The Economics of Search Warrants

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Abstract

We analyze the effects of the evidence standard for search warrants in an economic model of crime and search. If the warrant standard is initially below a certain positive threshold, increasing it actually reduces crime as well as searches. Moreover, the positive threshold is higher if searches are preventive than if they are not. If the warrant standard is above a positive threshold, increasing it tends to increase crime and reduce wrongful searches. However, if the police do not care too much about whether or not they search the innocent, increasing the standard also increases effort by the police to gather initial evidence non-invasively before seeking to perform invasive searches. Thus, increasing the standard might not greatly increase crime because greater police effort tends to reduce crime; but it might significantly reduce wrongful searches because greater police effort directly increases the accuracy of the police’s initial evidence. The results provide efficiency arguments for a right against unreasonable searches.

JEL Codes: K4, H1, D8.

Keywords: Crime, Police, Evidence, Search, Law, Effort, Efficiency.

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1. Introduction

In several countries, people have a legally-protected right against “unreasonable” searches and seizures by the police. In the U.S., the right against unreasonable searches is protected by the Fourth Amendment to the U.S. Constitution. In Canada, it is protected by Section 8 of the Canadian Charter of Rights and Freedoms. In Europe, it is protected by Article 8 of the European Convention for the Protection of Human Rights and Fundamental Freedoms.

Accordingly, police are usually required to obtain a warrant from the court before searching people or their property. The court does not grant a search warrant unless the police’s evidence against the individuals to be searched is strong enough to meet a certain standard. The rationale for the warrant requirement is that the police might not have the same preferences as society; in particular, the police might care mostly about reducing crime, while society might also care about preventing searches and privacy invasion. Society must then decide where to set the warrant requirement on the police. Setting the warrant standard potentially involves an important tradeoff for society. By preventing the police from searching when evidence is weaker, a higher warrant standard could reduce search and increase privacy, but it could also reduce security and increase crime.

In this paper, we analyze the effects of the evidence standard for search warrants in an economic model of crime and search. In the most basic version of the model, individuals choose whether to commit crime given their benefit from crime and the probability of being caught, and the police observe initial evidence about whether individuals committed crime and then search individuals if the evidence is strong enough to meet the given warrant standard. We solve for equilibrium crime and search probabilities and perform comparative statics with respect to the warrant standard. In particular, we examine the effects of the warrant standard on the probabilities of crime, search, and police errors—each of which is an
important element of social welfare.

The tradeoffs between crime and search and between type I and type II police errors that are involved in changing the warrant standard are weaker than might be expected. Crime does not always increase with the warrant standard. If the warrant standard is initially too low, increasing it actually reduces crime, because it reduces the probability of being searched sufficiently more for an innocent person than for a guilty person. Moreover, if an increase in the standard reduces crime, it also reduces the overall probability of search, while it has ambiguous effects on the probabilities of type I and II police errors. If the warrant standard is not too low initially, an increase in the standard increases crime. Moreover, it increases the probability that a person is guilty and not searched, and reduces the probability that a person is innocent and searched. However, if an increase in the standard increases crime, then it has an ambiguous effect on the overall probability of search. It directly reduces searches by making it harder for the police to obtain a warrant, but also indirectly increases searches by increasing crime.

These results are derived under the assumption that initial evidence comes to the police’s attention exogenously, and the police search if the evidence is strong enough to meet the given warrant standard. We also extend the basic model to endogenize police behavior. In particular, we consider the possibility that the police strategically choose their effort in gathering initial evidence without invading privacy before seeking warrants to perform invasive searches. Interestingly, in the extended model, if the police do not care too much about whether or not they search an innocent person, an increase in the warrant standard increases equilibrium effort by the police. But the more it increases effort by the police, the less it increases crime because greater police effort tends to reduce crime. Moreover, greater police effort directly increases the quality of the initial evidence. Thus, an increase in the
warrant standard might increase crime by only a small extent but reduce wrongful searches by a large extent.

In both the basic model and extended model with endogenous police effort, we assume that individuals derive their benefit from committing a crime even if they are searched. This assumption is reasonable if searches are used to solve crimes that have already been committed, such as murder or rape. But searches can also be used to stop crimes in progress or prevent future crimes—for example to search for stolen goods or drugs that have not yet been sold, or for evidence regarding future terrorist attacks. In this case, criminals may only benefit if they are not searched. In the last part of the paper, we assume that criminals do not benefit if they are searched, and we find that all the results of the paper continue to hold qualitatively, and one of the results becomes stronger. In particular, an increase in the warrant standard reduces crime as well as searches for a larger range of parameters when searches prevent individuals from deriving their benefit from committing a crime.

Overall, our analysis points to several possible efficiency justifications for a right against unreasonable searches. The rest of the paper is organized as follows. Section 2 discusses related literature. In Section 3, we present the basic model. In Section 4, we derive equilibrium crime and search, and examine the effects of the warrant standard on the components of welfare in the basic model. Section 5 extends the model to consider police effort. Section 6 extends the model to consider preventive searches. Section 7 summarizes the results, discusses policy implications, and suggests an avenue for further inquiry.

2. Related Literature

Starting with Becker (1968), an extensive literature on the economics of crime has developed. For a survey, see Ehrlich (1996). Merlo (2004) provides an introduction to recent economic
models of crime and policing. Benoit and Dubra (2004) derive conditions under which law-
abiding members of a police corps would find it in their interest to defend other members
of the corps who have broken the law. Eeckhout, Persico, and Todd (2008) demonstrate
that pre-announced periods of intense policing of arbitrary segments of the population, i.e.,
crackdowns, can be optimal with respect to crime deterrence. Our paper does not focus on
police crackdowns in particular, but considers more general police searches.

Knowles, Persico, and Todd (2001) analyze racial bias in motor vehicle searches. They
develop a model in which motorists, who differ by race, choose whether to carry contraband,
and police choose whether to search motorists observing their race. Police potentially have
a bias for searching a particular type of motorist. The authors show that, in equilibrium,
the rate at which contraband is seized (the hit rate) is the same across racial groups if the
police are not biased, whereas if the police are biased, it is lower for the group subject to
bias. Thus, the model provides a test for police bias, which the authors apply to test for bias
against African Americans in vehicle searches in Maryland. Hernandez-Murillo and Knowles
(2004), Dominitz and Knowles (2006), and Persico and Todd (2006) further extend the hit
rate test for police bias. Anwar and Fang (2006) develop an alternative test based on a
general framework in which the differing race of police officers is also taken into account.

Persico (2002) develops a policing model, in which individuals, who differ in both their
race and legal earning opportunities, choose whether to commit crime, and the police choose
whether to search individuals based on their race. The author defines police behavior as
being fair if the racial groups are searched with equal intensity. He then investigates the
conditions under which forcing the police to behave fairly reduces crime.

The above-mentioned profiling models show how police bias affects search patterns across
racial groups in order to empirically test for police bias, and how fairness constraints on
policing affect crime in order to evaluate the tradeoff between fairness and effectiveness of policing. In contrast, our model shows how the evidence standard for search warrants affects crime and searches in order to evaluate the tradeoff between security and privacy. While our model shares certain general features with the models in the literature on racial profiling in policing, it is different because it addresses a different set of issues. Thus, for instance, the papers in the profiling literature do not model the evidence or probable cause standard for searches to be legal. The focus of the profiling literature is on prejudice and fairness issues related to the right to equal protection of the laws, whereas our focus is on privacy and security issues related to the right against unreasonable searches.\textsuperscript{2}

A small but growing literature employs economic theory to analyze constitutional protections of individual rights (see Mialon and Rubin, 2008). For example, Breton and Wintrobe (1992) evaluate the right to free speech; Taylor (1995) studies the right to bear arms; Mialon (2005) examines the right to silence and the right to due process; Gay \textit{et al.} (1989) investigate the right to trial by jury; and Persson and Siven (2007), Andreoni (1991), and Shavell (1991) analyze issues related to the right against cruel and unusual punishment. However, economic theory has rarely been applied to analyze the right against unreasonable searches. Garoupa (2005) provides an efficiency justification for the European Human Rights Convention, based on a detailed analysis of enforcement technology with general privacy rights, but the author does not particularly focus on the right against unreasonable searches.

The only other economic models of protections against unreasonable searches are those developed by Dharmapala and Miceli (2003) and Mialon and Mialon (2008), which examine the effects of various remedies for illegal searches, i.e., searches when the evidence does not provide probable cause. Illegal searches are possible in cases where the police are not required

\textsuperscript{2} In the U.S., for example, the right to equal protection of the laws is protected by the Fourteenth Amendment to the Constitution, whereas the right against unreasonable searches is protected by the Fourth Amendment.
to obtain court-authorization before searching (e.g., when police are in “hot pursuit”). In contrast, the model developed in the present paper focuses on cases where the police are required to obtain a warrant from the court before searching. In such cases, the central question is not how to prevent illegal searches, but where to set the warrant requirement.

In the models put forth by Dharmapala and Miceli (2003) and Mialon and Mialon (2008), evidence is a random variable that can be in one of two states, either it provides probable cause or it does not; and the police choose whether or not to search when the evidence does not provide probable cause. The actual probable cause or warrant standard is fixed and exogenous in these models. In contrast, the present paper examines the effects of the warrant or probable cause standard itself, which requires a model with a continuum of evidence. We appear to be the first to formally analyze the effects and efficiency of the evidence standard for search warrants.

Several papers have analyzed the effects and efficiency of the evidence standard for conviction in criminal trials. In particular, Miceli (1990) analyzes the effects of the standard for conviction on prosecutorial effort to gather evidence. Yilankaya (2002) analyzes the effects of the conviction standard on the strategically interdependent evidence-gathering efforts by the prosecutor and defendant. Both papers analyze the efficiency of the conviction standard with respect to trial costs and court errors. In contrast, we analyze the effects of the evidence standard for search warrants on crime and effort by the police, and the efficiency of the warrant standard with respect to deterrence as well as search costs and police errors.

Garoupa (2004) analyzes in detail the effects of legal aid in criminal cases. He finds, *inter alia*, that legal aid in criminal cases might increase deterrence as long as it helps the innocent more than the guilty. In our basic model, we find that increasing the standard for search warrants reduces crime if the standard is initially low enough, because then increasing
the standard reduces the search probability sufficiently more for the innocent than for the guilty. While this result has a similar flavor to the above-mentioned result by Garoupa, our model is very different from Garoupa’s model, since we address a very different set of issues.

Lastly, Stephenson (2007) provides an insightful analysis of the effects of bureaucratic costs on agency expertise. He finds that increasing the costs to an agency of taking a regulatory action decreases the agency’s incentive to learn the truth about the benefit of the proposed action if the agency would not take the action upon learning nothing, and increases its incentive to learn the truth about the action’s benefit if it would take the action even upon learning nothing.

In our extended model, we examine the effects of the warrant standard on police effort to acquire initial evidence. Our extended model is different from Stephenson’s model, which is not particularly focused on the police search context. In our model, the police cannot search if they do not learn anything, unless the warrant standard is set at zero. Moreover, in our model, crime is endogenous, and thus the benefit from searching is endogenous, whereas in Stephenson’s model, the benefit of taking the regulatory action is exogenous. Our results are also very different from Stephenson’s results. We find that increasing the warrant standard increases the police’s effort to acquire initial evidence before seeking a warrant as long as the police do not care too much about searching the innocent, and the more an increase in the standard increases police effort, the less it increases crime.

3. Basic Model

Consider a unit population of citizens and a police force. Citizens differ according to their benefit or wage from crime, w. At time 1, Nature chooses each citizen’s w according to the c.d.f. F(w). At time 2, citizens choose whether or not to commit crime, C or ¬C. Their
choices at time 2 are not directly observable to the police. At time 3, Nature chooses the police’s initial evidence about whether or not a citizen has committed a crime, $\varepsilon$. The initial evidence $\varepsilon$ is distributed according to the c.d.f. $G(\varepsilon)$ if a citizen has committed a crime and according to the c.d.f. $H(\varepsilon)$ if a citizen has not committed a crime. The associated probability density functions are denoted by $g(\varepsilon)$ and $h(\varepsilon)$.

We assume that $g(\varepsilon)/h(\varepsilon)$ is increasing in $\varepsilon$, i.e., $g(\varepsilon)$ and $h(\varepsilon)$ satisfy the monotone likelihood ratio property (see Milgrom, 1981). This means that, as the evidence $\varepsilon$ increases, the likelihood of having $\varepsilon$ against a guilty citizen relative to the likelihood of having it against an innocent citizen increases. The monotone likelihood ratio property implies that $g(\varepsilon)/(1-G(\varepsilon)) > h(\varepsilon)/(1-H(\varepsilon))$ for all $\varepsilon$, i.e., the hazard rate is higher for the guilty than for the innocent (the hazard rate dominance property); this means that, if the evidence is below a given threshold, then if it is increased by a small amount, it is more likely to attain the threshold if a citizen is guilty than if a citizen is innocent. The hazard rate dominance property in turn implies that $G(\varepsilon) \leq H(\varepsilon)$ for all $\varepsilon$, i.e., $G(\varepsilon)$ first-order stochastically dominates $H(\varepsilon)$; this means that, for any given evidence level $\varepsilon$, the probability that police have evidence stronger than $\varepsilon$ is always higher if a citizen is guilty than if a citizen is innocent. For proofs of the two implications, see De Fraja (2005) and Krishna (2002, Appendix B). We employ each of the three properties throughout the paper.

The standard of evidence required for the police to obtain a search warrant from the court is denoted by $\bar{\varepsilon}$. Then, if $\varepsilon \geq \bar{\varepsilon}$, the police have a search warrant, while if $\varepsilon < \bar{\varepsilon}$, they do not have a search warrant. At time 4, the police observe the evidence $\varepsilon$ and either search or do not search. We assume that police follow a simple search rule in the basic model: they search if $\varepsilon \geq \bar{\varepsilon}$ and do not search if $\varepsilon < \bar{\varepsilon}$. Thus, the probability that a person who chooses to commit crime is searched is $1 - G(\bar{\varepsilon})$, and the probability that a person who chooses not
to commit crime is searched is $1 - H(\tilde{\varepsilon})$. The police are assumed to search whenever they have enough evidence for a court-authorized warrant, as they may always be tempted to search after the fact.\(^3\) On the other hand, if the police search without a court-authorized warrant, then their search is assumed to be automatically thrown out if the case proceeds to court, as the court can verify its records and see that no warrant was authorized.

If the police do not search, the citizen remains free. If the police search, then additional evidence is uncovered and the court determines whether to convict or acquit the citizen at time 5. We assume that the probability that a guilty person is convicted is $\alpha_1(\tilde{\varepsilon}, \theta)$ and that the probability that an innocent person is convicted is $\alpha_2(\tilde{\varepsilon}, \theta)$, where $\theta$ and $\tilde{\varepsilon}$ are parameters representing the accuracy of the court system and the evidence standard for conviction, respectively.

The conviction probabilities can be formally modeled as follows. Let the additional evidence uncovered by the search be $\varepsilon^*$, which is distributed according to the cumulative density function $G^*$ if a citizen is guilty and according to $H^*$ if a citizen is innocent, where $G^*$ first order stochastically dominates $H^*$. Total evidence is given by the function $t(\varepsilon, \varepsilon^*)$. This function is distributed according to the cumulative density function $W$ if a citizen is guilty and according to $Z$ if a citizen is innocent, where $W$ first order stochastically dominates $Z$. $W$ is a convolution of $G$ and $G^*$, and $Z$ is a convolution of $H$ and $H^*$.

Now, with probability $\theta$, the court observes the truth about whether the citizen is guilty or innocent, and with probability $1 - \theta$, it only observes the evidence $t(\varepsilon, \varepsilon^*)$. Then, the

\(^{3}\)The assumption that the police always search when the evidence is above the warrant standard, $\tilde{\varepsilon}$, is mainly for simplicity in the basic model. Of course, if the police have a cost of searching the innocent, then they might choose not to search even when the evidence is above $\tilde{\varepsilon}$ if $\tilde{\varepsilon}$ is very low. With a positive search cost for the police, there would be some positive cutoff level of evidence, call it $\tilde{\varepsilon}_p$, above which the police would choose to search and below which they would choose not to search. Then the police would search if and only if the evidence is higher than $\max\{\tilde{\varepsilon}_p, \tilde{\varepsilon}\}$. Most of the results of the basic model would be more or less the same with this added complication. See, for example, footnote 4 after Corollary 3. Moreover, we model the police’s costs and benefits of searching more explicitly in Section 5, where we extend the basic model to endogenize police effort to gather initial evidence.
probability that a guilty person is convicted is \( \alpha_1(\bar{\epsilon}, \theta) = \theta + (1 - \theta)(1 - W(\bar{\epsilon})) \), and the probability that an innocent person is convicted is \( \alpha_2(\bar{\epsilon}, \theta) = (1 - \theta)(1 - Z(\bar{\epsilon})) \). Note that the conviction probabilities do not depend on the search standard, \( \bar{\epsilon} \), because the court is not supposed to let the fact that a citizen has been searched influence its decision about whether to convict the citizen.\(^5\)

As modeled, \( \alpha_1(\bar{\epsilon}, \theta) > \alpha_2(\bar{\epsilon}, \theta) \) for any \( (\bar{\epsilon}, \theta) \), that is, for any level of the accuracy of the court system and for any evidence standard for conviction, the probability of being convicted is greater for a guilty than for an innocent citizen; \( \partial \alpha_1(\bar{\epsilon}, \theta)/\partial \bar{\epsilon} < 0 \) and \( \partial \alpha_2(\bar{\epsilon}, \theta)/\partial \bar{\epsilon} < 0 \), that is, the probabilities of being convicted decrease as the evidence standard for conviction increases; and \( \partial \alpha_1(\bar{\epsilon}, \theta)/\partial \theta > 0 \) and \( \partial \alpha_2(\bar{\epsilon}, \theta)/\partial \theta < 0 \), that is, an increase in the accuracy of the court system increases the conviction probability for guilty citizens who are searched and reduces the conviction probability for innocent citizens who are searched.

The cost of being convicted is \( \rho \), which represents the punishment, and the cost of being searched is \( \eta \), which represents the loss of privacy. Stigler (1980) and Posner (1981, 1983) discuss the nature and value of privacy. One definition of privacy is the ability to conceal personal information that others might use to one’s disadvantage. Concealment protects reputation, which is a valuable asset in most types of relationships. If citizens are searched by the police and the details of the search are subsequently made public, they may suffer a loss of reputation, which might result in the loss of a job or a spouse, for example. Discrediting

\(^4\) Note that \( \alpha_1(\bar{\epsilon}, \theta) = 1 \) if \( \bar{\epsilon} > \bar{\epsilon} \) and \( \alpha_1(\bar{\epsilon}, \theta) = \theta \) if \( \bar{\epsilon} < \bar{\epsilon} \), i.e., a guilty person who is searched is convicted with certainty if total evidence gathered by police is above the conviction standard, and convicted with probability \( \theta \) otherwise. If the court is perfectly accurate, i.e., \( \theta = 1 \), a guilty person who is searched is always convicted even when total evidence gathered by police is above the conviction standard. On the other hand, \( \alpha_2(\bar{\epsilon}, \theta) = 0 \) if \( \bar{\epsilon} < \bar{\epsilon} \) and \( \alpha_1(\bar{\epsilon}, \theta) = 1 - \theta \) if \( \bar{\epsilon} < \bar{\epsilon} \), i.e., an innocent person who is searched is acquitted with certainty if total evidence is above the conviction standard, and convicted with probability \( 1 - \theta \) otherwise. If the court is perfectly accurate, an innocent person who is searched is always acquitted even when total evidence gathered by police is above the conviction standard.

\(^5\) In many legal systems, judges and jurors are constrained or instructed to rule purely on the basis of the evidence and not also on the basis of other available background information, including prior beliefs. For interesting efficiency justifications for such exclusionary rules, see Demouguin and Fluet (2006 and 2007).
personal information obtained by the government could be used by the government and others to unfairly discriminate against citizens, which society might want to prevent. For these reasons, the Federal Privacy Act (Public Law No. 93-579) limits the government’s ability to obtain, retain and disseminate discrediting personal information.\(^6\)

We initially assume that a citizen who commits crime receives the benefit, \(w\), whether or not he is searched, which is likely to be the case when searches are used to solve crimes that have already been committed. In Section 6, we examine an extension of the model in which a citizen who commits crime derives the benefit only if he is not searched, which is likely to be the case if searches are used to stop crimes in progress or prevent future crimes.

4. Analysis

The basic model’s equilibrium probabilities of crime, search, and type I and II police errors are given in the following proposition:

**Proposition 1** The equilibrium probability of crime is

\[
C(\bar{\varepsilon}) = 1 - F(A(\bar{\varepsilon})),
\]  

where

\[
A(\bar{\varepsilon}) = (1 - G(\bar{\varepsilon}))(\eta + \alpha_1(\bar{\varepsilon}, \theta)\rho) - (1 - H(\bar{\varepsilon}))(\eta + \alpha_2(\bar{\varepsilon}, \theta)\rho).
\]  

The equilibrium probability of search is

\[
S(\bar{\varepsilon}) = C(\bar{\varepsilon})(1 - G(\bar{\varepsilon})) + (1 - C(\bar{\varepsilon}))(1 - H(\bar{\varepsilon})).
\]  

The equilibrium probability that a person is innocent and searched is

\[
WS(\bar{\varepsilon}) = (1 - C(\bar{\varepsilon}))(1 - H(\bar{\varepsilon})).
\]  

The equilibrium probability that a person is guilty and not searched is

\[
WN(\bar{\varepsilon}) = C(\bar{\varepsilon})G(\bar{\varepsilon}).
\]

\(^6\) Privacy has private and social value as it protects reputation and prevents unfair discrimination. Moreover, it can even be Pareto-improving in some contexts. Daughety and Reinganum (2006) show that privacy or incomplete information about abilities can yield a Pareto-improvement over complete information in a broad class of co-investment relationships. Incomplete information about abilities creates a motive to signal ability through greater effort, thereby mitigating the free-rider problem. Hui and Png (2006) provide a detailed survey of the economics of privacy, emphasizing the social benefits of privacy illuminated by recent work.
Proof. See Mathematical Appendix. ■

The effect of the warrant standard on equilibrium crime is then

\[ \frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = f(A(\bar{\varepsilon}))[g(\bar{\varepsilon})(\eta + \alpha_1(\bar{\varepsilon}, \theta)\rho) - h(\bar{\varepsilon})(\eta + \alpha_2(\bar{\varepsilon}, \theta)\rho)]. \tag{6} \]

Therefore, we have the following necessary and sufficient condition for crime to be increasing in the warrant standard:

Corollary 1

\[ \frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}} > 0 \iff \frac{g(\bar{\varepsilon})}{h(\bar{\varepsilon})} > \frac{\eta + \alpha_2(\bar{\varepsilon}, \theta)\rho}{\eta + \alpha_1(\bar{\varepsilon}, \theta)\rho}. \tag{7} \]

Crime need not be increasing in \( \bar{\varepsilon} \). By the monotone likelihood ratio assumption, \( g(\bar{\varepsilon})/h(\bar{\varepsilon}) \) decreases as \( \bar{\varepsilon} \) decreases. Therefore, if \( \bar{\varepsilon} \) is low, an increase in \( \bar{\varepsilon} \) may reduce crime. An increase in \( \bar{\varepsilon} \) decreases the probability of being searched for a guilty person, but also decreases the probability of being searched for an innocent person. If \( \bar{\varepsilon} \) is low, then a large proportion of the people being searched by the police are innocent, so increasing \( \bar{\varepsilon} \) may reduce the probability of being searched more for an innocent person than for a guilty person. If \( \bar{\varepsilon} \) reduces the probability of being searched sufficiently more for an innocent than for a guilty person, then it reduces the expected costs of being searched more for an innocent than for a guilty person, and thereby reduces crime.

It is reasonable to assume that \( g(0)/h(0) < (\eta + \alpha_2\rho)/\eta + \alpha_1(\bar{\varepsilon}, \theta)\rho \), i.e., a decrease in the warrant standard from a very small level to zero reduces the expected costs more for the innocent than for the guilty. Then, by the monotone likelihood ratio property, there is a unique \( \bar{\varepsilon}_c \) satisfying

\[ \phi(\bar{\varepsilon}_c; \eta, \theta, \rho, \bar{\varepsilon}) = \frac{g(\bar{\varepsilon}_c)}{h(\bar{\varepsilon}_c)} - \frac{\eta + \alpha_2(\bar{\varepsilon}, \theta)\rho}{\eta + \alpha_1(\bar{\varepsilon}, \theta)\rho} = 0. \tag{8} \]
We then have

\[
\frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = \begin{cases} < 0 & \text{if } \bar{\varepsilon} < \bar{\varepsilon}_c \\ = 0 & \text{if } \bar{\varepsilon} = \bar{\varepsilon}_c \\ > 0 & \text{if } \bar{\varepsilon} > \bar{\varepsilon}_c \end{cases}
\] (9)

Examining the effects of \(\eta, \theta, \rho,\) and \(\bar{\varepsilon}\) on \(\bar{\varepsilon}_c\), we obtain:

**Corollary 2**

\[
\frac{\partial \bar{\varepsilon}_c}{\partial \eta} > 0, \quad \frac{\partial \bar{\varepsilon}_c}{\partial \theta} < 0, \quad \frac{\partial \bar{\varepsilon}_c}{\partial \rho} < 0, \quad \text{and } \frac{\partial \bar{\varepsilon}_c}{\partial \bar{\varepsilon}} > 0.
\] (10)

**Proof.** See Mathematical Appendix. 

The cutoff \(\bar{\varepsilon}_c\) is increasing in the cost of privacy invasion \(\eta\), but is decreasing in the accuracy of the justice system \(\theta\) and the punishment \(\rho\). A greater cost of privacy invasion, a lower accuracy of the justice system, and a lower punishment each make it harder for an increase in the warrant standard to increase crime.

We now turn to the effect of the warrant standard on equilibrium search and privacy invasion. The effect of \(\bar{\varepsilon}\) on equilibrium search is:

\[
\frac{\partial S(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = (H(\bar{\varepsilon}) - G(\bar{\varepsilon})) \frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}} - [C(\bar{\varepsilon}) g(\bar{\varepsilon}) + (1 - C(\bar{\varepsilon})) h(\bar{\varepsilon})].
\] (11)

By the first order stochastic dominance property, \(H(\bar{\varepsilon}) - G(\bar{\varepsilon}) > 0\). Therefore, we have the following result.

**Corollary 3** If \(\bar{\varepsilon} < \bar{\varepsilon}_c\), then

\[
\frac{\partial S(\bar{\varepsilon})}{\partial \bar{\varepsilon}} < 0.
\] (12)

If \(\bar{\varepsilon} > \bar{\varepsilon}_c\), then

\[
\frac{\partial S(\bar{\varepsilon})}{\partial \bar{\varepsilon}} < 0 \iff (H(\bar{\varepsilon}) - G(\bar{\varepsilon})) \frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}} < C(\bar{\varepsilon}) g(\bar{\varepsilon}) + (1 - C(\bar{\varepsilon})) h(\bar{\varepsilon})].
\] (13)

\(^7\) In Section 6, we show that the parameter range over which an increase in \(\bar{\varepsilon}\) actually reduces crime is larger if searches are preventive. See Corollaries 6 and 7.
If $\bar{\varepsilon} < \bar{\varepsilon}_c$, so crime is decreasing in the standard, then search is also decreasing in the standard. Hence, if the standard is not high enough, increasing it not only reduces crime, but also reduces search, and therefore involves no tradeoff between crime and search.\(^8\)

If $\bar{\varepsilon} > \bar{\varepsilon}_c$, so crime is increasing in $\bar{\varepsilon}$, then the effect of $\bar{\varepsilon}$ on search is ambiguous. In this case, $\bar{\varepsilon}$ has two opposing effects on searches. It directly reduces searches by making it harder for police to obtain a warrant. However, it also indirectly increases search by increasing crime. The direct effect is the term on the right-hand side of (13). The indirect effect is the term on the left-hand side of (13), which is positive when $\partial C(\bar{\varepsilon})/\partial \bar{\varepsilon} > 0$. The overall effect of the warrant standard on searches depends on which of these two effects is dominant. If an increase in $\bar{\varepsilon}$ only weakly increases crime, the direct effect dominates, and searches decrease. However, if an increase in $\bar{\varepsilon}$ strongly increases crime, the indirect effect dominates, and searches increase.

Changes in $\bar{\varepsilon}$ involve a tradeoff between crime and search only if $\bar{\varepsilon}$ is sufficiently high that crime is increasing in $\bar{\varepsilon}$, but crime is not increasing in $\bar{\varepsilon}$ by so great an extent that the direct effect of $\bar{\varepsilon}$ on search dominates its indirect effect.

Turning to the effects of the standard on type I and type II police errors, we have:

\[
\frac{\partial WS(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = -\frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}}(1 - H(\bar{\varepsilon})) - \left(1 - C(\bar{\varepsilon})\right)h(\bar{\varepsilon}), \tag{14}
\]

\[
\frac{\partial WN(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = \frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}}G(\bar{\varepsilon}) + C(\bar{\varepsilon})g(\bar{\varepsilon}). \tag{15}
\]

Therefore, we have the following result.

\(^8\) The basic model implies that crime and search are both decreasing in $\bar{\varepsilon}$ if $\bar{\varepsilon} < \bar{\varepsilon}_c$. The results would be more or less the same if we relaxed the assumption that police always want to search if $\varepsilon > \bar{\varepsilon}$, and assumed instead that they want to search only if $\varepsilon$ is above some cutoff for the police, $\bar{\varepsilon}_p$, which would be a function of their cost of searching. Under this extension, if $\bar{\varepsilon}_p < \bar{\varepsilon}_c$ and $\bar{\varepsilon} \in [\bar{\varepsilon}_p, \bar{\varepsilon}_c]$, then as before, crime and search would both be decreasing in $\bar{\varepsilon}$; and if $\bar{\varepsilon}_p < \bar{\varepsilon}_c$ and $\bar{\varepsilon} < \bar{\varepsilon}_p$ or if $\bar{\varepsilon}_p > \bar{\varepsilon}_c$ and $\bar{\varepsilon} < \bar{\varepsilon}_c$, then changes in $\bar{\varepsilon}$ would have no effect at all on either crime or search. Therefore, if police want to search only if $\varepsilon > \bar{\varepsilon}_p$, crime and search are both still non-increasing in $\bar{\varepsilon}$ if $\bar{\varepsilon} < \bar{\varepsilon}_c$. In general, if $\bar{\varepsilon} < \bar{\varepsilon}_c$, increasing $\bar{\varepsilon}$ up to $\bar{\varepsilon}_c$ cannot increase crime or search and may decrease both crime and search.
Corollary 4 If \( \bar{\varepsilon} < \bar{\varepsilon}_c \), then

\[
\begin{align*}
\frac{\partial WS(\bar{\varepsilon})}{\partial \bar{\varepsilon}} &< 0 \iff -\frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}} (1 - H(\bar{\varepsilon})) < h(\bar{\varepsilon})(1 - C(\bar{\varepsilon})) h(\bar{\varepsilon}), \\
\frac{\partial WN(\bar{\varepsilon})}{\partial \bar{\varepsilon}} &> 0 \iff C(\bar{\varepsilon}) g(\bar{\varepsilon}) > -\frac{\partial C(\bar{\varepsilon})}{\partial \bar{\varepsilon}} G(\bar{\varepsilon}).
\end{align*}
\] (16) (17)

If \( \bar{\varepsilon} > \bar{\varepsilon}_c \), then

\[
\frac{\partial WS(\bar{\varepsilon})}{\partial \bar{\varepsilon}} < 0 \text{ and } \frac{\partial WN(\bar{\varepsilon})}{\partial \bar{\varepsilon}} > 0.
\] (18)

If \( \bar{\varepsilon} > \bar{\varepsilon}_c \), so crime is increasing with the warrant standard, then the probability that a person is innocent and searched decreases, and the probability of that a person is guilty and not searched increases, with the warrant standard. If \( \bar{\varepsilon} < \bar{\varepsilon}_c \), so crime is decreasing with the standard, then the effect of the standard on the probabilities of type I and type II errors are ambiguous. Because an increase in the standard decreases crime, it increases the number of innocent people, which tends to increase the probability that a person is innocent and searched and decrease the probability that a person is guilty and not searched. However, an increase in the standard also tends to directly decrease the probability that a person is innocent and searched and increase the probability that a person is guilty and not searched because it makes it harder for the police to search.

As long as \( \bar{\varepsilon} < \bar{\varepsilon}_c \), increasing the warrant standard unambiguously reduces both crime and searches, while it has ambiguous effects on police errors. It might therefore be better for society to increase the warrant standard at least up to \( \bar{\varepsilon}_c \). That is, a basic right against unreasonable searches might be efficient, at least with respect to deterrence.

If \( \bar{\varepsilon} > \bar{\varepsilon}_c \), an increase in the standard increases both crime and the probability that a person is guilty and not searched, and reduces the probability that a person is innocent and searched. The overall socially optimal standard balances the social costs of crime and criminals on the loose against the social costs of wrongful searches. Thus, for example, if
the crime is more severe (e.g., murder), so the social costs of the crime and criminals on
the loose are higher, then the efficient warrant standard is lower. On the other hand, if the
search technology is more invasive (e.g., DNA technology), the efficient standard is higher.

5. Police Effort

In the basic model, initial evidence comes to the police’s attention exogenously, and the
police search is the given evidence is above the given warrant standard. In reality, the police
choose their effort level in gathering evidence and investigating crimes by means other than
invasive searches prior to seeking a warrant to perform an invasive search. If the police
choose higher effort, they might be more likely to have evidence above the warrant standard
if a citizen is actually guilty and below the standard if a citizen is innocent. We now modify
the basic model to endogenize police effort.

As in the basic model, at time 1, Nature chooses each citizen’s benefit from crime, \( w \),
and at time 2, each citizen chooses whether or not to commit a crime. However now, at
time 3, the police choose their effort level, denoted by \( e \), in establishing a case prior to
asking for a warrant to perform an invasive search, not knowing for certain whether or not
a citizen is guilty. At time 4, as in the basic model, Nature chooses the evidence \( \varepsilon \) against
the citizen. However now, the evidence is distributed according to the cumulative density
function \( G(\varepsilon|e) \) if the citizen is guilty and the evidence is distributed according to \( H(\varepsilon|e) \) if
the citizen is innocent.

We have that \( \partial G(\varepsilon|e)/\partial \varepsilon = g(\varepsilon|e) > 0 \) and \( \partial H(\varepsilon|e)/\partial \varepsilon = h(\varepsilon|e) > 0 \) for all \( e \); that is,
an increase in the warrant standard reduces the probability that the police have evidence
that provides probable cause for a warrant, whether the citizen is guilty or innocent, for
any level of police effort. We assume that \( \partial G(\varepsilon|e)/\partial e < 0 \) and \( \partial H(\varepsilon|e)/\partial e > 0 \) for all \( \varepsilon \);
that is, an increase in police effort increases the probability that the police have probable cause against a guilty citizen and reduces the probability that the police have probable cause against an innocent citizen for any level of the warrant standard. Also, \( \partial g(\bar{\varepsilon}|e)/\partial e < 0 \) and \( \partial h(\bar{\varepsilon}|e)/\partial e > 0 \); that is, if the police choose higher effort, an increase in the warrant standard reduces the probability that the evidence provides probable cause less if the citizen is guilty and more if the citizen is innocent.

At time 5, as in the basic model, the police search if the evidence is above the warrant standard, \( \bar{\varepsilon} \), and if a citizen is searched, then the citizen is convicted with probability \( \alpha_1 \) if the citizen committed crime and is convicted with probability \( \alpha_2 \) if the citizen did not commit crime. Also as in the basic model, the citizen’s cost of being searched is \( \eta \) and cost of being convicted is \( \rho \).

The police are assumed to incur a constant marginal cost of effort \( c_e \). The police’s utility from searching a guilty person is 1, their utility from not searching an innocent person is \( k \), their utility from searching an innocent person is \( -k \), and their utility from not searching a guilty person is \( -1 \). The parameter \( k \geq 0 \) measures the extent to which the police care about not searching an innocent person. If \( k = 0 \), the police are indifferent between searching and not searching an innocent person.\(^9\)

The above formulation of the police’s utility also motivates the search warrant requirement that is imposed on the police. If the police care about protecting the security of citizens but not about invading their privacy, but society cares about protecting the privacy as well as the security of citizens, then society may want to require the police to obtain court-authorization before searching rather than letting the police determine whether the search is necessary on their own.

\(^9\) One could make police utilities proportional to the probabilities of convicting a guilty person and convicting an innocent person instead of making them proportional to the probabilities of searching a guilty person and searching an innocent person. However, this would not affect the qualitative results reported in this section.
Proposition 2 characterizes equilibrium police effort and crime in the extended model:

**Proposition 2** The equilibrium effort by the police \( e^* \) is characterized by

\[
\lambda(\bar{\varepsilon}, e^*) = C^E(\bar{\varepsilon}, e^*) (-2 \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} + (1 - C^E(\bar{\varepsilon}, e^*) (2k \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e}) - c_e = 0,
\]

(19)

and the equilibrium probability of crime is

\[
C^E(\bar{\varepsilon}, e^*) = 1 - F(A^E(\bar{\varepsilon}, e^*)),
\]

(20)

where

\[
A^E(\bar{\varepsilon}, e^*) = (1 - G(\bar{\varepsilon}|e^*)) (\eta + \alpha_1 \rho) - (1 - H(\bar{\varepsilon}|e^*)) (\eta + \alpha_2 \rho).
\]

(21)

**Proof.** See Mathematical Appendix.  ■

The effect of the warrant standard on equilibrium effort by the police is then

\[
\frac{de^*}{d\bar{\varepsilon}} = \frac{2}{SOC} \left\{ - \frac{\partial C^E(\bar{\varepsilon}, e^*)}{\partial e} \left( \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} + k \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} \right) - C^E(\bar{\varepsilon}, e^*) \frac{\partial g(\bar{\varepsilon}|e^*)}{\partial e} + k (1 - C^E(\bar{\varepsilon}, e^*)) \frac{\partial h(\bar{\varepsilon}|e^*)}{\partial e} \right\},
\]

(22)

where

\[
SOC = 2 \left\{ - \frac{\partial C^E(\bar{\varepsilon}, e^*)}{\partial e} \left( \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} + k \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} \right) - C^E(\bar{\varepsilon}, e^*) \frac{\partial^2 G(\bar{\varepsilon}|e^*)}{\partial e^2} + k (1 - C^E(\bar{\varepsilon}, e^*)) \frac{\partial^2 H(\bar{\varepsilon}|e^*)}{\partial e^2} \right\} < 0.
\]

(23)

The effect of the warrant standard on equilibrium crime is

\[
\frac{dC^E(\bar{\varepsilon}, e^*)}{d\bar{\varepsilon}} = f(A^E) \left\{ g(\bar{\varepsilon}|e^*) (\eta + \alpha_1 \rho) - h(\bar{\varepsilon}|e^*) (\eta + \alpha_2 \rho) \right\} + \frac{de^*}{d\bar{\varepsilon}} \frac{\partial C^E(\bar{\varepsilon}, e^*)}{\partial e},
\]

(24)

where

\[
\frac{\partial C^E(\bar{\varepsilon}, e^*)}{\partial e} = f(A^E) \left\{ \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} (\eta + \alpha_1 \rho) - \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} (\eta + \alpha_2 \rho) \right\} < 0.
\]

(25)

We then obtain the following results:
Corollary 5 $de^*/d\tilde{\varepsilon} > 0$ if

\[ 0 < k < -\frac{\partial G(\tilde{\varepsilon}|e^*)}{\partial e}f\frac{\partial H(\tilde{\varepsilon}|e^*)}{\partial e} \]  

(26)

and $g(\tilde{\varepsilon}|e) > \eta + \alpha_2 \rho \frac{\partial C_E(\tilde{\varepsilon}, e^*)}{\partial e}$ around $e^*$.  

(27)

When $de^*/d\tilde{\varepsilon} > 0$, $dC_E(\tilde{\varepsilon}, e^*)/d\tilde{\varepsilon} > 0$ if

\[ 0 < \frac{de^*}{d\tilde{\varepsilon}} < -\frac{f(A^E) [g(\tilde{\varepsilon}|e)(\eta + \alpha_1 \rho) - h(\tilde{\varepsilon}|e)(\eta + \alpha_2 \rho)]}{\partial C^E(\tilde{\varepsilon}, e^*)/\partial e}. \]  

(28)

Proof. See Mathematical Appendix. ■

If the police care sufficiently little about whether or not they search an innocent person ($k$ is sufficiently low), and if the warrant standard is not too low to begin with, then an increase in the warrant standard increases police effort. The police cannot rely as much on Nature to produce evidence strong enough to meet the warrant standard if the standard is higher, and therefore they put in greater effort. Moreover, when an increase in the warrant standard increases police effort, it increases crime as well only if it does not increase police effort by too much. The extent to which an increase in the warrant standard increases crime is proportional to

\[ -\frac{f(A^E) [g(\tilde{\varepsilon}|e)(\eta + \alpha_1 \rho) - h(\tilde{\varepsilon}|e)(\eta + \alpha_2 \rho)]}{\partial C^E(\tilde{\varepsilon}, e^*)/\partial e} - \frac{de^*}{d\tilde{\varepsilon}} > 0. \]  

(29)

The more an increase in the standard increases police effort, the less it increases crime.

Similarly, the more an increase in the standard increases police effort, the less it increases the probability that a person is guilty and not searched. Given endogenous police effort, the effect of the warrant standard on the probability that a person is guilty and not searched is

\[ \frac{\partial WNE(\tilde{\varepsilon})}{\partial \tilde{\varepsilon}} = C^E(\tilde{\varepsilon}, e^*)g(\tilde{\varepsilon}|e^*) + \frac{\partial C^E(\tilde{\varepsilon}, e^*)}{\partial \tilde{\varepsilon}}G(\tilde{\varepsilon}|e^*) \]

(30)

\[ + \frac{de^*}{d\tilde{\varepsilon}} \{ \frac{\partial G(\tilde{\varepsilon}|e^*)}{\partial e} C^E(\tilde{\varepsilon}, e^*) + \frac{\partial C^E(\tilde{\varepsilon}, e^*)}{\partial e} G(\tilde{\varepsilon}|e^*) \}. \]
If an increase in the warrant standard increases police effort \((d e^*/d \bar{\varepsilon} > 0)\), then an increase in police effort tends to directly reduce the probability that a person is both guilty and not searched by increasing the accuracy of the police’s evidence \(((\partial G(\bar{\varepsilon}|e^*)/\partial e)C^E(\bar{\varepsilon}, e^*) < 0)\). It also tends to indirectly reduce the probability that a person is both guilty and searched to the extent that it reduces crime so that there are fewer guilty people around \(((\partial C^E(\bar{\varepsilon}, e^*)/\partial e)G(\bar{\varepsilon}|e^*) < 0)\).

Moreover, an increase in the warrant standard might significantly reduce the probability that a person is innocent and searched because greater police effort directly increases the quality of police evidence. The effect of the standard on wrongful searches, given endogenous effort, is

\[
\frac{\partial WSE(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = -(1 - C^E(\bar{\varepsilon}, e^*))h(\bar{\varepsilon}|e^*) - \frac{\partial C^E(\bar{\varepsilon}, e^*)}{\partial \bar{\varepsilon}}(1 - H(\bar{\varepsilon}|e^*)) + \frac{d e^*}{d \bar{\varepsilon}} \left\{ -\frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e}(1 - C^E(\bar{\varepsilon}, e^*)) - \frac{\partial C^E(\bar{\varepsilon}, e^*)}{\partial e}(1 - H(\bar{\varepsilon}|e^*)) \right\}. \tag{31}
\]

As in the basic model, for any given effort level by the police, an increase in the warrant standard tends to directly reduce wrongful searches by making it harder for the police to obtain a warrant \((- (1 - C^E(\bar{\varepsilon}, e^*))h(\bar{\varepsilon}|e^*) < 0)\), and it also tends to indirectly reduce wrongful searches to the extent that it increases crime \((- (\partial C^E(\bar{\varepsilon}, e^*)/\partial \bar{\varepsilon})(1 - H(\bar{\varepsilon}|e^*)) < 0)\). In addition, if an increase in the standard increases police effort \((d e^*/d \bar{\varepsilon} > 0)\), then an increase in effort tends to directly reduce wrongful searches by increasing the accuracy of evidence \((- (\partial H(\bar{\varepsilon}|e^*)/\partial e)(1 - C^E(\bar{\varepsilon}, e^*) < 0)\). An increase in effort also tends to indirectly increase wrongful searches to the extent that it reduces crime so that more people are innocent \((- (\partial C^E(\bar{\varepsilon}, e^*)/\partial e)(1-H(\bar{\varepsilon}|e^*)) > 0)\). However, as long as the direct effect of effort dominates the indirect effect, an increase in the standard reduces wrongful searches even more when we take its effect on effort into account.

An increase in the warrant standard might increase crime and the probability that a person is guilty and not searched by a small extent, but reduce the probability that a person
is innocent and searched by a large extent, because it might increase police effort, and greater police effort reduces crime and increases the accuracy of the police’s evidence. Thus a right against unreasonable searches might be considerably more efficient if police effort is taken into consideration.

6. Preventive Searches

In both the basic model and extended model with endogenous police effort, we have assumed that citizens who commit a crime derive their benefit from the crime, \( w \), whether or not they are searched. This is a reasonable assumption if searches are used to solve crimes that have already been committed, such as murder or rape. But searches may also be used to disrupt crimes in progress or prevent future crimes. For example, police may use searches to find stolen goods, drugs, or guns that have not yet been sold, or to find evidence regarding future terrorist attacks. In this case, criminals might not benefit if they are searched.

We now assume that citizens who commit crime derive their benefit, \( w \), only if they are not searched, i.e., searches are “preventive.” We find that the results of the previous sections continue to hold qualitatively, and one of the results becomes stronger.

**Proposition 3** In the basic model, if searches are preventive, then the equilibrium probability of crime is

\[
C^P(\bar{\varepsilon}) = 1 - F \left( A^P(\bar{\varepsilon}) \right),
\]

where

\[
A^P(\bar{\varepsilon}) = \frac{(1 - G(\bar{\varepsilon})) (\eta + \alpha_1(\bar{\varepsilon}, \theta) \rho) - (1 - H(\bar{\varepsilon})) (\eta + \alpha_2(\bar{\varepsilon}, \theta) \rho)}{G(\bar{\varepsilon})}.
\]

The equilibrium probability of search is

\[
S^P(\bar{\varepsilon}) = C^P(\bar{\varepsilon})(1 - G(\bar{\varepsilon})) + (1 - C^P(\bar{\varepsilon}))(1 - H(\bar{\varepsilon})).
\]

**Proof.** See Mathematical Appendix.
The effect of the warrant standard on equilibrium crime is now
\[
\frac{\partial C^P(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = f(A^P(\bar{\varepsilon}))[g(\bar{\varepsilon})(\eta + \alpha_1(\bar{\varepsilon}, \theta)\rho) - (G(\bar{\varepsilon})h(\bar{\varepsilon}) + (1 - H(\bar{\varepsilon}))g(\bar{\varepsilon}))(\eta + \alpha_2(\bar{\varepsilon}, \theta)\rho)].
\] (35)

We now have the following condition for crime to be increasing in the warrant standard:

**Corollary 6**

\[
\frac{\partial C^P(\bar{\varepsilon})}{\partial \bar{\varepsilon}} > 0 \iff \frac{g(\bar{\varepsilon})}{h(\bar{\varepsilon})G(\bar{\varepsilon}) + g(\bar{\varepsilon})(1 - H(\bar{\varepsilon}))} > \frac{\eta + \alpha_2(\bar{\varepsilon}, \theta)\rho}{\eta + \alpha_1(\bar{\varepsilon}, \theta)\rho}.
\] (36)

Comparing the conditions in Corollary 1 and Corollary 6, we have

\[
\frac{g(\bar{\varepsilon})}{h(\bar{\varepsilon})G(\bar{\varepsilon}) + g(\bar{\varepsilon})(1 - H(\bar{\varepsilon}))} < \frac{g(\bar{\varepsilon})}{h(\bar{\varepsilon})} \text{ for all } \bar{\varepsilon}
\] (37)

\[
\iff \frac{g(\bar{\varepsilon})}{1 - G(\bar{\varepsilon})} > \frac{h(\bar{\varepsilon})}{1 - H(\bar{\varepsilon})} \text{ for all } \bar{\varepsilon},
\]

which holds since the hazard rate is higher for the guilty than for the innocent by assumption.

If we maintain the reasonable assumption that \(g(0)/h(0) < (\eta + \alpha_2\rho)/(\eta + \alpha_1\rho)\), it follows that \(g(0)/[h(0)G(0) + g(0)(1 - H(0))] < (\eta + \alpha_2\rho)/(\eta + \alpha_1\rho)\). Moreover,

\[
\frac{g(\bar{\varepsilon})}{h(\bar{\varepsilon})G(\bar{\varepsilon}) + g(\bar{\varepsilon})(1 - H(\bar{\varepsilon}))} \text{ is increasing in } \bar{\varepsilon} \iff g'(\bar{\varepsilon})h(\bar{\varepsilon}) - g(\bar{\varepsilon})h(\bar{\varepsilon})' > 0,
\] (38)

which holds by the monotone likelihood ratio property. Therefore, there is a unique \(\bar{\varepsilon}_c^P\) satisfying

\[
\phi(\bar{\varepsilon}_c^P; \eta, \theta, \rho, \bar{\varepsilon}) = \frac{g(\bar{\varepsilon}_c^P)}{h(\bar{\varepsilon}_c^P)G(\bar{\varepsilon}_c^P) + g(\bar{\varepsilon}_c^P)(1 - H(\bar{\varepsilon}_c^P))} - \frac{\eta + \alpha_2(\bar{\varepsilon}_c^P, \theta)\rho}{\eta + \alpha_1(\bar{\varepsilon}_c^P, \theta)\rho} = 0.
\] (39)

We then have

\[
\frac{\partial C^P(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = \begin{cases} 
< 0 & \text{if } \bar{\varepsilon} < \bar{\varepsilon}_c^P \\
= 0 & \text{if } \bar{\varepsilon} = \bar{\varepsilon}_c^P \\
> 0 & \text{if } \bar{\varepsilon} > \bar{\varepsilon}_c^P
\end{cases}
\] (40)

Lastly, from (37), it follows directly that \(\bar{\varepsilon}_c^P > \bar{\varepsilon}_c\), where \(\bar{\varepsilon}_c\) is defined in (8) and \(\bar{\varepsilon}_c^P\) is defined in (39). Thus, we have the following result:
Corollary 7 An increase in the warrant standard reduces crime for a larger range of parameters if searches are preventive than if they are not.

The expected benefit from committing crime is lower if searches are preventive than if they are not. Therefore, if the warrant standard is low, an even larger proportion of the people being searched by the police is innocent in the case of preventive searches, so increasing the warrant standard is even more likely to reduce the expected costs of being searched more for an innocent than for a guilty person, and thereby actually reduce crime. For this reason, crime is decreasing in the warrant standard for a larger range of parameters if searches are preventive. Moreover, when crime is decreasing in the warrant standard, the probability of search is also decreasing in the warrant standard. Hence, an increase in the warrant standard actually reduces crime as well as searches for a larger range of parameters if searches are preventive than if they are not.

It is easy to see that the qualitative results of Corollaries 2, 3, and 4 hold with $\tilde{\varepsilon}_c$ simply replaced by $\tilde{\varepsilon}_c^P$ if searches are preventive. Lastly, in Appendix B, we show that all the results of the extended model with endogenous police effort (namely, Proposition 2 and Corollary 5) also continue to hold qualitatively when searches are preventive.

7. Conclusion

We developed an economic model of crime and search to analyze the effects of the evidence standard for search warrants. Our main findings may be summarized as follows. If the warrant standard is initially below a certain positive threshold, then increasing it actually reduces crime as well as searches. Moreover, the positive threshold is higher if searches are preventive than if they are not. Thus, a basic right against unreasonable searches might be efficient, at least with respect to deterrence, and especially in the case of preventive searches.
If the warrant standard is initially above a positive threshold, then increasing it increases crime. However, if the police do not care too much about whether or not they search the innocent, increasing the standard also increases police effort to gather initial evidence non-invasively. Therefore, increasing the standard may not increase crime much since greater police effort tends to reduce crime. Moreover, greater police effort directly increases the accuracy of the police’s initial evidence. Thus, an increase in the standard might significantly reduce wrongful searches without significantly increasing crime. This also provides an economic justification for a right against unreasonable searches.

The results have direct policy implications. Countries often respond to internal or external security threats by weakening the search and seizure constraints on their police forces. For example, after the terrorist attacks on the U.S. in 2001, the U.S. government enacted the USA PATRIOT Act of 2001 (Public Law 107-56), several provisions of which explicitly lowered the warrant standards in terrorism as well as other cases. The model suggests that, even if the standards were initially high enough that the reduction in the standards increased security, the reduction in the standards might not have increased security much because it might have reduced police effort to gather initial evidence non-invasively. Moreover, it might have increased wrongful searches a lot, both directly and through its effect on police effort.

The model could be extended in at least one other interesting way. Throughout our analysis, we assumed that the police do not search when the evidence is below the warrant standard. The police might search illegally or without a warrant if there is a chance that an illegal search would not be thrown out. The court might not always throw out an illegal search, especially if the search uncovers incriminating evidence about a suspect in a serious crime. In such cases, an increase in the probable cause standard might increase the probability of an illegal search and, therefore, might increase crime even less.
A Appendix A: Mathematical Appendix

Proof of Proposition 1. For a citizen with benefit \( \omega \) from a crime, the expected utility from committing the crime is

\[
EU(C) = G(\bar{\epsilon})(\omega) + (1 - G(\bar{\epsilon}))(\omega - \eta - \alpha_1(\bar{\epsilon}, \theta)\rho),
\]

(A1)

and the expected payoff from not committing the crime is

\[
EU(\neg C) = (1 - H(\bar{\epsilon}))(\eta - \alpha_2(\bar{\epsilon}, \theta)\rho).
\]

(A2)

There exists a citizen with benefit \( \omega^* \) for whom

\[
EU(C) = EU(\neg C) \iff \omega^* = (1 - G(\bar{\epsilon}))(\eta + \alpha_1(\bar{\epsilon}, \theta)\rho) - (1 - H(\bar{\epsilon}))(\eta + \alpha_2(\bar{\epsilon}, \theta)\rho).
\]

(A3)

The equilibrium fraction of citizens who commit the crime is then

\[
C(\bar{\epsilon}) = \text{prob}(\omega > \omega^*) = 1 - F(A(\bar{\epsilon})),
\]

(A4)

where,

\[
A(\bar{\epsilon}) = (1 - G(\bar{\epsilon}))(\eta + \alpha_1(\bar{\epsilon}, \theta)\rho) - (1 - H(\bar{\epsilon}))(\eta + \alpha_2(\bar{\epsilon}, \theta)\rho).
\]

(A5)

Therefore, the equilibrium probability of search is

\[
S(\bar{\epsilon}) = C(\bar{\epsilon})(1 - G(\bar{\epsilon}))(1 - C(\bar{\epsilon}))(1 - H(\bar{\epsilon})),
\]

and the equilibrium probability that a person is guilty and not searched is

\[
WN(\bar{\epsilon}) = C(\bar{\epsilon})G(\bar{\epsilon}).
\]

Q.E.D.

Proof of Corollary 2. Applying the implicit function theorem to (8), we obtain

\[
\frac{\partial \bar{\epsilon}_c}{\partial \eta} = B(\bar{\epsilon}_c)[\frac{\rho(\alpha_1(\bar{\epsilon}, \theta) - \alpha_2(\bar{\epsilon}, \theta))}{(\eta + \alpha_1(\bar{\epsilon}, \theta)\rho)^2}]
\]

(A6)

\[
\frac{\partial \bar{\epsilon}_c}{\partial \theta} = B(\bar{\epsilon}_c)[\frac{\partial \alpha_2(\bar{\epsilon}, \theta)}{\partial \theta}(\eta + \alpha_1(\bar{\epsilon}, \theta)\rho) - \frac{\partial \alpha_1(\bar{\epsilon}, \theta)}{\partial \theta}(\eta + \alpha_2(\bar{\epsilon}, \theta)\rho)]
\]

(A7)
\[ \frac{\partial \bar{\varepsilon}_c}{\partial \rho} = B(\bar{\varepsilon}_c) \left[ \frac{\eta \left( \alpha_2(\bar{\varepsilon}, \theta) - \alpha_1(\bar{\varepsilon}, \theta) \right)}{(\eta + \alpha_1(\bar{\varepsilon}, \theta)^2} \right], \quad \tag{A8} \]

\[ \frac{\partial \bar{\varepsilon}_c}{\partial \bar{\varepsilon}} = B(\bar{\varepsilon}_c) \left[ \frac{\frac{\partial \alpha_2(\bar{\varepsilon}, \theta)}{\partial \bar{\varepsilon}} \rho(\eta + \alpha_1(\bar{\varepsilon}, \theta) \rho) - \frac{\partial \alpha_1(\bar{\varepsilon}, \theta)}{\partial \bar{\varepsilon}} \rho(\eta + \alpha_2(\bar{\varepsilon}, \theta) \rho)}{(\eta + \alpha_1(\bar{\varepsilon}, \theta)^2} \right], \quad \tag{A9} \]

where

\[ B(\bar{\varepsilon}_c) = \frac{h(\bar{\varepsilon}_c)^2}{g'(\bar{\varepsilon}_c)h(\bar{\varepsilon}_c) - h'(\bar{\varepsilon}_c)g(\bar{\varepsilon}_c)}. \quad \tag{A10} \]

By the monotone likelihood ratio assumption, \( g'h - h'g > 0 \), so \( B(\bar{\varepsilon}_c) > 0 \). Moreover, \( \alpha_1(\bar{\varepsilon}, \theta) > \alpha_2(\bar{\varepsilon}, \theta) \) for any \((\bar{\varepsilon}, \theta)\); \( \partial \alpha_1(\bar{\varepsilon}, \theta)/\partial \bar{\varepsilon} < 0 \) and \( \partial \alpha_2(\bar{\varepsilon}, \theta)/\partial \bar{\varepsilon} < 0 \); and \( \partial \alpha_1(\bar{\varepsilon}, \theta)/\partial \theta > 0 \) and \( \partial \alpha_2(\bar{\varepsilon}, \theta)/\partial \theta < 0 \). Therefore, \( \frac{\partial \bar{\varepsilon}_c}{\partial \eta} > 0 \), \( \frac{\partial \bar{\varepsilon}_c}{\partial \theta} < 0 \), \( \frac{\partial \bar{\varepsilon}_c}{\partial \rho} < 0 \), and \( \frac{\partial \bar{\varepsilon}_c}{\partial \bar{\varepsilon}} < 0 \). Q.E.D.

**Proof of Proposition 2.** For a citizen with benefit \( \omega \) from a crime, the expected utility from committing the crime is

\[ EU(C) = G(\bar{\varepsilon}|e)(\omega) + (1 - G(\bar{\varepsilon}|e)) (\omega - \eta - \alpha_1 \rho), \quad \tag{A11} \]

and the expected payoff from not committing the crime is

\[ EU(\neg C) = (1 - H(\bar{\varepsilon}|e)) (\omega - \eta - \alpha_2 \rho) \quad \tag{A12} \]

for a given effort level \( e \) by the police. There exists a citizen with benefit \( \omega^* \) for whom

\[ EU(C) = EU(\neg C) \Leftrightarrow \omega^* = (1 - G(\bar{\varepsilon}|e)) (\eta + \alpha_1 \rho) - (1 - H(\bar{\varepsilon})) (\eta + \alpha_2 \rho). \quad \tag{A13} \]

The equilibrium fraction of citizens who commit the crime is then

\[ C^E(\bar{\varepsilon}, e) = 1 - F(A^E(\bar{\varepsilon}, e)), \quad \tag{A14} \]

where

\[ A^E(\bar{\varepsilon}, e) = (1 - G(\bar{\varepsilon}|e)) (\eta + \alpha_1 \rho) - (1 - H(\bar{\varepsilon}|e)) (\eta + \alpha_2 \rho). \quad \tag{A15} \]

For a given probability of crime \( C^E(\bar{\varepsilon}) \), the police’s problem is

\[ \max_e C^E(\bar{\varepsilon})[(1 - G(\bar{\varepsilon}|e)) - G(\bar{\varepsilon}|e)] + (1 - C^E(\bar{\varepsilon}))[-(1 - H(\bar{\varepsilon}|e)) k + H(\bar{\varepsilon}|e) k] - c_e e. \quad \tag{A16} \]
The first order condition is

\[ C^E(\bar{\varepsilon})(-2 \frac{\partial G(\bar{\varepsilon}|e)}{\partial e}) + (1 - C^E(\bar{\varepsilon},e))(2k \frac{\partial H(\bar{\varepsilon}|e)}{\partial e}) - c_e = 0. \]  

(A17)

Combining (A14), (A15), and (A17), we obtain

\[ \lambda(\bar{\varepsilon}, e^*) = C^E(\bar{\varepsilon}, e^*)(-2 \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e}) + (1 - C^E(\bar{\varepsilon}, e^*))(2k \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e}) - c_e = 0, \]  

(A18)

and

\[ C^E(\bar{\varepsilon}, e^*) = 1 - F((1 - G(\bar{\varepsilon}|e^*))\eta + \alpha_1 \rho) - (1 - H(\bar{\varepsilon}|e^*))\eta + \alpha_2 \rho), \]  

(A19)

which characterize equilibrium effort and crime. Q.E.D.

**Proof of Corollary 5.** Combining (22) and (24), we get

\[ \frac{de^*}{d\bar{\varepsilon}} = \frac{1}{\Delta} \left\{ -f(A^E)\left( \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} + \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} k \right) \right\} \]  

\[ * \left[ g(\bar{\varepsilon}|e^*)(\eta + \alpha_1 \rho) - h(\bar{\varepsilon}|e^*)(\eta + \alpha_2 \rho) \right] \]  

\[ - C^E(\bar{\varepsilon}, e^*) \frac{\partial g(\bar{\varepsilon}|e^*)}{\partial e} + k(1 - C^E(\bar{\varepsilon}, e^*)) \frac{\partial h(\bar{\varepsilon}|e^*)}{\partial e^2} \}, \]  

(A20)

where

\[ \Delta = 1 + 2(-1) f(A^E)\left( \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} + \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} k \right) \]  

\[ * \left[ \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} (\eta + \alpha_1 \rho) - \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} (\eta + \alpha_2 \rho) \right] \]  

(A21)

and

\[ SOC = 2\left\{ - \frac{\partial C^E(\bar{\varepsilon}, e^*)}{\partial e} \left( \frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} + k \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} \right) \right\} \]  

\[ - C^E(\bar{\varepsilon}, e^*) \frac{\partial^2 G(\bar{\varepsilon}|e^*)}{\partial e^2} + k(1 - C^E(\bar{\varepsilon}, e^*)) \frac{\partial^2 H(\bar{\varepsilon}|e^*)}{\partial e^2} \}. \]  

(A22)

The term $SOC$ is negative, and the term $\Delta$ is positive as long as $\frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} + \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} k < 0 \iff k < -\frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} / \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e}$ because $\frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e} (\eta + \alpha_1 \rho) - \frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e} (\eta + \alpha_2 \rho) < 0$. Because $\frac{\partial g(\bar{\varepsilon}|e^*)}{\partial e} < 0$
and $\frac{\partial h(\bar{\varepsilon}|\bar{e}^*)}{\partial e} > 0$ by assumption, we have $C^E(\bar{\varepsilon}, e^*)(-\frac{\partial g(\bar{\varepsilon}|e^*)}{\partial e}) + (1 - C^E(\bar{\varepsilon}, e^*))(k\frac{\partial h(\bar{\varepsilon}|e^*)}{\partial e}) > 0$.

Moreover, if $\frac{g(\bar{\varepsilon}|e)}{h(\bar{\varepsilon}|e)} > \frac{(\eta + \alpha_2 \rho)}{(\eta + \alpha_1 \rho)}$ around $e^*$, then we have $g(\bar{\varepsilon}|e^*)((\eta + \alpha_1 \rho) - h(\bar{\varepsilon}|e^*)((\eta + \alpha_2 \rho)) > 0$.

Therefore, $\frac{de^*}{d\bar{\varepsilon}} > 0$ if $0 < k < -\frac{\partial G(\bar{\varepsilon}|e^*)}{\partial e}/\frac{\partial H(\bar{\varepsilon}|e^*)}{\partial e}$ and $\frac{g(\bar{\varepsilon}|e)}{h(\bar{\varepsilon}|e)} > \frac{(\eta + \alpha_2 \rho)}{(\eta + \alpha_1 \rho)}$ around $e^*$. On the other hand, if $\frac{de^*}{d\bar{\varepsilon}} > 0$, from (24), $\frac{dc^E(\bar{\varepsilon}, e^*)}{d\bar{\varepsilon}} > 0$ as long as $0 < \frac{de^*}{d\bar{\varepsilon}} < -\frac{f(A^F)[g(\bar{\varepsilon}|e^*)((\eta + \alpha_1 \rho) - h(\bar{\varepsilon}|e^*)((\eta + \alpha_2 \rho))]}{\partial C^E(\bar{\varepsilon}, e^*)/\partial e}$.

Q.E.D.

**Proof of Proposition 3.** For a citizen with benefit $\omega$ from a crime, the expected utility from committing the crime is

$$EU(C) = G(\bar{\varepsilon})(\omega) + (1 - G(\bar{\varepsilon}))(\eta - \alpha_1(\bar{\varepsilon}, \theta) \rho),$$

(A23)

and the expected payoff from not committing the crime is

$$EU(\neg C) = (1 - H(\bar{\varepsilon}))(\eta - \alpha_2(\bar{\varepsilon}, \theta) \rho).$$

(A24)

There exists a citizen with benefit $\omega^*$ for whom

$$EU(C) = EU(\neg C) \iff \omega^* = \frac{(1 - G(\bar{\varepsilon}))(\eta + \alpha_1(\bar{\varepsilon}, \theta) \rho) - (1 - H(\bar{\varepsilon}))(\eta + \alpha_2(\bar{\varepsilon}, \theta) \rho)}{G(\bar{\varepsilon})}.$$  \hspace{1cm} (A25)

The equilibrium fraction of citizens who commit the crime is then

$$C^P(\bar{\varepsilon}) = \text{prob}(\omega > \omega^*) = 1 - F\left(A^P(\bar{\varepsilon})\right),$$

(A26)

where,

$$A^P(\bar{\varepsilon}) = \frac{(1 - G(\bar{\varepsilon}))(\eta + \alpha_1(\bar{\varepsilon}, \theta) \rho) - (1 - H(\bar{\varepsilon}))(\eta + \alpha_2(\bar{\varepsilon}, \theta) \rho)}{G(\bar{\varepsilon})}.$$ \hspace{1cm} (A27)

The equilibrium probability of search is then $S^P(\bar{\varepsilon}) = C^P(\bar{\varepsilon})(1 - G(\bar{\varepsilon})) + (1 - C^P(\bar{\varepsilon}))(1 - H(\bar{\varepsilon}))$.

Q.E.D.
Appendix B: Police Effort and Preventive Searches

In this Appendix, we show that the results of the model with endogenous police effort, which are reported in Proposition 2 and Corollary 5 in Section 5, continue to hold qualitatively in the case of preventive searches. Consider the model with both endogenous police effort and preventive searches. For a given effort level $e$ by the police, there exists a citizen with benefit $\omega^*$ for whom

$$EU(C) = EU(\neg C) \Leftrightarrow \omega^* = \frac{1}{G}\{(1 - G)(\eta + \alpha_1 \rho) - (1 - H)(\eta + \alpha_2 \rho)\},$$  \hspace{2cm} (B1)$$

where $G \equiv G(\bar{\epsilon}|e)$, and $H \equiv H(\bar{\epsilon}|e)$.

For a given probability of crime $C_{EP}(\bar{\epsilon})$, the police’s problem is then

$$\max_e C_{EP}(\bar{\epsilon})[\{(1 - G) - G\} + (1 - C_{EP}(\bar{\epsilon}))\{(1 - H)k + Hk\} - c_e e].$$  \hspace{2cm} (B2)$$

The first order condition is

$$C_{EP}(\bar{\epsilon})(-2 \frac{\partial G}{\partial e}) + (1 - C_{EP}(\bar{\epsilon}, e))(2k \frac{\partial H}{\partial e}) - c_e = 0.$$  \hspace{2cm} (B3)$$

Therefore, the equilibrium effort by the police $e^*$ is characterized by

$$\lambda(\bar{\epsilon}, e^*) = C_{EP}(\bar{\epsilon}, e^*)(-2G'(e^*)) + (1 - C_{EP}(\bar{\epsilon}, e^*)) (2kH'(e^*)) - c_e = 0,$$  \hspace{2cm} (B4)$$

and equilibrium crime is

$$C_{EP}(\bar{\epsilon}, e^*) = 1 - F\left(\frac{1}{G^*}\{(1 - G^*(\eta + \alpha_1 \rho) - (1 - H^*) (\eta + \alpha_2 \rho)\}\},$$  \hspace{2cm} (B5)$$

where $G'(e^*) \equiv \frac{\partial G(\bar{\epsilon}|e^*)}{\partial e}$, $H'(e^*) \equiv \frac{\partial H(\bar{\epsilon}|e^*)}{\partial e}$, $G^* \equiv G(\bar{\epsilon}|e^*)$ and $H^* \equiv H(\bar{\epsilon}|e^*)$.

The effect of the warrant standard on equilibrium effort by the police is then

$$\frac{de^*}{d\bar{\epsilon}} = -\frac{1}{SOC}\{-2\frac{\partial C_{EP}(\bar{\epsilon}, e^*)}{\partial \bar{\epsilon}}(G'(e^*) + kH'(e^*)) - 2C_{EP}(\bar{\epsilon}, e^*) \frac{\partial g(\bar{\epsilon}|e^*)}{\partial e} + 2k(1 - C_{EP}(\bar{\epsilon}, e^*)) \frac{\partial h(\bar{\epsilon}|e^*)}{\partial e}\}.$$  \hspace{2cm} (B6)$$

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The effect of the warrant standard on equilibrium crime is
\[
\frac{dC^{EP}(\bar{\epsilon}, e^*)}{d\bar{\epsilon}} = \frac{f(A^{EP})}{G^{*2}}\{DE\} + \frac{de^*}{d\bar{\epsilon}} \frac{\partial C^{EP}(\bar{\epsilon}, e^*)}{\partial \epsilon},
\]
where
\[
DE \equiv g(\bar{\epsilon}|e^*)(\eta + \alpha_1 \rho) - (h(\bar{\epsilon}|e^*)G^* + (1 - H^*)g(\bar{\epsilon}|e^*)))(\eta + \alpha_2 \rho)
\]
is the direct effect of the search warrant standard on crime. The second term of (B7) is the indirect effect of search warrant standard on crime through its effect on the police effort.

The effect of police effort on equilibrium crime is
\[
\frac{\partial C^{EP}(\bar{\epsilon}, e^*)}{\partial \epsilon} = \frac{f(A^{EP})}{G^{*2}}(G'(e^*)(\alpha_1 - \alpha_2) - (H'(e^*)G^* - G'(e^*)H^*)(\eta + \alpha_2 \rho) < 0.
\]
Combining (B7), (B8), and (B9), we get
\[
\frac{de^*}{d\bar{\epsilon}} = \frac{1}{\Delta(\text{SOC})}\left\{-\frac{f(A^{EP})}{G^{*2}}(G'(e^*) + H'(e^*)k)(DE)ight. \\
-C^{EP}(\bar{\epsilon}, e^*)\frac{\partial g(\bar{\epsilon}|e^*)}{\partial \epsilon} + k(1 - C^{EP}(\bar{\epsilon}, e^*))\frac{\partial h(\bar{\epsilon}|e^*)}{\partial \epsilon}\right\},
\]
where
\[
\Delta = 1 + \left(\frac{-2}{\text{SOC}}\right)\frac{f(A^{EP})}{G^{*2}}(G'(e^*) + H'(e^*)k) \\
\times [G'(e^*)(\alpha_1 - \alpha_2) - (H'(e^*)G^* - G'(e^*)H^*)(\eta + \alpha_2 \rho)].
\]
The term \( \Delta \) is positive, as long as \((G'(e^*) + H'(e^*)k) < 0 \Leftrightarrow k < \left(\frac{-\partial G(\bar{\epsilon}|e^*)}{\partial \epsilon} / \partial H(\bar{\epsilon}|e^*)\right) \) because \(G'(e^*)(\alpha_1 - \alpha_2) - (H'(e^*)G^* - G'(e^*)H^*)(\eta + \alpha_2 \rho) < 0 \) and \( \text{SOC} < 0 \). Since \(\frac{\partial g(\bar{\epsilon}|e^*)}{\partial \epsilon} < 0 \) and \(\frac{\partial h(\bar{\epsilon}|e^*)}{\partial \epsilon} > 0 \) by assumption, we have \(-2C^{EP}(\bar{\epsilon}, e^*)(-\frac{\partial g(\bar{\epsilon}|e^*)}{\partial \epsilon}) + (1 - C^{EP}(\bar{\epsilon}, e^*))k\frac{\partial h(\bar{\epsilon}|e^*)}{\partial \epsilon} > 0 \). If \( DE > 0 \Leftrightarrow \frac{\partial g(\bar{\epsilon}|e^*)}{\partial \epsilon} \frac{\eta + \alpha_2 \rho} {\eta + \alpha_1 \rho} > \frac{\eta + \alpha_2 \rho} {\eta + \alpha_1 \rho} \) around e* and 0 < k < \left(-\frac{\partial G(\bar{\epsilon}|e^*)}{\partial \epsilon} / \partial H(\bar{\epsilon}|e^*)\right), then we have \( \frac{de^*}{d\bar{\epsilon}} > 0 \). Therefore, \( \frac{de^*}{d\bar{\epsilon}} > 0 \) if
\[
0 < k < \left(-\frac{\partial G(\bar{\epsilon}|e^*)}{\partial \epsilon} / \partial H(\bar{\epsilon}|e^*)\right) \frac{\partial h(\bar{\epsilon}|e^*)}{\partial \epsilon} - \frac{\eta + \alpha_2 \rho} {\eta + \alpha_1 \rho} \)
\]
around e*.

On the other hand, if \( \frac{de^*}{d\bar{\varepsilon}} > 0 \), from (B7), \( \frac{dC^{EP}(\bar{\varepsilon}, e^*)}{d\bar{\varepsilon}} > 0 \) as long as the direct effect \( \left( \frac{f(A^{EP})}{G^{*2}} \{DE\} \right) \) dominates the indirect effect \( \left( \frac{de^*}{d\bar{\varepsilon}} \frac{\partial C^{EP}(\bar{\varepsilon}, e^*)}{\partial e} \right) \). Therefore, when \( de^* / d\bar{\varepsilon} > 0 \),
\[
dC^{EP}(\bar{\varepsilon}, e^*) / d\bar{\varepsilon} > 0 \text{ if }\]
\[
0 < \frac{de^*}{d\bar{\varepsilon}} < -\frac{f(A^{EP})}{G^{*2}} \left\{ \frac{g(\bar{\varepsilon}|e^*)(\eta + \alpha_1 \rho) - (h(\bar{\varepsilon}|e^*)G^* + (1 - H^*)g(\bar{\varepsilon}|e^*)(\eta + \alpha_2 \rho))}{\partial C^{EP}(\bar{\varepsilon}, e^*) / \partial e} \right\}. \quad (B14)
\]
Hence, the results of the model with endogenous police effort continue to hold qualitatively in the case of preventive searches.
REFERENCES


