), our orthogonality tests are based on the insight that first-order conditions can be helpful in detecting bubbles. In Section III, we examine the role of serially correlated adjustment shocks.
Our main orthogonality tests do not require us to measure the fundamental stock price, but tests in Section III B are based on forecasts of fundamentals, which raise the possibility of forecast error. In Section IV, we use several approaches to see whether forecast error is an important factor in determining the results in Section III B. In Section V, we introduce an additional orthogonality test for whether there was an active financing mechanism. In Section VI, we take a much different approach to assessing whether a bubble may have affected investment; we estimate standard investment equations, adding a measure of the bubble as an additional regressor. In Section VII, we summarize the empirical results and present our interpretation of the data.

I. Informal Evidence

A. Principal Enterprise Data

If its shares are overvalued, a firm may have an incentive to issue new shares to take advantage of the "arbitrage" opportunity. Thus, one sign of overvaluation would be a dramatic increase in new share issues.\textsuperscript{4} Equity issues increased steadily during the latter part of the 1980s and peaked in 1988-89 at a level more than twice as high as the mean over the sample period. Moreover, if firms suspect that there are investors with overly optimistic beliefs about their shares, it makes sense for them to take advantage of the situation by issuing bonds which include an option to purchase shares. As Malkiel (1995, p.35) notes, Japanese "corporations began to float bonds in the European markets with interest rates as low as one percent, by offering warrants to buy the companies' stock." In fact, a large proportion of the bonds issued by Japanese firms in the late 1980s had an equity option. In 1987, 1988, and 1989, convertible bonds and bonds with warrants accounted for 81%, 87% and 89%, respectively, of the volume of new, publicly offered corporate bonds. By comparison, the proportions were 21% in 1979 and 39% in 1990 (Yamamoto (1993), Table 6.3, p.222).

Figure 1 plots the sum of Principal Enterprises' equity and bond issues (divided by total investment).\textsuperscript{5} As the figure shows, combined issues of equity and bonds were very high in the late 1980s relative to the late 1970s and early 1980s. Issues of equity and bonds were much lower in 1990 and 1991. The scale of the vertical axis is noteworthy: in 1989, enough funds were raised from equity and bond issues to cover more than 88% of total investment spending. This was an exceptionally large amount; over the sample period, these sources covered an average
of less than 30% of investment spending. Figure 1 also shows long-term borrowing by Principal Enterprises from banks and insurance companies (divided by total investment). The figure is consistent with a gradual reduction in firms' reliance on banks but displays a time pattern during the late 1980s and early 1990s quite different from equity and bond issues.

If firms believe that high stock prices simply reflected excellent growth opportunities, they would plow the funds they raise back into investment. On the other hand, if they do not believe their investment opportunities fully justify the high stock prices, then they would put funds into cash and securities. Figure 2 shows changes in holdings of cash and securities (divided by total investment) for Principal Enterprises. This ratio rises during the late 1980s to reach a peak which coincides with the peak of the stock market in 1989 and then falls dramatically in 1990 and 1991. Again, the scale of the vertical axis is noteworthy. At the peak of the stock market, firms were putting almost as much into cash and securities as they were into total investment (92% in 1988 and 89% in 1989). This use of funds, combined with marked changes in the source of funds toward equity and equity-related securities, suggests that firms believed that investment opportunities, while solid, were not exceedingly robust and equities were overvalued.

B. Aggregate Data

Whether this possible overvaluation impacted real investment depends on the financing mechanism. If the financing mechanism was active, investment should have boomed during the late 1980s. For the aggregate economy, the ratio of net investment to output rose sharply in the late 1980's and fell in the early 1990's. Of course, the investment boom in the late 1980s could have been due to changes in market interest rates or other factors affecting investment demand. To control for these factors, we estimate a non-structural investment equation with the investment/capital ratio \( \frac{I_t}{K_t} \), effective exchange rate, and percentage changes in the discount factor, the relative price of investment, and real output. (The equation was estimated with annual aggregate data, using two lags of each variable. To allow for the possibility of structural changes in the late 1980's, we reestimated the equation for each year from 1984 through 1992.) On average, \( \frac{I_t}{K_t} \) was 19% higher than predicted by the non-structural equation in the years 1987-1991 and 7% lower in 1992. Figure 3 plots \( \frac{I_t}{K_t} \) minus the predicted \( \frac{I_t}{K_t} \) (as a percent of the predicted \( \frac{I_t}{K_t} \)) for the full sample period. The pattern illustrated in Figure 3 is consistent
with the view that the putative bubble boosted investment through an active financing mechanism in the late 1980s.

II. Orthogonality Tests: Theory and Main Empirical Results

A. Theory

We use an innovative approach to test whether bubbles exist and, if so, whether they affect fixed investment. Our approach is in the spirit of the orthogonality tests which have been pioneered by Hall (1978), used frequently in the asset pricing (Hansen and Singleton, 1983) and consumption literatures (Deaton (1992)), and applied in the investment literature (Hubbard and Kashyap, 1992; Whited, 1992). As noted above, our orthogonality tests are also in the spirit of West's (1987) insight, namely that bubbles disturb the link between the stock price and the present value of future dividends but do not necessarily affect Euler equations. Our use of orthogonality tests based on a system of first-order conditions also distinguishes our work from the previous papers (referenced in endnote 3) assessing whether bubbles affect investment. In our case, by exploiting information from the firm’s optimization problem, we are able to distinguish hypotheses of economic interest without making strong parametric assumptions about the bubbles process. Unlike previous tests for the existence of bubbles, our approach uses information from the investment decisions of firms and does not require measurement of fundamentals and hence the bubble component of equity prices.

In our first case, we assume that there is no bubble in the stock market. The first-order conditions for optimal investment by a profit-maximizing firm facing convex costs when adjusting its capital stock imply that:

\[ Q_t^* \& C_{t,t} \uparrow a_t \]

\[ \left( F_{K,t} \& C_{K,t} \right) \& (p_t \% C_{t}) \% R_t \left( p_t \% C_{t,t} \right) \uparrow e_t \]

where \( Q_t^* \) is the expected present value of future marginal products of capital (\( Q^*_t \)) less the price of
investment goods relative to the price of output \( (p_I) \) (i.e., \( Q^*_t = ?^*_t - p_I ) \). \( C_{It} \) and \( C_{Kt} \) are the first derivatives of the cost of adjusting the capital stock with respect to investment \( (I_t) \) and the capital stock \( (K_t) \). \( \alpha_t \) is the adjustment cost shock, \( \epsilon_t \) is the Euler equation error (which contains the expectational error and the adjustment cost shock), \( F_{K,t} \) is the marginal product of capital, and \( R^*_t \) is the discount factor, equal to \( (1-d)/(1+r_t) \) where \( d \) is the depreciation rate and \( r_t \) is the real interest rate. In the empirical work, we employ second-order Taylor series expansions for \( C_{It} \), \( C_{Kt} \), and \( F_{K,t} \), described below in sub-Section B. Equation (1) cannot be estimated because it contains the unobserved \( Q^*_t \). Based on the result from Hayashi (1982), that under stock market efficiency and other conditions, we can equate \( Q^*_t \) to \( Q_{SM}^t \), where \( Q_{SM}^t \) is stock market \( Q \); i.e., the market value of the firm (based on the stock market) divided by the capital stock (adjusted for the timing of depreciation) less the price of investment goods. We refer to the first equation as the \( Q \) equation and the second as the Euler equation.  

In the second case we consider, there are bubbles in the stock market, but the bubble \( (B_t) \) has no effect on the discount factor which firms use in evaluating investment projects; that is, the financing mechanism is inactive, and the \( Q \) equation becomes:

\[
(3) \quad Q_{SM}^t \& C_{It} \& \alpha_t \% B_t
\]

Intuitively, the reason \( B_t \) appears on the right hand side is because \( Q_{SM}^t \) now equals \( Q^*_t + B_t \), so we need to add \( B_t \) to both sides of (1) to express the relationship between investment and \( Q_{SM}^t \) (which, unlike \( Q^*_t \), is directly observable). The Euler equation is unchanged. Note that the bubble now appears on the right hand side of the \( Q \) equation but not the Euler equation.

In the third case, there are bubbles in the stock market and they affect the discount factor, so \( R^*_t \) in (2) is replaced by \( R_t = R^*_t + f(B_t) \), where \( f(B_t) \) is a positive and increasing function of \( B_t \). That is, the financing mechanism is active, and the \( Q \) and Euler equations become:

\[
(4) \quad Q_{SM}^t \& C_{It} \& \alpha_t \% B_t \& f(B_t)
\]
\[
(5) \quad (F_{K,t} \& C_{K,t}) \& (p_I \% C_{It}) \% R_t^{(p)} \% C_{It}^{(p)} \% \alpha_t \% \alpha_{(p)}
\]

where \( ?^B_t(B_t) \) is the wedge between two present values of future profitability, one evaluated at the market interest rate and the other evaluated at the lower discount rate induced by a bubble. This
wedge is a function of the bubble because an active financing mechanism implies that firms use the
discount factor \( R_t^* + f(B_t) \) in evaluating future cash flows. \( ?^{B_t}(B_t) \) is an increasing function of \( B_t \) and
will be positive when \( B_t > 0 \). For purposes of the tests, the key point to note is that, when the
financing mechanism is active, the bubble appears on the right-hand side of both the Q and Euler
equations. \(^9\)

The instrumental variables procedure we use is designed to detect correlation between
variables in the time \( t-1 \) information set and the right-hand side of the Q and Euler equations.
More precisely, the tests described by Hansen (1982) and Eichenbaum, Hansen, and Singleton
(1988) can determine which subsets of the orthogonality conditions associated with the Q or Euler
equations are suspect. \(^10\) The orthogonality tests can therefore be used to distinguish
between the three economically interesting cases we have described. If there are no bubbles in the
stock market, both the Q and Euler equations will pass the specification test. If the Q equation is
rejected but the Euler equation passes, this is consistent with the existence of bubbles but provides
no evidence that bubbles affect investment. If both the Q and Euler equations are rejected, this is
consistent with the existence of bubbles and an effect of bubbles on the discount rate and
therefore on investment. Of course, the model rejections could also be consistent with a non-
bubble misspecification of the basic investment model. In Sections III and IV, we introduce a
number of tests designed to check this possibility.

B. The Standard Investment Model

The Euler and Q equations rely on two technological functions — the marginal product of
capital and marginal adjustment costs, which are parameterized by second-order Taylor
expansions:

\[
MPK_t = (F_{K,t} & C_{K,t})^2 \int_0^2 \beta_j (Y_t/K_t)^j \%_j (I_t/K_t) + C_{I,t}^2 \int_0^2 a_j (I_t/K_t)^j
\]

Before turning to the empirical results, we briefly summarize the data used in the
orthogonality tests; details can be found in Appendix A. (All appendices are available from the
authors.) Real investment \( (I_t) \), real output \( (Y_t) \), and the price of investment goods relative to the
price of output \( (p_t^I) \) are drawn from standardized national income account data published by the
OECD. The constant dollar capital stock \((K_t)\) is net of depreciation (at rate \(d\) equal to 0.10), and is constructed with \(I_t\) using the perpetual inventory method. Stock market \(Q\) \((Q_{SM}^t)\) is measured by the TOPIX index of equity share prices on the Tokyo Stock Exchange adjusted for the replacement values of fixed capital and land. Linear projection \(Q\) \((Q_{LP}^t)\) is calculated by the technique in Section III.B. The flow of liquidity \((LIQ_t)\) is taken from the Financial Statements of Principal Enterprises (which covers only large firms), and equals retained earnings plus depreciation expenses (in real terms) divided by \(K_t\). The discount factor \((R_t^*)\) depends on \(d\) and the real discount rate \((r_t)\); the latter equals the nominal money market rate less the inflation rate in output. All of the instrumental variables are drawn from the above data, except for the growth rate in \(M1_t\). All of the forecasting variables (listed below equation 11) are drawn from the above data, except for the real effective exchange rate \((ER_t, a\ weighted-average of the currencies of 24 countries per yen) and the relative price of oil \((p_{oil}^t, the price of North Sea Brent crude oil at customs clearance in yen relative to the price of output). After accounting for all leads and lags, we can compute estimates for the period 1966-1992, with the exception of any models containing \(LIQ_t\) for which estimates can be computed for 1966-1991.

Table 1 presents the orthogonality tests for non-linear three-stage least squares estimates of the Standard model with aggregate data for 1966-1992. The estimates are based on two instrument sets, Primary and Money, whose specific elements are listed in the note to Table 1.\(^{11}\) For both choices of instrument sets, the orthogonality tests strongly reject the \(Q\) equation. Using the Primary instrument set, the Euler equation is also rejected at standard significance levels but, using the Money instrument set, the data fail to reject the Euler equation. As shown in the subsection A, the rejection of the \(Q\) equation suggests that there was a bubble in the stock market, and the rejection of the Euler equation using the Primary instrument set provides some evidence that the bubble affected investment.
C. Robustness Tests

i. Accounting for Liquidity

Much recent theoretical and empirical research has emphasized the possibility that at least some firms face finance constraints. To capture the impact of capital market frictions, we follow the recent literature and insert liquidity as an additional regressor in both the Q and Euler equations. The orthogonality tests in columns 3 and 4 of Table 1 continue to reject the Q equation, suggesting the existence of bubbles. For the Euler equation, the p-values are .06 and .24 for the Primary and Money instrument sets, respectively. Thus, the evidence is clear for bubbles but mixed on whether bubbles affect investment.

ii. Heteroscedasticity

The NL3SLS estimator is chosen instead of Hansen's (1982) Generalized Method of Moments (GMM) estimator, which allows for conditional heteroscedasticity, because the covariance matrix of orthogonality conditions required for GMM estimation is nearly singular, as indicated by its condition number. The condition number is x, where 10^x is the ratio of the largest to the smallest eigenvalue for the covariance matrix. A matrix is ill-conditioned if the condition number is large relative to the inverse of the precision of the algorithm, equal to 12 in the computer program used here. For the Standard model, the condition numbers equalled or exceeded 12 for both instrument lists.

Diagnostic tests show evidence of conditional heteroscedasticity in the Q equation. An ARCH test (specifically a regression of squared residuals on a constant and the first lag of squared residuals) rejects the null hypothesis of no conditional heteroscedasticity with a p-value of .000. (This test is based on the residuals from the first column of Table 1. All of the results in this subsection are qualitatively similar for the residuals from the second column of Table 1.)

The presence of conditional heteroscedasticity is not surprising in light of the evidence in favour of a bubble. As equation (3) shows, when there is a bubble, it will be contained in the Q equation residual. This suggests that transforming the variables in the Q equation (by dividing them by the absolute value of B_{t-1}) should largely eliminate heteroscedasticity. In fact, the p-value for the ARCH test statistic for the Q residuals from the transformed model is .578. Using the transformed model, the "Test Q" and "Test Euler" statistics (and p-values) are 25.762 (.012) and
13.637 (.058), respectively.

iii. Single Equation Estimates

If there are bubbles in the stock market and this leads to correlation between the instruments and $u_t^Q$, as the tests suggest, then estimating the system of $Q$ and Euler equations could lead to inconsistency in the Euler equation.$^{13}$ One way to address this issue is by estimating the $Q$ and Euler equations separately. The results are qualitatively very similar to those from the Standard model in Table 1. The $Q$ equation is always rejected (with p-values of about .01) and the Euler equation is rejected using the Primary instrument set (p-value=.038).

III. The Role of Serially Correlated Adjustment Cost Shocks

Several different orthogonality tests presented in Section II point strongly toward the existence of a bubble and suggest that the financing mechanism was active. However, this evidence could be consistent with a situation where bubbles are absent and shocks to the adjustment cost function ($a_t$) are serially correlated. More generally, a variety of forms of misspecification might lead to something resembling serial correlation of $a_t$. In this section, we examine whether serially correlated $a_t$ could account for the rejection of the $Q$ equation. The three tests presented in sub-section A do not rely on forecasts of the fundamentals. In sub-section B, we take a stand on fundamentals, and introduce two additional tests robust to serially correlated adjustment cost shocks.

A. Tests Independent of Forecasts of Fundamentals

A straightforward and non-parametric way to check the interpretation of the orthogonality tests is to plot the $Q$ residuals. If there is a stock market bubble, the residual from the $Q$ equation will contain the bubble term (cf. equations (3) and (4)) and we might expect the $Q$ residual to rise during the 1980s, as the stock market headed towards its peak, and decline in the early 1990s. Figure 3 plots the $Q$ residuals (from both the Standard and Liquidity models, using the Primary instrument set) and shows exactly this pattern. Something special appears to have occurred in the mid to latter 80's that is unlikely to be attributed to serial correlation in $a_t$.

If $a_t$ is MA(1), it is straightforward to show that the orthogonality tests reported above will be valid with instruments in the information set at $t$-2. When we repeat the orthogonality
tests for the Standard model in Table 1 with this change in the timing of the instruments, the Q equation is rejected with p-values of .012 and .004 for the Primary and Money instrument sets, respectively.

As a final check on our finding of a stock market bubble in the presence of a serially correlated \( a \), we derive in Appendix B a ratio of residual variance test from the structure of the Q and Euler error terms. This test is robust to either an AR(1) or MA(1) processes for \( a \). The ratio of residual variance statistic is \( RRV / \text{Var}[u^Q] / \text{Var}[u^E] \), where \( u^Q_t \) and \( u^E_t \) are the residuals from the Q and Euler equations, respectively. Under the assumption that the adjustment cost shocks are not excessively positively serially correlated (serial correlation coefficient < .80), then, if there is no bubble, \( RRV < 20 \); on the other hand, if there is a bubble that varies substantially, \( RRV \gg 20 \). This reversal occurs because \( u^Q_t \) directly reflects the bubble but \( u^E_t \) does not. As Table 1 shows, \( RRV \) is larger than 45 in all but one case.

**B. Tests Based on Forecasts of Fundamentals**

All of the above analysis is independent of forecasts of stock market fundamentals. We now introduce a VAR-based forecast of Q. When there is a bubble in the stock market, \( V_t = V^*_t + V^B_t \), where \( V^*_t \) and \( V^B_t \) are the fundamental and bubble components of stock market value, respectively. We define a bubble (normalized by the adjusted capital stock) as follows:

\[
B_t = \frac{V^B_t}{(1-d)K_t}, \frac{V^*_t + V^B_t}{(1-d)K_t}
\]

where the term \((1-d)\) adjusts the capital stock for the timing assumption that existing capital depreciates this period while new capital begins depreciating next period.

Key to measuring the bubble is estimating the fundamental stock market value. We rely on the result of Hayashi (1982) that stock market efficiency (and Hayashi’s other conditions) implies that

\[
V^*_t = \frac{\eta^*_t}{(1-d)K_t}
\]

where \( \eta^*_t \) is the expected present value of future marginal products of capital defined as:
\( \gamma_t = E_t \left[ \frac{4}{j \Delta t} R_t \left( \left( M_{t+j} \right) \right) \right] \)

where \( M_{t+j} = (F_{K,t+j} - C_{K,t+j}) \) (i.e., the marginal product of capital, including the reduction in adjustment costs due to increased \( K \)), \( R^* = (1-d)/(1+r_t) \) and \( E_t \) is the expectations operator conditional on the firm’s information set.

To measure \( \gamma_t \), we forecast \( M_t \) and \( R^*_t \) directly, following the procedure described by Abel and Blanchard (1986). Let \( \tilde{\gamma}_t \) be the ex post realization of the RHS of (8). Linearizing \( \tilde{\gamma}_t \) around \( R^*_t = \bar{R} \) and \( M_{t+j} = \bar{M} \), where a bar indicates the sample mean:

\[
\tilde{\gamma}_t \cdot \tilde{M}(1 \& \bar{R}) \% \tilde{M}(1 \& \bar{R}) \% S \tilde{R}(j \& \bar{R}) \% S \tilde{R}(j \& \bar{M})
\]

We then assume that \( M_t(z_t) \) and \( R^*_t(z_t) \) and other variables (\( z_t \) to \( z^5_t \)) useful in forecasting these two components of \( \tilde{\gamma}_t \) have the following vector autoregressive structure:

\[
\begin{bmatrix}
z^1_t \\
z^2_t \\
z^3_t \\
z^4_t \\
z^5_t
\end{bmatrix}
= 
\begin{bmatrix}
A^{11}(L) & A^{15}(L) \\
A^{21}(L) & A^{25}(L) \\
0 & 0 \\
0 & 0 \\
A^{51}(L) & A^{55}(L)
\end{bmatrix}
\begin{bmatrix}
z^1_t \\
z^2_t \\
z^3_t \\
z^4_t \\
z^5_t
\end{bmatrix}
+ 
\begin{bmatrix}
u^1_t \\
u^2_t \\
u^3_t \\
u^4_t \\
u^5_t
\end{bmatrix}
\]

We consider two sets of forecasting variables. Set 1 includes \( M_t, R^*_t, \) inflation, real exchange rate, and relative price of oil. Set 2 includes \( M_t, R^*_t, \) inflation, relative price of investment goods, and output growth. \( A^q(L) \) are polynomials in the lag operator. The previous equation can be rewritten in companion matrix form as:

\[
z_t \cdot A_{\&\Delta t} \% u_t
\]

where \( z_t = (z^1_t, z^1_{t-1},..., z^5_t, z^5_{t-1}) \) and \( u_t = (u^1_t, 0,...,u^5_t, 0) \). It is then possible to construct the expected present value of future profitability without using the stock market and thus exclude the bubble component. We refer to this as \( Q^{LP} \) since it is formed as a linear projection on a subset of the variables in the information set at \( t-1 \):
\[ Q_{t}^{LP} = E \left[ \sum_{j=0}^{\infty} \prod_{s=0}^{j-1} R_{t+s}^{*} M_{t+j} | z_{t-1} \right] - \rho_{t}^{I} \]

\[ = \left( \frac{1}{M(1 - \bar{R}^{*})} \right)^{-1} + \frac{1}{M(1 - \bar{R}^{*})} e_{2} \left( 1 - A \bar{R}^{*} \right)^{-1} A z_{t-1} \]

\[ + e_{1} \left( 1 - \bar{R}^{*} \right)^{-1} A z_{t-1} \right) - \rho_{t}^{I} \]

where \( e_{i} \) is a vector whose \( i^{th} \) element is 1 and all the other elements of which are 0. Since \( Q_{t}^{*} = \bar{Q}_{t}^{*} - \rho_{t}^{I} \) and \( Q_{t}^{LP} \) forecasts \( Q_{t}^{*} \), \( Q_{t}^{LP} = \bar{Q}_{t}^{*} - \bar{Q}_{t}^{*} - \rho_{t}^{I} \), where \( \bar{Q}_{t} \) is the forecast error in \( Q \) which arises because \( z_{t-1} \) is a subset of the firm’s information set at time \( t \). Combining (6), (7), and (12) and adding and subtracting \( \rho_{t}^{I} \), we can measure the bubble \( B_{LP}^{t} \) as the difference between stock market \( Q \) (\( Q_{t}^{SM} \), which will include the bubble, if it exists) and linear projection \( Q \) (\( Q_{t}^{LP} \), which will exclude the bubble): \( B_{LP}^{t} / Q_{t}^{SM} - Q_{t}^{LP} = B_{t} + \bar{Q}_{t} \).

For the moment, we assume that \( \bar{Q}_{t} = 0 \). We devote the next section to the implications of \( \bar{Q}_{t} \). Figure 4 plots \( Q_{t}^{SM} \) and \( Q_{t}^{LP} \), where \( Q_{t}^{LP} \) is based on the forecasting variables (\( z_{t}^{1}, ..., z_{t}^{5} \) listed below (11)) in Set 1. (The plot is similar using the forecasting variables in Set 2.) The plot of \( Q_{t}^{SM} \) is consistent with the hypothesis of a stock market bubble which began during the 1980s, reached a peak in 1989, and deflated during the early 1990s. \( Q_{t}^{LP} \) shows no similar pattern during the 1980s and 1990s.\(^{16} \)

Our explicit measurement of the bubble term allows us to calculate two additional tests robust to serially correlated \( a_{t} \). The first test begins with the observation that stock market bubbles will be reflected in the \( Q \) equation residual (cf. equations (3) and (4)). Consequently, there should be a positive correlation between the \( Q \) equation residual and the bubble. We construct several bubble measures: one set is based on the linear projections discussed above and another set is based on various dividend-based models of asset prices (see the foot of Table 1 and Appendix C for details). In fact, there is strong correlation (about .9) between the \( Q \) residual and these different measures of the bubble, as shown in Panel B of Table 1. This is considerably higher than the correlation of the \( Q \) residual with other macroeconomic variables. For example, the correlations with output growth, M1 growth, and the real interest rate are .126, -.266, and .351, respectively, for the Liquidity model using the Money instrument set. The strong positive
correlation of the Q residual with the bubble provides an additional piece of evidence that stock market bubbles are responsible for the rejection of the Q equation.17

The second test compares the Q residuals from estimates of the two-equation system using either \(Q_{\text{SM}}\) or \(Q_{\text{LP}}\). Denote the residual based on stock market Q (from Table 1) as \(u_{t}^{\text{QSM}}\) and the residual based on linear projection Q (12) as \(u_{t}^{\text{QLP}}\). These Q equation residuals allow us to distinguish between bubbles and serial correlation of adjustment cost shocks. When a bubble exists and the financing mechanism is active, these residuals and their difference (\(s_{t}\)) can be written as:

\[
\begin{align*}
  u_{t}^{\text{QSM}} &= \alpha_{t} \%B_{t} \& \gamma^{B}_{t}(B_{t}) \\
  u_{t}^{\text{QLP}} &= \alpha_{t} \& \gamma^{B}_{t}(B_{t}) \\
  s_{t} &= u_{t}^{\text{QSM}} + u_{t}^{\text{QLP}} - B_{t} 
\end{align*}
\]

In \(s_{t}\), the adjustment cost shock and \(\gamma^{B}_{t}(B_{t})\) cancel, and hence \(s_{t}\) is unaffected by any form of serial correlation in \(\alpha_{t}\). Under the null hypothesis that \(B_{t} = 0\), \(s_{t}\) will be serially uncorrelated even if there is serial correlation in \(\alpha_{t}\); if \(B_{t} \neq 0\), serial correlation of \(B_{t}\) will lead \(s_{t}\) to be serially correlated. We assess serial correlation in three separate models regressing \(s_{t}\) on \(s_{t-1}, s_{t-2}, \) or \(s_{t-3}\). The coefficients (standard errors) are .854 (.142), .661 (.256), and .574 (.360), respectively. These results are robust to serial correlation in \(\alpha_{t}\), and hence provide additional support for the conclusion that a bubble existed in the Japanese stock market.

**IV. The Role of Forecast Errors (\(\gamma_{t}\))**

A variety of formal and informal tests robust to serial correlation in \(\alpha_{t}\) continue to suggest the existence of a bubble. However, part of that evidence (reported in Section III.B) could be affected by \(\gamma_{t}\), which arises because \(Q_{\text{LP}}\) is constructed with an information set which is less extensive than the information set used by firms. Facing a similar issue, Campbell and Shiller (1987) pointed out that using the stock price could be very helpful. To the extent that the stock market rationally reflects available information, adding it to the set of forecasting variables should reduce any gap between \(Q_{t}^{\text{SM}}\) and \(Q_{t}^{\text{LP}}\) that arises because \(z_{t}\) fails to capture information available
to market participants. We therefore add stock market Q as a forecasting variable (in place of inflation), and calculate a market-information-augmented Q, $Q_t^{MIA}$. As shown in Figure 4, the inclusion of market information in the linear projection does not explain the dramatic rise in $Q_t^{SM}$ in the late 1980s and its rapid fall in the early 1990s. We then use $Q_t^{MIA}$ to compute a market-information-augmented bubble series ($B_t^{MIA}$) that should be less affected by $\hat{\nu}_t$ than $B_t^{LP}$. As reported in Panel B of Table 1, the correlation of $B_t^{MIA}$ with $u_t^{QSM}$ is very high. This suggests that the correlation of the bubble series with the Q equation residuals is not attributable to $\hat{\nu}_t$.

Two further tests use the identifying assumption that a stock market bubble, if it existed, would have been most pronounced following 1983. First, note that when $\hat{\nu}_t \neq 0$, the equations for the $s_t$ test become:

\begin{align*}
(14') & \quad u_t^{QLP} = a_t - \psi_t - \lambda_t B_t \\
(15') & \quad s_t = u_t^{QSM} - u_t^{QLP} = B_t + \psi_t
\end{align*}

If serial correlation in $\hat{\nu}_t$ is present and driving the results for the $s_t$ test, we would expect the serial correlation in $s_t$ to occur evenly over the sample period. This expectation is soundly rejected. The first-order serial correlation coefficients (and standard errors) for $s_t$ as defined in (15') are .347 (.211) for 1967-82 and .718 (.182) for 1983-92. While $\hat{\nu}_t$ surely exists, it is not the dominant force driving the results with the $s_t$ test.

Our second test continues to use the identifying assumption that the effects of a bubble will be most strongly felt in the latter part of the sample. By contrast, the impact of $\hat{\nu}_t$ will be more or less uniform over the sample. We recompute the correlations between $u_t^{QSM}$ and various bubble measures and macroeconomic variables for the sample periods 1966-82 and 1983-92. These correlations are presented in Table 2, and show a clear pattern. For the earlier period, the correlations are generally insignificant and, in some cases, even negative. However, for the latter part of the sample, the correlations for all bubble measures exceed .90. This pattern is consistent with bubbles, but difficult to rationalize as being due to $\hat{\nu}_t$.

The results presented in this section suggest one of two possibilities. Serially correlated $\hat{\nu}_t$ just happened to emerge in the 1980s or there was a stock market bubble. We can not absolutely reject the former interpretation, but the weight of evidence here and elsewhere in this
V. An Additional Orthogonality Test of the Financing Mechanism

An additional orthogonality test of the financing mechanism can be based on the linear projection of Q if we are willing to maintain that the forecast error is white noise. Suppose that there is a bubble, but the financing mechanism is inactive, so $B_t \sim \mathcal{O}$ and $f(B_t) = 0$. Under these assumptions, the Q equation becomes:

$$Q_t^{LP} \ & C_{t,t} \ & a_t \ & ?_t$$

Intuitively, (16) is like (1). Even though there is a bubble in the stock market, $B_t$ does not appear on the right hand side of (16) because $Q_t^{LP}$ is not contaminated by stock market prices and therefore corresponds to $Q_t^*$. Using $Q_t^{LP}$ does not change the Euler equation; it is still as shown in (2) above. Thus when the financing mechanism is inactive and we measure the present value of future profitability with a linear projection, the bubble will not appear on the right hand side of either the Q or Euler equations.

However, if the financing mechanism is active, so $f(B_t) \sim \mathcal{O}$, the Q equation (estimated with linear projection Q, $Q_t^{LP}$) becomes:

$$Q_t^{LP} \ & C_{t,t} \ & a_t \ & ?_t \ & ?^B_t(B_t)$$

and the Euler equation corresponds to (5). The bubble term now appears indirectly on the right-hand side of the Q equation through $?^B_t(B_t)$. Thus if the financing mechanism is active, the orthogonality tests will tend to reject the Q equation estimated with $Q_t^{LP}$.

The Q equation is rejected at the 0.01 level using either the Primary instruments or the Money instruments. (The test was run using $Q_t^{LP}$ constructed with the forecasting variables in Sets 1 and 2, as well as the market-information-augmented version of Set 1; the highest p-value across all of these tests was 0.007.) These results imply that there was a bubble and that it affected investment.
VI. Parametric Estimates

Given the strong evidence in sections II-V for the existence of bubbles, we focus exclusively in this section on the question of whether bubbles affect investment. Our empirical strategy is straightforward: we regress investment on the bubble and one or more variables which reflect the standard determinants of investment. To make the coefficients easier to interpret, we divide all variables by their standard deviations.

In closed-economy investment models, the variables which determine investment are lagged changes in interest rates, the price of investment goods, and output, as well as the lagged I/K ratio. In Table 3, we regress I/K on the bubble and these other variables. We consider two measures of the bubble, based on different sets of forecasting variables. For both measures of the bubble, the data suggest that the bubble has a statistically significant effect on investment. When liquidity is added to the specification, it enters significantly but has little impact on the estimated effect of the bubble.

Imports and exports are a larger proportion of output in Japan than in the U.S. and the real exchange rate or the price of oil might be important determinants of investment. In Table 4, we therefore add these two variables (both separately and in combination) to the specification. Both always enter with negative coefficients, suggesting that increases in either the value of the yen or in the relative price of oil may depress investment in Japan (although neither effect is ever statistically different from zero). After accounting for these potential open-economy determinants of investment, the t-statistics on the bubble are still generally close to two. If anything, accounting for these open-economy variables tends to raise the point estimates of the bubble's impact on investment.

As developed by Jorgenson and others, the neoclassical investment model suggests that investment depends on distributed lags of the change in output, the change in the cost of capital, and the level of the dependent variable. In Table 5, we add the bubble to such a specification. Using either bubble measure, the bubble has a highly significant effect on investment. The point estimates are similar to those in Table 3. When liquidity is added to the specification, the coefficient on liquidity is not significantly different from zero. The estimated effect of the bubble is similar regardless of whether or not liquidity is included.
A final model which has enjoyed frequent success in empirical studies of investment is the flexible accelerator. It is similar to the neoclassical model, except that the cost of capital terms are omitted. As columns 3 and 4 of Table 5 show, when the bubble is included in such a specification, it is a highly significant determinant of investment. The point estimates are similar to those in Table 3 and are minimally affected by the inclusion of liquidity.

VII. Summary and Conclusions

This paper has presented a wide range of empirical tests examining two questions: 1) do bubbles exist and, if so, 2) do they have an effect on business fixed investment?

If firms perceive an overvaluation of their stock, they have an incentive to issue equity and equity-linked securities. Figure 1 shows that issues of equity and bonds surged in the late 1980s and fell precipitously in the early 1990s. At the peak in 1989, the funds raised from these security issues were enough to cover almost 90% of the expenditures on business fixed investment by the principal Japanese enterprises, roughly three times the usual proportion.

The stock market boom of the late 1980s coincided with high levels of business fixed investment. When we use a non-structural forecasting equation which controls for macroeconomic factors which might have affected investment, we find that the investment/capital ratio was about 20% higher than predicted by these factors in the late 1980's, but lower than predicted following the crash.

When we turn from informal evidence to tests based on the first-order conditions for investment, the Eichenbaum, Hansen, and Singleton (1988) tests reject the orthogonality conditions which would hold if there were no bubbles. This rejection seems to be driven by the existence of bubbles. The correlation between various measures of bubbles and the residual from a standard first-order condition (which assumes no bubbles) is high. Additional robustness tests suggest the existence of bubbles. The orthogonality tests can also be used to examine whether bubbles affect fixed investment, specifically by lowering the cost of capital. In most cases, the data reject the first-order condition which would hold if bubbles did not affect investment.

The parametric estimates reinforce the evidence from the orthogonality tests. In most specifications, the data reject the null hypothesis that the bubble had no effect on investment. The
point estimates from standard closed-economy specifications of investment imply that the bubble boosted business fixed investment by approximately 6-9% in the years 1987-89. This amounts to about 1-2% of GDP in each of these three years. When we control for open-economy variables (such as the real exchange rate and the price of oil), the estimated effects tend to be larger.

A variety of types of evidence therefore suggests that there was a bubble in Japanese equity markets and that the bubble affected business fixed investment.

**Endnotes**


4. Analysing seasoned U.S. equity offerings, Loughran and Ritter (1995) find evidence "consistent with a market in which there are swings in investor sentiment which persist for many months. Companies respond by issuing equity during these windows of opportunity."

5. The data are for the period 1966 to 1991, and are drawn from the Financial Statements Of Principal Enterprises published by the Bank of Japan in *Economic Statistics Annual*. By focusing on Principal Enterprises (the 500-600 largest non-financial firms), we can examine the pattern of financial flows over time. In Figures 1 and 2, total investment is defined as the sum of fixed and inventory investments. See Appendix A for further details.

6. Kiyotaki and West (1996), using different techniques, also find that capital growth was stronger than predicted in 1986-91 and weaker than predicted in 1991-94.

7. For an example of a standard investment model from which equations of this form can be
derived, see Chirinko (1993a, Section III).

8. It should be noted that instrumental variables estimates may be sensitive to normalization (see West and Wilcox (1994) and Chirinko (1993b) for evidence for the Euler and Q equations, respectively). The normalization is shown in the equations (e.g., in the Q equation (1), $Q_t^*$ is the left-hand-side variable; it has a coefficient of one). In the Euler equation (2), $(-p_t^p R_{t+1}^p + R_{t+1}^p p_t^p)$ has a coefficient of one, and the other variables ($F_{K,t}, C_{K,t}$, and $C_{I,t}$) depend on estimated parameters.

9. If $B_t \neq 0$ and $B_t$ takes the form of a "rational bubble" (in the sense, for example, of Tirole (1982)), a technical issue may arise because of the presence of $B_t$ in the error term. This could affect the asymptotic distribution of the test statistics under the alternative hypothesis, as discussed in West (1985, 1987). Under the null hypothesis of no bubble, of course, the asymptotic distribution is unaffected by this issue.

10. The $J$ test evaluates the two-equation system, equals the criterion for the overall system, and is distributed $\chi^2$ with degrees of freedom equal to the number of orthogonality conditions (i.e., the number of instruments times the number of equations less the number of parameters in the system). The Eichenbaum, Singleton, and Hansen test evaluates subsets of orthogonality conditions, and equals the difference between the criteria for the overall system valid under the null and the equations valid under the alternative. These two estimates are both computed with elements of the weighting matrix for the overall system (i.e., the equations valid under the null). The test statistic is distributed $\chi^2$ with degrees of freedom equal to the number of instruments less the number of parameters that appear only in the "suspect" equation. A significant test statistic reveals that the misspecification in the econometric system is traceable to the equation valid only under the null.

11. The difference between these instrument sets is that $Q_{t-2}, I_{t-2}/K_{t-2},$ and $Y_{t-2}/K_{t-2}$ appearing in Primary are replaced by three lags of the growth rate of $M_{1,t}$, which we included in light of
suggestions that monetary policy may have played a role in the "bubble economy." The exclusion of $Y_{t-2}/K_{t-2}$ in the Money set appears to decrease the power of the test.


13. We thank Ken West for raising this point.

14. West (1987, p. 573-77) uses a related procedure. In order to construct the fundamental stock market price, he linearizes the present value of future dividends and uses past dividends to forecast future dividends. Abel and Blanchard (1986) use a larger information set, including variables such as past rates of return and marginal products of capital.

15. $z_i^t$, $i=1,...,5$, are expressed as deviations from their sample means. $M_t$ is based on the Taylor series expansion described in Section II.B. The VAR was computed with a lag length of two.

16. Analyzing price/earnings and price/dividend ratios, Ueda (1990) compares market to fundamental values and also finds a marked divergence in the latter 1980s. Using quarterly data for 14 industries, Ogawa et. al. (1994) compare stock market to linear projection $Q$, where the latter is estimated with univariate autoregressions of average profitability and discount rates. They find deviations in the latter 1980s similar to our Figure 3 which they interpret as “evidence for the bubbles or fads in the stock price” (p.28).

17. In Section IV, we consider whether this correlation could be due to $\tau_i$. 
References


Hayashi, Fumio. "Tobin's Marginal q and Average q: A Neoclassical Interpretation."


Poterba, James M., and Summers, Lawrence H. "Mean Reversion in Stock Prices: Evidence and


Table 1

Panel A: Orthogonality Tests Using Stock Market Q

<table>
<thead>
<tr>
<th></th>
<th>Standard Model</th>
<th></th>
<th>Liquidity Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Primary</td>
<td>Money</td>
<td>Primary</td>
<td>Money</td>
</tr>
<tr>
<td>Test System</td>
<td>37.228</td>
<td>28.115</td>
<td>29.953</td>
<td>25.286</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.002)</td>
<td>(0.031)</td>
<td>(0.008)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Test Q</td>
<td>36.573</td>
<td>27.986</td>
<td>29.862</td>
<td>25.254</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.000)</td>
<td>(0.006)</td>
<td>(0.002)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Test Euler</td>
<td>15.741</td>
<td>7.593</td>
<td>12.006</td>
<td>8.040</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.028)</td>
<td>(0.370)</td>
<td>(0.062)</td>
<td>(0.235)</td>
</tr>
<tr>
<td>RRV</td>
<td>107.5</td>
<td>47.2</td>
<td>59.2</td>
<td>19.8</td>
</tr>
<tr>
<td>MPK: Mean</td>
<td>0.209</td>
<td>0.254</td>
<td>0.383</td>
<td>0.260</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>[0.067]</td>
<td>[0.116]</td>
<td>[0.110]</td>
<td>[0.148]</td>
</tr>
<tr>
<td>MAC: Mean</td>
<td>-0.075</td>
<td>-0.048</td>
<td>0.417</td>
<td>0.503</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>[0.157]</td>
<td>[0.278]</td>
<td>[0.034]</td>
<td>[0.041]</td>
</tr>
</tbody>
</table>

Panel B: Residual Analysis

<table>
<thead>
<tr>
<th></th>
<th>B^1</th>
<th></th>
<th>B^2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.876</td>
<td>0.828</td>
<td>0.882</td>
<td>0.852</td>
</tr>
<tr>
<td>B^CS</td>
<td>0.981</td>
<td>0.953</td>
<td>0.939</td>
<td>0.896</td>
</tr>
<tr>
<td>B^W</td>
<td>0.976</td>
<td>0.945</td>
<td>0.926</td>
<td>0.879</td>
</tr>
<tr>
<td>B^MRS</td>
<td>0.982</td>
<td>0.965</td>
<td>0.961</td>
<td>0.926</td>
</tr>
<tr>
<td>B^MIA</td>
<td>0.933</td>
<td>0.801</td>
<td>0.914</td>
<td>0.839</td>
</tr>
<tr>
<td>Y Growth</td>
<td>-0.168</td>
<td>-0.072</td>
<td>0.035</td>
<td>0.126</td>
</tr>
<tr>
<td>M1 Growth</td>
<td>-0.413</td>
<td>-0.341</td>
<td>-0.311</td>
<td>-0.266</td>
</tr>
<tr>
<td>r</td>
<td>0.408</td>
<td>0.395</td>
<td>0.366</td>
<td>0.351</td>
</tr>
</tbody>
</table>

In Panel A, "Test System" is the Hansen (1982) test statistic of overidentifying restrictions; "Test Q" and "Test Euler" are the test statistics of Eichenbaum, Hansen, and Singleton (1988) for the validity of the Q and Euler equations, respectively. (See footnote 10 for details of the construction of these statistics.) P-values are in parentheses. NL3SLS estimates for 1966-1992.
for the Standard model. The Liquidity model adds liquidity to both the Q and Euler equations. Liquidity (LIQ) is defined as retained earnings plus depreciation, normalized by the capital stock. Because of data availability, the Liquidity model estimates are for 1966-1991. The Primary instrument set includes a constant, time, time squared, the real interest rate lagged, the second lagged forward differences in the real price of investment goods and I/K, and the first and second lags of Q, I/K, Y/K. The Money instrument set replaces the second lags of Q, I/K, and Y/K with three lags of the growth rate of M1. The MAC statistic is defined as: $\text{MAC}_t = C_{I,t}/(p_{I,t}^2 + C_{I,t})$. For MPK and MAC, the means and standard deviations are calculated over the sample period. The RRV is the ratio of the Q equation residual to the Euler equation residual, as described in Section III.D. and Appendix B.

In Panel B, the entries represent the correlation between the variable shown and the Q equation residual for each specification. The .05 critical value for the correlations is .381. $B^1$, $B^2$, $B^{CS}$, $B^W$, and $B^{MRS}$ are measures of bubbles. $B^1$ and $B^2$ equal $Q^{SM} - Q^{LP}$, where $Q^{LP}$ comes from (12); for $B^1$, we use Set 1 of the forecasting variables $z^1, ..., z^5$ as outlined in (10)-(12) and for $B^2$ we use Set 2 of the forecasting variables. $B^{CS} = P - P^{CS}$, where $P^{CS}$ is based on Campbell and Shiller (1988). $B^W = P - P^W$, where $P^W$ is based on the West (1987) equations for fundamental price. $B^{MRS} = P - P^{MRS}$, where $P^{MRS}$ equals $P^o$ in equation (21) of Mankiw, Romer, and Shapiro (1991). $B^{MIA}$ equals $Q^{SM} - Q^{LP}$ and is similar to $B^i$ except that stock market Q is used as an additional forecasting variable in place of inflation. Details are contained in Appendix C (available from the authors). The residuals come from the estimations presented in Panel A.
Table 2

Residual Analysis By Sub-Periods

<table>
<thead>
<tr>
<th></th>
<th>Standard Model</th>
<th>Liquidity Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^1$</td>
<td>-0.38</td>
<td>0.915</td>
</tr>
<tr>
<td>$B^2$</td>
<td>-0.298</td>
<td>0.972</td>
</tr>
<tr>
<td>$B^{CS}$</td>
<td>0.206</td>
<td>0.992</td>
</tr>
<tr>
<td>$B^W$</td>
<td>0.234</td>
<td>0.99</td>
</tr>
<tr>
<td>$B^{MRS}$</td>
<td>0.335</td>
<td>0.992</td>
</tr>
<tr>
<td>$B^{MIA}$</td>
<td>-0.056</td>
<td>0.974</td>
</tr>
<tr>
<td>Y Growth</td>
<td>0.291</td>
<td>0.527</td>
</tr>
<tr>
<td>M1 Growth</td>
<td>0.027</td>
<td>0.66</td>
</tr>
<tr>
<td>$r$</td>
<td>0.303</td>
<td>-0.231</td>
</tr>
</tbody>
</table>

The entries represent the correlation between the variables shown and the Q equation residual. The Primary instrument set is used in computing these residuals. Results are similar using the Money instrument set. See the notes to Table 1 for a description of different measures of bubbles.
Table 3
Parametric Estimates
Closed-Economy Specifications

<table>
<thead>
<tr>
<th></th>
<th>Bubble Measure: $B_1$</th>
<th>Bubble Measure: $B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without LIQ</td>
<td>With LIQ</td>
</tr>
<tr>
<td>$\delta Y_{t-1}$</td>
<td>0.437 (0.141)</td>
<td>0.277 (0.106)</td>
</tr>
<tr>
<td>$\delta R_{t-1}$</td>
<td>-0.053 (0.067)</td>
<td>-0.047 (0.043)</td>
</tr>
<tr>
<td>$\delta p^I_{t-1}$</td>
<td>-0.033 (0.091)</td>
<td>0.039 (0.063)</td>
</tr>
<tr>
<td>$(I/K)_{t-1}$</td>
<td>0.658 (0.145)</td>
<td>0.506 (0.111)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.671 (0.393)</td>
<td>0.814 (0.293)</td>
</tr>
<tr>
<td>LIQ$_t$</td>
<td></td>
<td>0.398 (0.128)</td>
</tr>
<tr>
<td>$B_t$</td>
<td>0.143 (0.045)</td>
<td>0.149 (0.036)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.913</td>
<td>0.961</td>
</tr>
<tr>
<td>$LM_1$</td>
<td>-0.926</td>
<td>-0.518</td>
</tr>
</tbody>
</table>

The dependent variable is $I/K$. The numbers in parentheses are standard errors which account for heteroscedasticity. Ordinary Least Squares estimates for 1966-1991. All model variables are divided by their standard deviations. $B_1$ and $B_2$ are two measures of the bubble; see Table 1 for further explanation. LIQ is liquidity, defined as retained earnings plus depreciation, normalized by the capital stock. $LM_1$ is a modified Lagrange Multiplier statistic that evaluates the null hypothesis of no first-order residual serial correlation (against AR(1) and MA(1) alternatives), and is distributed t under the null.
Table 4
Parametric Estimates
Open-Economy Specifications

<table>
<thead>
<tr>
<th></th>
<th>Bubble Measure: $B^1$</th>
<th></th>
<th>Bubble Measure: $B^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta Y_{t-1}$</td>
<td>0.461 (0.132)</td>
<td>0.433 (0.142)</td>
<td>0.452 (0.125)</td>
</tr>
<tr>
<td>$\Delta R_{t-1}$</td>
<td>-0.050 (0.061)</td>
<td>-0.055 (0.071)</td>
<td>-0.056 (0.071)</td>
</tr>
<tr>
<td>$\Delta p_{t-1}$</td>
<td>0.001 (0.095)</td>
<td>-0.030 (0.097)</td>
<td>0.021 (0.103)</td>
</tr>
<tr>
<td>$(I/K)_{t-1}$</td>
<td>0.631 (0.147)</td>
<td>0.641 (0.161)</td>
<td>0.560 (0.178)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.923 (1.256)</td>
<td>0.792 (0.642)</td>
<td>2.768 (1.636)</td>
</tr>
<tr>
<td>ER_{t-1}</td>
<td>-0.165 (0.151)</td>
<td>-0.216 (0.153)</td>
<td>-0.206 (0.156)</td>
</tr>
<tr>
<td>$p_{oil}^{t-1}$</td>
<td>-0.024 (0.074)</td>
<td>-0.092 (0.100)</td>
<td>-0.014 (0.078)</td>
</tr>
<tr>
<td>$B_t$</td>
<td>0.266 (0.150)</td>
<td>0.134 (0.058)</td>
<td>0.271 (0.142)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.917</td>
<td>0.908</td>
<td>0.916</td>
</tr>
<tr>
<td>$LM_1$</td>
<td>-1.537</td>
<td>-0.829</td>
<td>-1.500</td>
</tr>
</tbody>
</table>

See the notes to Table 3. The numbers in parentheses are standard errors which account for heteroscedasticity. ER is the real effective exchange rate; $p_{oil}^{t-1}$ is the relative price of oil.
Table 5  
Parametric Estimates  
Neoclassical and Accelerator Specifications

<table>
<thead>
<tr>
<th></th>
<th>Neoclassical</th>
<th>Accelerator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Y_t$</td>
<td>0.309 (0.055)</td>
<td>0.322 (0.057)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.311 (0.084)</td>
</tr>
<tr>
<td>$\Delta Y_{t-1}$</td>
<td>0.360 (0.038)</td>
<td>0.275 (0.082)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.272 (0.089)</td>
</tr>
<tr>
<td>$\Delta C_t$</td>
<td>0.077 (0.030)</td>
<td>0.079 (0.029)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.079 (0.029)</td>
</tr>
<tr>
<td>$\Delta C_{t-1}$</td>
<td>0.018 (0.020)</td>
<td>0.019 (0.023)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.019 (0.023)</td>
</tr>
<tr>
<td>$(I/K)_{t-1}$</td>
<td>0.562 (0.060)</td>
<td>0.604 (0.078)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.598 (0.087)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.713 (0.200)</td>
<td>0.671 (0.217)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.679 (0.223)</td>
</tr>
<tr>
<td>LIQ_t</td>
<td>0.047 (0.106)</td>
<td>0.018 (0.126)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.018 (0.126)</td>
</tr>
<tr>
<td>$B_t$</td>
<td>0.138 (0.026)</td>
<td>0.125 (0.022)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.125 (0.024)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.982</td>
<td>0.980</td>
</tr>
<tr>
<td></td>
<td>1.495</td>
<td>0.979</td>
</tr>
<tr>
<td>LM_t</td>
<td>1.249</td>
<td>1.271</td>
</tr>
<tr>
<td></td>
<td>1.495</td>
<td>1.399</td>
</tr>
</tbody>
</table>

See the notes to Table 3. The numbers in parentheses are standard errors which account for heteroscedasticity. $\Delta Y_t$ is the percentage change in real output. $\Delta C_t$ is the percentage change in the cost of capital, defined as the relative price of investment goods multiplied by the sum of the real interest rate and the economic depreciation rate, the latter equal to .10. $B_t$ is $B^1_t$; results are very similar for $B^2_t$. 
This Data Appendix contains five sections: Legend (of the symbols); Data Sources; Data Definitions -- Aggregate; Derivation of the Q Series; and Data Definitions -- Principal Enterprises.

**LEGEND**

BOP --- beginning of the period.

C --- column.

EOP --- end of the period.

FLO --- flow for the period.

L --- line.

MOP --- middle of the period.

NSA --- not seasonally adjusted.

P --- page.

SA --- seasonally adjusted.

T --- table.

$ --- in the glossary corresponds to a "Z" in the computer code.

{ } --- label on the FAME database.

**DATA SOURCES**


BIS --- Bank for International Settlements. Data from the FAME database.

DATA SOURCES (continued)

DOTS --- International Monetary Fund, Direction of Trade Statistics. Various issues.


ESA --- Bank of Japan, Economic Statistics Annual. Land price data kindly provided by Takeo Hoshi; other data obtained from various issues.


FAME --- Forecasting Analysis and Modeling Environment database maintained by the Board of Governors of the Federal Reserve System.


IFS --- International Monetary Fund, International Financial Statistics. Data from the FAME database.


QNA --- Organisation for Economic Co-operation and Development, Quarterly National Accounts. Data from the FAME database via BIS.

DATA DEFINITIONS -- AGGREGATE

Data are quarterly unless indicated otherwise and, where appropriate, stated in trillions of yen. Flow data are stated at annual rates. For annual data, flow data are averages of the quarterly series, and ratios (products) are computed as the ratio (product) of the annual series.


d --- Depreciation rate on capital: 0.10, as well as alternative pre-set values. Kiyotaki and West (1996, p. 20) report that 0.10 "is approximately the depreciation rate implied by the balance sheet data" (for nonfinancial Japanese corporations).

EPS^1 --- Earnings per share, First Section of the Tokyo Stock Exchange (the approximately 1,200 stocks constituting the TOPIX index), NSA, 1972-1994, BOP: ASR.

EPS^2 --- Earnings per share, Second Section of the Tokyo Stock Exchange (approximately 400 stocks), NSA, 1972-1994, BOP: ASR.
EX --- Real effective exchange rate, weighted-average of the currencies of 24 countries per yen (1973:03 = 100), NSA, 1963:1-1995:4, MOP: defined in a three-step procedure: 1) calculate the real exchange rates (nominal exchange rate less the inflation rate in the producer price index) for the Japanese yen and each of the currencies of the 24 major destination countries for Japanese exports (IFS); 2) base the above series to March 1973 = 100; 3) compute a weighted geometric average of these 24 indices with weights determined by Japanese export volume (DOTS). For 1963-1969, the series equals the value in 1970:1.


GDEBT --- Central government debt, NSA, 1964:4-1992:3, MOP: BIS {FTBAJP01.Q}.


I$ --- Gross private capital formation, machinery and equipment, nonresidential buildings and construction, and breeding stocks, etc., current prices, NSA, 1961:1-1992:4, FLO: QNA {REGAJP01.Q}.


IGOVT-M --- Nominal government bond yield, monthly, NSA, 1966:10-1992:12, MOP: IFS, L 61 {SM15861}. Arithmetic average yield to maturity of all government bonds with seven years to maturity. Monthly series are compiled from end-of-month prices quoted on the Tokyo stock exchange, as reported in ESM.

ILENDEOP --- Nominal lending interest rate, NSA, 1957:1-1992:4, EOP: final month of ILEND-M. Weighted arithmetic average of contracted interest rates charged by all banks on both short-term and long-term loans and discounts. Prior to February 1978, member banks of the Second Association of Regional Banks were excluded from this calculation.

ILEND-M --- Nominal lending interest rate, monthly, NSA, 1957:1-1992:12, MOP: IFS, L 60p {SM15860p}. Weighted arithmetic average of contracted interest rates charged by all banks on both short-term and long-term loans and discounts, as reported in ESM. Prior to February 1978, member banks of the Second Association of Regional Banks were excluded from this calculation.

ILENDMOP --- Nominal lending interest rate, monthly, NSA, 1957:1-1992:12, MOP: average of ILEND-M. Weighted arithmetic average of contracted interest rates charged by all banks on both short-term and long-term loans and discounts. Prior to February 1978, member banks of the Second Association of Regional Banks were excluded from this calculation.


K  ---  Net stock of capital in machinery and equipment, nonresidential buildings and construction, and breeding stocks, etc., constant 1985 prices, NSA, 1960:4-1992:4, EOP: $K_t = I_t + (1-d)K_{t-1}$, where $K_{60:4} = 30.54$ is from DECH.


LP-AVG  ---  Index of land prices for all urban districts, overall average of commercial, industrial, and residential (1985 = 1), NSA, 1955:1-1992:3, EOP: ESA. These data are based on source series with base years of 1990, 1980, and 1955 that have been rescaled to a 1985 base year. The source data are annual for 1955-1964 and biannual for 1965-1992. The missing data are generated by linear interpolation.

LP-COM  ---  Index of land prices for all urban districts, commercial (1985 = 1), NSA, 1955:1-1992:3, EOP: ESA. These data are based on source series with base years of 1990, 1980, and 1955 that have been rescaled to a 1985 base year. The source data are annual for 1955-1964 and biannual for 1965-1992. The missing data are generated by linear interpolation.


M1$ --- M1, sum of currency outside banks and demand deposits other than those of the central government, current prices, NSA, 1957:1-1992:4, EOP: IFS, L 34 {Q15834}.

M1$-SA --- M1, sum of currency outside banks and demand deposits other than those of the central government, current prices, SA, 1957:1-1992:4, EOP: IFS, L 34b {Q15834__b}.


$I^{LP}$ --- Index of land prices constructed as a weighted average of commercial and industrial land prices (1985 = 1), NSA, 1955:1-1992:3, BOP: \( wt \cdot LP-COM(-1) + (1-wt) \cdot LP-IND(-1) \), where \( wt = 0.60 \). This weight is derived as the average of the ratio of the non-residential fixed capital stocks for commercial buildings to the sum of this stock and that for industrial buildings in the United States for 1960, 1970, 1980, and 1989 (FRT). The ratio was the same if calculated with current-cost or constant-cost stocks. Note that the correlation between LP-COM and LP-IND is 0.992, suggesting that \( I^{LP} \) is insensitive to the choice of weights.


PP\textsuperscript{80} --- TOPIX price per share of equity on the Tokyo Stock Exchange, annual, 1949-1995, MOP: average of SPMOP (extended with IFS, L 62). The index has a value of 16.94 (= 367.60 / 21.70) in 1980, which is the price of a share consistent with the dividends per share and the dividend yield in 1980.


PB\textsuperscript{1} --- Price/Book Value ratio, First Section of the Tokyo Stock Exchange (the approximately 1,200 stocks constituting the TOPIX index), NSA, 1972-1994, BOP: ASR.

PB\textsuperscript{2} --- Price/Book Value ratio, Second Section of the Tokyo Stock Exchange (approximately 400 stocks), NSA, 1972-1994, BOP: ASR.

PE\textsuperscript{1} --- Price/Earnings ratio, First Section of the Tokyo Stock Exchange (the approximately 1,200 stocks constituting the TOPIX index), NSA, 1972-1994, BOP: ASR.

PE\textsuperscript{2} --- Price/Earnings ratio, Second Section of the Tokyo Stock Exchange (approximately 400), NSA, 1972-1994, BOP: ASR.

PE\textsuperscript{225} --- Price/Earnings ratio for the Nikkei 225 (drawn from and constituting approximately 50% of the market value and approximately 75% of the trading volume of the First Section of the Tokyo Stock Exchange), NSA, 1954-1993, MOP: DTS.

Q --- The Brainard-Tobin Q, NSA, 1961:1-1992:4, BOP: see the next section of this Data Appendix for details.

r --- Real discount rate, NSA, 1962:1-1992:4, MOP: [(1+IMONYMOP) / (1+inflation rate in P\textsuperscript{Y})] - 1.0.

r\textsubscript{real} --- Real interest rate, annual, 1957-1994, MOP: IMONYMOP (extended with IFS, L60b) - the inflation rate calculated with the CPI.

SE\textsuperscript{1} --- Stockholders' equity per share, First Section of the Tokyo Stock Exchange (the approximately 1,200 stocks constituting the TOPIX index), NSA, 1972-1994, BOP: ASR.

SE\textsuperscript{2} --- Stockholders' equity per share, Second Section of the Tokyo Stock Exchange (approximately 400 stocks), NSA, 1972-1994, BOP: ASR.

SP-JAPAN --- Index of real equity share prices for Japan (used in Figure 1), 1963-1992, MOP: SPMOP / PY, rebased to 1963 = 1.00.


SP-USA --- Index of real equity share prices for the United States (used in Figure 1), 1963-1992, MOP: S&P Composite Stock Price Index (ERP, Table B-96) divided by the Implicit Price Deflator for GDP (ERP, Table B-3), rebased to 1963 = 1.00.


t^A --- Average income tax rate, 1972:1-1992:4, MOP: (TAXC$ + TAXL$) / Y$, where, in a given year, Y$ is the annual value and thus the quarterly values are equal. For the period 1976-1981, correlation with t^H at an annual frequency is 0.462; for the period 1970-1981, correlation with t^H defined only with TAXC$/Y$ at an annual frequency is only 0.116.

t^H --- Marginal income tax rate reflecting national, local and enterprise tax rates, annual, 1956:1-1981:4: HAY T 10.1 and eqn. (28); in a given year, quarterly values set equal to the annual value.

t^I --- Relative marginal tax rate on investment, annual, 1956:1-1981:4, MOP: (t^H z^H - t^H) / (1-t^H); in a given year, quarterly values set equal to the annual value.


WP-M --- Index of wholesale prices (1985 = 1), monthly, NSA, 1957:1-1992:12, MOP: IFS, L 63 {SM15863}. The index has a base in 1985, and covers 1,185 domestic products and imported goods, weighted with 1985 transaction values. Data reported in ESM.


$^H$   --- Present value of depreciation allowances on a marginal investment, annual, 1956:1-1981:4: HAY T 10.1 and eqn. (26); in a given year, quarterly values set equal to the annual value.
DERIVATION OF THE Q SERIES

The traditional expression for Q is as follows (ignoring time subscripts):

\[ Q = \frac{(P^S S + P^B B - DB - NSFA - P^L L - P^I I)}{(P^L L + P^K K)}, \quad (A-1) \]

where \( P \) represents a price index, \( S, B, L, \) and \( K \) represent equities, bonds, land, and fixed + inventory capital, respectively, \( DB \) represents the depreciation bond, and \( NSFA \) represents net short-term financial assets. Substituting the price of investment \( (I) \) for the price of capital, equation \( (A-1) \) can be rewritten as follows,

\[ Q = \frac{[(P^S S + P^B B - DB - NSFA)}{(P^L L + P^I I)] - 1.0. \quad (A-2) \]

We assume that the ratio of \( L \) to \( K \) is constant. (While this assumption would be inappropriate for the economy as a whole, we assume that some of the fixed stock of land has been converted to business use.) Using the HAIN dataset (which contains sample averages across all firms of the balance sheet for 687 Japanese firms), we estimate that the ratio of the value of land to the value of the stocks of fixed capital equals 0.639 in 1985 (the year in which all price indices equal 1.0). Equation \( (A-2) \) can be rewritten as follows,

\[ Q = \frac{[(P^S S + P^B B - DB - NSFA)}{(P^L L * 0.639 + P^I I)]] - 1.0. \quad (A-3) \]

We assume that the variation in the term in brackets can be approximated as follows,

\[ Q = \frac{P^S}{(P^L * 0.639 + P^I I)} - 1.0. \quad (A-4) \]

The validity of the approximation in \( (A-4) \) rests on the assumption that \( P^S, P^L, \) and \( P^I \) are the dominant source of the variability in \( (A-3) \). Since the parameters and statistics (apart from incidental constants) reported in the text depend only on the variability in \( Q \), this approximation (insofar as it is valid) will not distort the econometric results.

The validity of these approximations can be examined with the data in HAIN. The correlation between \( (A-1) \) and \( (A-3) \) is 0.995, implying that the proportionality assumption
concerning land and capital is not inappropriate. The correlation between (A-1) calculated from HAIN and (A-4) calculated from annualized values of the current dataset is 0.801. Thus, the primary variation in Q appears to be captured by the variation in the relative asset prices for equity, land, and capital. Lastly, the correlation between the q series of HAIN (which includes tax variables, as well as some additional adjustments) and (A-4) calculated from annualized values of the current dataset is 0.877.

Four adjustments are needed to create a Q series consistent with the theoretical model. First, all prices are stated relative to the price of output \( (P^Y) \); this scaling is represented by rewriting the price indices in lower case. Second, the theory suggests the following definition of Q,

\[
Q / \ ? - p^I,
\]

(A-5)

where \(?\) is the shadow price of capital. To relate this unobserved shadow price to financial variables, we rely on a modification of the result of Hayashi (1982),

\[
? K = (p^S S + p^B B - DB - NSFA - p^L L),
\]

(A-6)

and rewrite (A-5) as follows,

\[
Q / \ ? - p^I = [(p^S S + p^B B - DB - NSFA) / K] - (p^L * 0.639 + p^I)
\]

(A-7)

where, as above, we have used the estimate that the ratio of the value of land to the value of the stocks of fixed capital equals 0.639. Third, we assume that the variation in the expression in brackets can be captured by \( p^S \). Fourth, this approximation affects the level of Q, and a scalar is chosen so that the mean value of Q equals zero,

\[
Q = (p^S - (p^I * 0.639 + p^I) - \mu_Q.
\]

(A-8)

The correlation between (A-4) and (A-8) is 0.996.

Taken together, this limited evidence suggests that the approximate Q series derived here
for the quarterly dataset is a reasonable approximation to the "true" Q.
DATA DEFINITIONS -- PRINCIPAL ENTERPRISES

Data definitions for the series from the Financial Statements Of Principal Enterprises are discussed below. These annual data are taken from ESA, are usually stated in 100 million yen, and represent the supply of and demand for funds as determined by an annual survey. Enterprises covered by this survey are as a rule those listed on the stock market with a capital of 1 billion yen, excluding finance and insurance companies. This criteria has remained fixed over the sample; the number of firms covered in 1966 and 1991 were 514 and 638, respectively. Data are for the fiscal year beginning April 1; that is, three-quarters of data for a given fiscal year pertain to that calendar year. For the period 1975-1991, the data appear in ESA by fiscal year; for the period 1966-1974, the data appear in ESA for the first and last halves of the fiscal year, and are added together.


INVI --- Inventory investment, 1966-1991, FLO: ESA.


LIQ --- Cash flow, 1966-1991, FLO: [(RET + DEPC / P^Y) / K] * SCALE, where SCALE equals 4.750, which is the ratio of the means of IS for the aggregate economy and the nominal investment for the principal enterprises.

LONGBOR --- Long-term borrowings (from banks and insurance companies with maturities greater than one year), 1966-1991, FLO: ESA.


RESD --- Residual for the demand side of the financial accounts (based on the convention that the supply side of the financial accounts is correct), 1966-1991, FLO: DEMAND - CASH - INVT - SEC - TRADECR.

RET --- Adjusted retained earnings (or voluntary reserve), 1966-1991, FLO: RET' + RESRET.

RET' --- Retained earnings (or voluntary reserve), 1966-1991, FLO: ESA.


RCASH --- Ratio of cash and deposits to total investment, 1966-1991, FLO: CASH / INVT.

REXTEQ --- Ratio of external equity issues to total investment, 1966-1991, FLO: EXTEQ / INVT.

RINTEQ --- Ratio of internal equity to total investment, 1966-1991, FLO: INTEQ / INVT.


RTOTB --- Ratio of total borrowings to total investment, 1966-1991, FLO: TOTB / INVT.

RTRADE --- Ratio of net trade debits to total investment, 1966-1991, FLO: TRADE / INVT.

RTRADECR --- Ratio of gross trade credits to total investment, 1966-1991, FLO: TRADECR / INVT.

RTRADEDR --- Ratio of gross trade debits to total investment, 1966-1991, FLO: TRADEDR / INVT.

RRESD --- Ratio of demand residual to total investment, 1966-1991, FLO: RESD / INVT.

RRESRET --- Ratio of retained earnings residual to total investment, 1966-1991, FLO: RESRET / INVT.


SHORTBOR --- Short-term borrowings (including bills discounted, from banks and the commercial paper market with maturities less than one year), 1966-1991, FLO: ESA.


TRADECR --- Trade credits (receivables, including bills discounted), 1966-1991, FLO: ESA.

TRADEDR --- Trade debits (payables), 1966-1991, FLO: ESA.
APPENDIX B

THE RATIO OF RESIDUAL VARIANCES TEST

This appendix derives the Ratio of Residual Variances test presented in Table 1, a test robust to serial correlation in the adjustment cost shock. The analysis proceeds in three parts by evaluating the Ratio of Residual Variances statistic (RRV) under the null hypothesis of no bubbles and the alternative hypotheses of a bubble and a financing mechanism that is either inactive or active.

The formal model implies that, in the absence of bubbles, the errors for the Q and Euler equations ($u^Q_t$ and $u^E_t$, respectively) are

\[u^Q_t = a_t, \quad (B-1)\]

\[u^E_t = e_{t+1} - R^*_t a_{t+1} + a_t, \quad (B-2)\]

where $a_t$ is the adjustment cost shock, $e_{t+1}$ is the serially uncorrelated expectation error, and $R^*_t$ is the stochastic discount factor that we assume is known at time $t$. Let $M[.]$, $V[.]$, and $C[.]$ represent the mean, variance, and covariance operators, respectively. The variances of the Q and Euler errors are

\[V[u^Q] = V[a], \quad (B-3)\]

\[V[u^E] = V[a] + V[e] + V[R^* a] - 2 E[e_{t+1} R^*_t a_{t+1}] + 2 E[e_{t+1} a_t] - 2 E[R^*_t a_{t+1} a_t], \quad (B-4)\]
where $E[.]$ is the expectations operator.

The variance of the Euler residuals (B-4) can be evaluated in terms of a specific serially correlated process. We assume that $a_t$ is AR(1) (the analysis is quite similar if the adjustment cost shock follows an MA(1) process, $a_t = m_t + T m_{t-1}$; in this case, $? \text{is replaced by } T/(1+T)$), and we maintain the following assumptions:

(A1) \[ a_t = ?a_{t-1} + w_t, \text{ where } w_t \text{ is an innovation}, \]

(A2) \[ R^*_t \text{ independent of } a_{t+1}, \]

(A3) \[ M[a] = 0, \]

(A4) \[ M[e] = 0. \]

The following lemmas evaluate individually the four terms on the second and third lines of (B-4).

**Lemma 1:**

\[ V[R^* \ a] = M[a]^2 V[R^*] + M[R^*]^2 V[a] + V[a] V[R^*] \text{ (by (A2))}, \]

\[ = V[a] (M[R^*]^2 + V[R^*]) \text{ (by (A3))}. \]

**Lemma 2:**

\[-2 E[e_{t+1} \ R^*_t \ a_{t+1}] = -2 \{ M[e] M[R^*_t] M[a] + C[e_{t+1}, R^*_t] M[a] \]

\[+ C[e_{t+1}, a_{t+1}] M[R^*_t] + C[R^*_t, a_{t+1}] M[e] \}, \]

\[= -2 C[e_{t+1}, a_{t+1}] M[R^*_t] \text{ (by (A3) and (A4))}. \]

We define $e_{t+1}$,
\[ e_{t+1} / R_t^* a_{t+1} - E_t(R_t^* a_{t+1}) \]

\[ + R_t^*(p_{t+1}^I + C_{I,t+1}) - E_t(R_t^*(p_{t+1}^I + C_{I,t+1})) \],

where \( E_t(\cdot) \) represents expectations taken with respect to information in period \( t \).

Since \( a_{t+1} = \beta a_t + w_{t+1} \), only \( w_{t+1} \) will be correlated with \( e_{t+1} \) because \( a_t \) is in the period \( t \) information set. Hence,

\[ -2 C[e_{t+1}, a_{t+1}] M[R^*] = -2 C[e_{t+1}, w_{t+1}] M[R^*]. \]

\( C[e_{t+1}, w_{t+1}] \) can be decomposed into two terms. First, the covariance between \( w_{t+1} \) and the expectational error of the adjustment cost shock is

\[ C[R_t^* a_{t+1} - E_t(R_t^* a_{t+1}), w_{t+1}] = \]

\[ E(R_t^*(a_{t+1} - E_t(a_{t+1})) w_{t+1}) = \]

\[ E((\beta a_t + w_{t+1} - \beta a_t) w_{t+1}) M[R^*] = \]

\[ V[w] M[R^*] = V[a] (1 - \beta^2) M[R^*]. \]

Second, the covariance between \( w_{t+1} \) and the expectational error of the price and marginal adjustment cost of investment is defined by \( G \),

\[ G / C[R_t^*(p_{t+1}^I + C_{I,t+1}) - E_t(R_t^*(p_{t+1}^I + C_{I,t+1})), w_{t+1}]. \]

These two terms are multiplied by \(-2 M[R^*]\), and we obtain
\[-2 \mathbb{E}[e_{t+1} R_t^* a_{t+1}] = -2 V[a] (1 - \frac{\tau^2}{2}) M[R^*]^2 - 2 M[R^*] G.\]

Lemma 3:

\[2 \mathbb{E}[e_{t+1} a_t] = 0 \text{ since } a_t \text{ is in the information set at time } t \text{ and by (A4).}\]

Lemma 4:

\[-2 \mathbb{E}[R_t^* a_{t+1} a_t] = -2 \{M[R^*] M[a] M[a] + C[R_t^* a_{t+1}] M[a] + C[R_t^* a_t] M[a] + C[a_t a_{t+1}] M[R^*]\}

\[= -2 \{C[a_t a_{t+1}] M[R^*]\} \text{ (by (A3))}\]

\[= -2 \{V[a] M[R^*]\} \text{ (by (A1)).}\]

With Lemmas 1-4, (B-4) can be written as follows,
\[ V[u^E] = V[a] + V[e] + V[a] \{ M[R^*]^2 + V[R^*] \} \]

\[ - 2 V[a] (1-?^2) M[R^*]^2 - 2 M[R^*] G - 2 V[a] \times M[R^*], \]

\[ = V[a] \{ 1 + M[R^*]^2 + V[R^*] - 2 (1-?^2) M[R^*]^2 - 2 \times M[R^*] \} + V[e] - 2 M[R^*] G, \]

\[ = V[a] \{ 1 - M[R^*]^2 + V[R^*] + 2 M[R^*] \{ ?^2 M[R^*] - ? \} \} + V[e] - 2 M[R^*] G. \]

We evaluate the latter two terms in Lemma 5,

**Lemma 5:**

We define \( EE_{t+1} / R^*_t(p_{t+1}^I + C_{I,t+1}) - E_t{R^*_t(p_{t+1}^I + C_{I,t+1})} \), note that \( a_{t+1} = w_{t+1} \) (per Lemma 2), and obtain

\[ V[e] - 2 M[R^*] G = V[R^*_t w_{t+1} + EE_{t+1}] - 2 M[R^*] G, \]

\[ = V[R^*_t w] + V[EE] + 2 C[R^*_t w_{t+1}, EE_{t+1}] \]

\[ - 2 M[R^*] G, \]

\[ = (R^*_t)^2 V[w] + V[EE] + 2 R^*_t C[w_{t+1}, EE_{t+1}] \]

\[ - 2 M[R^*] G. \]

We evaluate \( R^*_t \) at its sample mean, and rewrite \( V[w] \) in terms of \( V[a] \),

\[ = M[R^*_t]^2 (1-?^2) V[a] + V[EE] + 2 M[R^*] G - 2 M[R^*] G. \]
\[ = M[R^*]^2 (1 - \gamma^2) V[a] + V[EE]. \]

Substituting the result for Lemma 5 into (B-5), we obtain

\[
V[u_E^E] = V[a] \{ 1 - M[R^*]^2 + V[R^*] + 2 M[R^*] \{ \gamma^2 - \gamma \} + M[R^*]^2 (1 - \gamma^2) \}
\]

+ V[EE]. \hspace{1cm} (B-6)

\[
V[u_E^E] = V[a] \{ 1 + V[R^*] + M[R^*] \{ M[R^*]^2 - \gamma^2 \} \} + V[EE].
\]
The population moments for $R^*_t$ in (B-6) are determined by the sample moments --

$$M[R^*_t] = .883 \text{ and } V[R^*_t] = .006,$$

and (B-6) becomes

$$V[u^E] = V[a] \{1.006 + .883 \cdot (.883^2 - 2)\} + V[EE]. \quad (B-7)$$

We define the Ratio of Residual Variances statistic (RRV) as

$$\frac{RRV}{V[u^Q] / V[u^E]} = V[a] / \left(V[a] \{1.006 + .883 \cdot (.883^2 - 2)\} + V[EE]\right). \quad (B-8)$$

Under the null hypothesis of no bubbles, the RRV calculated from the regression residuals will be less than or equal to a critical $(RRV^*)$ determined by substituting (B-3) and (B-7) into (B-8),

$$\frac{RRV}{RRV^*} = V[a] / \left(V[a] \{1.006 + .883 \cdot (.883^2 - 2)\} + V[EE]\right). \quad (B-9)$$

We set $V[EE]$ to zero (which raises $RRV^*$ relative to its true value) and cancel the $V[a]$'s,

$$\frac{RRV}{RRV^*} = \{1.006 + .883 \cdot (.883^2 - 2)\}^{-1}. \quad (B-10)$$

The critical value in (B-10) depends on only one unknown parameter, $\gamma$, the measure of serial correlation in the adjustment cost shock. For large positive values of $\gamma$, $RRV^*$ is sensitive to estimates of the mean of $R^*$. Hence, $RRV^*$ is computed based on the sample mean of $R^*$ ($M[R^*_t]=0.883$), a one-standard deviation increase in the sample mean (labelled $M[R^*_t]+S[R^*_t]$, where $S[R^*_t]=0.006^{1/2}$), and the maximum value of $R^*$ ($M[R^*_t]_{MAX}=0.999$).

The results are plotted in Figure B-1 and, for $\gamma > 0.75$, are presented in Table B-1. For an extensive range of $\gamma$s, the critical value of the RRV statistic is rather low, but increases sharply for large positive values of the serial correlation parameter. This increase is magnified as the
mean of $R^*$ rises. As $\#$ approaches 0.80, the $RRV^*$s begin to rise sharply and, as shown in the latter column for $M[R^*]_{\text{MAX}}$, exceed 20. For the remainder of the analysis, we consider the situation where $-1.00 < \# < 0.80$ and hence that $RRV^*$ equals 20.
There are two alternative hypotheses to consider depending on whether the financing mechanism is active or inactive. Under the alternative hypothesis of bubbles and an inactive financing mechanism,

\[ u^Q_t = a_t + B_t, \]  

(B-11)
\[ u^E_t = e_{t+1} - R^*_t a_{t+1} + a_t, \] (B-12)
It is important to note that $V[u^E_t]$ is independent of the bubble but $V[u^Q_t]$ is affected by $B_t$. A sufficiently large bubble, coupled with the plausible assumption that the bubble and the adjustment cost shock are uncorrelated (or at least not negatively correlated), implies the following inequality,

$$RRV > RRV^* = 20. \quad (B-13)$$

A rejection of $(B-10)$ can be interpreted in terms of a bubble in the $Q$ equation that raises $V[u^Q_t]$.

Under the alternative hypothesis of bubbles and an active financing mechanism,

$$u^Q_t = a_t + B_t - ^B(B_t), \quad (B-14)$$

$$u^E_t = e^*_t + R^*_t a_{t+1} + a_t - f(B_t)(a_{t+1} + p^I_{t+1} + C_{1,t+1}). \quad (B-15)$$

Note that $^t ? / ^t ? ^B(B_t)$ and $^B(B_t)$ and $(f(B_t) ? ? ^{t+1})$, and we maintain that these covariances are quantitatively small and can be considered of second-order importance. (Even if this condition does not hold, the important point is that the $V[u^Q_t]$ increases directly with $B_t$ while the additional terms introduced in $(B-16)$ have a less direct effect on $V[u^E_t]$.) The following condition will then be enough to assure that

$$u^E_t = e^*_t + R^*_t a_{t+1} + a_t - f(B_t) ? ? ^{t+1}. \quad (B-16)$$

Analysis of the variances of $(B-14)$ and $(B-16)$ is made difficult by the covariances introduced by $^B(B_t)$ and $(f(B_t) ? ? ^{t+1})$, and we maintain that these covariances are quantitatively small and can be considered of second-order importance. (Even if this condition does not hold, the important point is that the $V[u^Q_t]$ increases directly with $B_t$ while the additional terms introduced in $(B-16)$ have a less direct effect on $V[u^E_t]$.) The following condition will then be enough to assure that
the above analysis of the inactive financing mechanism applies to the active financing mechanism

\[ V[\beta^B] \leq V[f(B)^*]. \quad (B-17) \]

To quantify the conditions under which (B-17) holds, we begin by treating \( f(B)^* \) as a random variable, and rewrite (B-17) using the alternative variance formula,

\[ V[\beta^B] \leq E[(f(B)^*)^2] - E[f(B)^*]^2. \quad (B-18) \]

Given the expectations on the right-side of (B-18), the value of \( V[\beta^B] \) validating (B-18) is not lowered if we set \( -E[f(B)^*]^2 \neq 0 \) to zero,

\[ V[\beta^B] \leq E[(f(B)^*)^2]. \quad (B-19) \]

Two properties of \( f(B) \) and \( \beta \) are worth noting: 1) \( \beta \neq 0 \); 2) \( f(B) \) varies over a restricted range, \( 0 \leq f(B) \leq f_{\max} \). By Hölder's inequality (Davidson, 1994, p. 138), we can rewrite (B-19) as

\[ V[\beta^B] \leq E[(f_{\max}^*)^2] = E[(f_{\max})^2]. \quad (B-20) \]

Subtracting and adding \( (f_{\max}^*)^2 E[(\beta)^2] \) to (B-20) and using the alternative variance formula, we obtain,

\[ V[\beta^B] \leq (f_{\max}^*)^2 \left\{ V[\beta] + M[\beta]^2 \right\}. \quad (B-21) \]

Divide both sides of (B-21) by \( V[\beta] \),

\[ \frac{V[\beta^B]}{V[\beta]} \leq (f_{\max}^*)^2 \left\{ 1 + M[\beta]^2/V[\beta] \right\}. \quad (B-22) \]

We assume that the \( \text{COV}[^*^*, \beta^B] \$0 \) (which seems quite likely since both terms contain the same streams of current and future marginal products of capital). Hence, \((1/V[^*]) \$ (1/V[\beta])\), and the
following inequality holds,

\[
\left\{ 1 + M[?]^2/V[?] \right\} \# \left\{ 1 + M[?]^2/V[?]^* \right\}.
\]

(B-23)

Combining (B-22) and (B-23), we obtain the following inequality determining the critical ratio,

\[ V[?^B]/V[?] \]
\[ \frac{V[?^B]}{V[?]} \geq (f_{\text{max}})^2 \left\{ 1 + \frac{M[?]}{V[?^*]} \right\}. \]  

(B-24)

To evaluate (B-24), we need values for the three right-side variables. Since \( \gamma_t \) tends to gravitate toward \( p_t^I \), \( M[p_t^I] = 1.088 \) is a reasonable estimate of \( M[?] \), and we set \( M[?] = 1.10 \).

We estimate \( V[?^*] \) using the series for \( \gamma_t^* \) from our VAR forecasting equation; \( V[?^*] = 0.0163 \).

We assume that \( f_{\text{max}} = 0.04 \) (which is twice the mean of the real interest rate). With these estimates and assumptions, the inequality will be satisfied if

\[ \frac{V[?^B]}{V[?]} \geq .12. \]  

(B-25)

Assuming that (B-25) holds and, as with the inactive financing mechanism, the bubble is sufficiently large then, under the alternative hypothesis of bubbles and an active financing mechanism,

\[ \text{RRV} > \text{RRV}^* = 20. \]  

(B-26)

In sum, as long as \( \gamma \) is less than 0.80, the RRVs do not depend on the serial correlation parameter, and hence are robust to either AR(1) or MA(1) processes for \( a_t \). A RRV statistic greater than 20 indicates the existence of a bubble when the financing mechanism is either active or inactive. For the four specifications in Table 1, the RRVs substantially exceed 20 in three cases and the remaining RRV is slightly less than 20, suggesting that bubbles, rather than serial correlation of \( a_t \), are responsible for the rejection of the Q equation.

Reference

Davidson, James (1994), *Stochastic Limit Theory: An Introduction For Econometricians*, New
York: Oxford University Press.
Appendix C

Bubble Measures

In this appendix, we briefly describe the construction of the bubble measures $B_t^{CS}$, $B_t^W$, and $B_t^{MRS}$. The papers on which these variables are based provide additional background information.

$B_t^{CS}$

Define the log of the stock market return as:

$$h_t / \log(P_{t+1}^\text{P}) \& \log(P_t)$$  \hspace{1cm} (C1)

Campbell and Shiller (1988) show that this can be approximated quite closely as:

$$h_t - k \cdot d_t \& \frac{d_t}{d_t-1}$$  \hspace{1cm} (C2)

where $d_t/d_{t-1}=\delta_t$, lower case letters denote logs, $k$ is a constant, and $\delta$ is a coefficient which emerges from the approximation. We follow Campbell and Shiller (1988) in setting $\delta=\exp(g-h)$, where $h$ is the sample mean stock return and $g$ is the sample mean dividend growth rate.

This equation can be solved forward subject to a terminal condition ($\lim_{i\to\infty} \delta_{t+i}=0$) to obtain:

$$d_t \cdot \int_{j=0}^{4} \delta_t \cdot (h_\delta \& d_\delta) \& \frac{k}{1+\delta}$$  \hspace{1cm} (C3)

Campbell and Shiller (1988) assume that expected stock market returns differ from the expected returns on some other asset by an amount that does not vary over time:

$$E(h_t - E_r_t \% c)$$  \hspace{1cm} (C4)

Taking expectations on both sides of (C3), substituting in (C4), and decomposing $r_t$ and $\delta d_t$ into
demeaned variables (indicated by a tilde) plus their means (indicated by a bar), we obtain:

\[ d_t \tilde{\bar{\eta}} E_{j \bar{0}}^4 \bar{\eta}(\bar{r}_{t+\bar{g}} \& \bar{d}_{t+\bar{g}}) \% E_{j \bar{0}}^4 \bar{\eta}(\bar{r} \& \bar{d}) \% \frac{c \& k}{1 \& \bar{d}} \]  \hspace{1cm} (C5)

Motivated by equation (3) in Campbell and Shiller (1988), we set \( k = -\log(\bar{d}) - (1-\bar{d}) \). Based on equation (C4) above, we set \( c = \bar{h} - \bar{r} \).

To construct an optimal linear forecast of the dividend-price ratio, we estimate a VAR of \( d_t, r_{t-1}, \) and \( \bar{d}_{t-1} \) on annual data using two lags of each variable. Using companion matrix notation, our forecast of the log dividend price ratio is:

\[ d_t^C \ll \ll e2^j A \rho^\delta z_t \% \bar{\rho} \& \bar{d} \% \frac{c \& k}{1 \& \bar{d}} \]  \hspace{1cm} (C6)

where \( e2 \) is a vector which picks out \( r_{t-1} - \bar{d}_{t-1} \). The value of \( d_t^C \) which we obtain is used to construct a measure of fundamental price and thus a measure of \( B_t \):

\[ P_t^{CS} \ll e \& d_t^C D_t \]  \hspace{1cm} (C7)

\[ B_t^{CS} \ll P_t \& P_t^{CS} \]  \hspace{1cm} (C8)
West (1987) estimates the regression:

\[ P_t = b(P_{t-1} \% D_{t-1}) + \mu_t \]  

(C9)

[West’s equation (1')] and an AR(q) process for dividends:

\[ D_t = \mu_t + \sum_{i=1}^{q} \phi_i D_{t-i} + \varepsilon_t \]  

(C10)

[West’s equation (11a)], for several possible lag lengths. (We use annual data and, following West (1987), we choose the lag length based on the Akaike criterion, which is minimized for q=1.) We then calculate

\[ P_{t-1}^W = m \% d_1 \% D_{t-1} \% ... \% d_q \% D_{t-q} \]  

(C11)

[West’s equation (12a)]. The values of m and d_1, ..., d_q are calculated from the following formulas:

\[ 0 = m \& b (1 \& b)^{\& d} F (b)^{\& d} \mu \]  

(C12)

\[ 0 = d_j \& [F (b)^{\& d} \& 1] \]  

(C13)

\[ 0 = d_j \& F (b)^{\& d} \int_{k=1}^{q} b^{\& d} f_k \int_{j=2}^{q} \]  

(C14)
\[
F (b) = \left[ \sum_{j=1}^{q} b_j f_j \right]^{d_l}
\]

[West’s equations (13a)], using our estimate of \(b\) from our equation (C9) and our estimate of \(\mu\) and \(f_1, ..., f_q\) from our equation (C10). Using \(P^W_t\), we construct \(B^W_t = P_t - P^W_t\).

\[B^{MRS}\]

Mankiw, Romer, and Shapiro (1991) consider a simple measure of the fundamental price which we use to construct \(B^{MRS}_t = P_t - P^{MRS}_t\). Specifically, we set \(P^{MRS}_t = P^0_t\), where:

\[
P^0_t = \left( \frac{1}{1 + fD} \right)^{\frac{1}{2}} D^{f\delta_l}
\]

(See equation (21) of their paper.) We consider several possible values for \(f\), the equity premium, and report results for \(f = .01\). (Larger values of \(f\) imply that \(B^{MRS}_t\) is even larger in the late 1980's but do not materially affect the reported correlations between \(B^{MRS}_t\) and the \(Q\) equation residual.)
APPENDIX D

POINT ESTIMATES, STANDARD ERRORS, AND t-STATISTICS
FOR THE MODELS IN TABLE 1

The statistics in this table are based on the following parameterizations of the marginal adjustment costs \( C_{I,t} \) and the marginal product of capital \( MPK_t \):

\[
C_{I,t} = \sum_{j=0}^{2} a_j (I/K)_t^j
\]

\[
MPK_t = \sum_{j=0}^{2} \beta_j (Y/K)_t^j + \gamma_j (I/K)_t^j
\]

\( \gamma_0 \) and \( \gamma_\varepsilon \) are the parameters on the liquidity variable appearing in the Q and Euler equations, respectively, in Table 1 (columns 3 and 4).

Point estimates are in column 1. Standard errors (which account for heteroscedasticity) are in column 2. t-statistics are in column 3.

| Table 1 (col. 1), Base Model, Primary Instruments, Two Equations: |
|-------------------|----------------|----------------|
| a0                | -.2141         | .1735          | -1.234         |
| a1                | 4.165          | .9778          | 4.260          |
| a2                | -17.19         | 5.629          | -3.055         |
| \( \beta_0+\gamma_0 \) | .9266         | .3316          | 2.794          |
| \( \beta_1 \)    | -.9185         | .2630          | -3.492         |
| \( \beta_2 \)    | .2288          | .0973          | 2.351          |
| \( \gamma_1 \)   | -1.680         | 2.070          | -.8115         |
| \( \gamma_2 \)   | 9.460          | 5.485          | 1.725          |

| Table 1 (col. 1), Base Model, Primary Instruments, Q Equation: |
|-------------------|----------------|----------------|
| a0                | -6.743         | 3.798          | -1.776         |
| a1                | 72.52          | 39.11          | 1.854          |
| a2                | -185.3         | 93.99          | -1.972         |

| Table 1 (col. 1), Base Model, Primary Instruments, Euler Equation: |
|-------------------|----------------|----------------|
| a0                | -1.354         | .2758          | -4.909         |
| a1                | -1.667         | 1.402          | -1.189         |
| a2                | 16.14          | 21.33          | .7569          |
| \( \beta_0+\gamma_0 \) | -.1008       | .1963          | -.5137         |
| $\beta_1$ | 3.713E-02 | 0.0802 | 0.0463 |
| $\beta_2$ | 0.0536 | 0.0252 | 2.124 |
| $\gamma_1$ | 0.1569 | 1.236 | 0.1269 |
| $\gamma_2$ | -1.569 | 3.041 | -0.5157 |
### Table 1 (col. 2), Base Model, Money Instruments, Two Equations:

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### Table 1 (col. 3), Liquidity Model, Primary Instruments, Euler Equation:

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<td>$\gamma_1$</td>
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<td>$\gamma_2$</td>
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<td>$\zeta$</td>
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### Table 1 (col. 4), Liquidity Model, Money Instruments, Two Equations:

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<tr>
<td>$\beta_2$</td>
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### Table 1 (col. 4), Liquidity Model, Money Instruments, Q Equation:

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### Table 1 (col. 4), Liquidity Model, Money Instruments, Euler Equation:

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</table>
\( \varepsilon \) & -0.0377 & 0.0235 & -1.60