Optimal Monetary Policy and the Equivalency between the One-Period AD-AS Model and the Forward-Looking New Keynesian Model

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Abstract

We show how a one-period aggregate demand - aggregate supply (AD-AS) model yields equivalent expressions for inflation and output under optimal monetary policy as a forward-looking New Keynesian model. Furthermore, we analyze the advantage the AD-AS model has over the New Keynesian specification when evaluating policy efficiency.

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1 Introduction

In this short paper we show how a one-period aggregate demand - aggregate supply (AD-AS) model yields the exact same reduced form representations for optimal inflation and output under optimal monetary policy as the more elaborate New Keynesian model, in which aggregate behavior results from a maximization process by firms and consumers. Furthermore, we argue that the simple AD-AS model has a practical advantage over the forward-looking New Keynesian model when it comes to evaluating monetary policy efficiency.

Section 2 studies the monetary policy maker’s problem using the stylized one-period model, whose outcome is the reduced form representation for the deviations of inflation and output from their respective targets, assuming policy makers behave optimally. Section 3 compares these outcomes with the one resulting from the forward-looking New Keynesian model developed by Roberts (1995) and employed by Clarida, Galí and Gertler (CGG, 1999) in their analysis of optimal monetary policy, and argues why the simple AD-AS model is better suited for assessing how close actual monetary policy is to the optimal. Section 4 provides a summary of the findings.

2 Policy Maker’s Decision Problem in a One-Period AD-AS Model

We assume that the primary concern of the monetary authority is to achieve stabilization of the economy through the reduction in the variability of inflation and output growth. Consistent with most contemporary analyses of government policy we summarize the policy maker’s objective through the following one-period quadratic loss function:

\[ L = \lambda (\pi_t - \pi_T)^2 + (1 - \lambda) (y_t - y_T)^2 ; \ 0 \leq \lambda \leq 1, \]  

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\[ 1 \]
where $\pi$ is inflation; $y$ is (log) aggregate output; $\pi^T$ and $y^T$ are the target levels of inflation and output; and $\lambda$ is the relative weight given to squared deviations of inflation and output from their desired levels.

Minimization of this loss requires knowledge of the determinants of deviations of inflation and output from their respective targets. We assume that two random shocks push $y$ and $\pi$ away from $y^T$ and $\pi^T$. First, an aggregate demand shock ($d$) moves inflation and output in the same direction, while an aggregate supply shock ($s$) moves inflation and output in opposite directions. Since policy is only capable of moving inflation and output in the same direction its effect is analogous to that of an aggregate demand shock.

We define aggregate demand (AD) as the negative relationship between $(y_t - y_t^T)$ and $(\pi_t - \pi_t^T)$ that is shifted by the demand shock and the deviations of the policy instrument from its equilibrium value ($\bar{r}$):\(^1\),\(^2\)

$$y_t - y_t^T = -\omega(\pi_t - \pi_t^T) - \phi(\bar{r}_t - d_t) ; \omega > 0 , \phi > 0 ,$$

(2)

where $\omega$ is the inverse of the slope of the aggregate demand function and $-\phi$ is the real interest rate semi-elasticity. Analogously, aggregate supply (AS) is the positive relationship between inflation deviations and output deviations that is shifted by the supply shock:

$$\pi_t - \pi_t^T = \gamma(y_t - y_t^T) - s_t ; \gamma > 0 ,$$

(3)

where $\gamma$ is the slope of the aggregate supply function. The aggregate disturbances $d$ and $s$ have been normalized to yield the simple representation of the AD-AS model.

Combining (2) and (3) we obtain expressions for $(y_t - y_t^T)$ and $(\pi_t - \pi_t^T)$ as a function of

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\(^1\)The equilibrium value of the interest rate is defined as the value needed such that output would equal its potential (or target) level.

\(^2\)A similar version of this model can be found in Cecchetti (1998). See also Krause (2003) for a theoretical derivation of the model using a rational expectations optimization process in the presence of imperfect information.
the structural parameters, the aggregate shocks and the policy instrument:

\[ y_t - y^T_t = -\phi(\bar{r}_t - d_t) + \omega s_t \]
\[ \pi_t - \pi^T_t = -\phi \gamma (\bar{r}_t - d_t) - s_t \]

Minimizing the quadratic loss function, subject to the constraints imposed by the structure of the economy, yields a simple linear policy rule of the form:

\[ \bar{r}_t = ad_t + bs_t \]

where the optimal values for the coefficients \( a \) and \( b \) are given by:

\[ a^* = 1 \]
\[ b^* = \frac{-\lambda \gamma + (1 - \lambda) \omega}{\phi \lambda \gamma^2 + (1 - \lambda)} \]

Thus, an optimal policy rule has two parts: first, the authorities completely neutralize the effect of demand shocks, and second, they accommodate supply shocks depending on the structural parameters \( (\omega, \gamma, \phi) \) and their preferences \( (\lambda) \).

Substituting (6)-(8) into (4) and (5) enables us to obtain the reduced form representations for the optimal deviations of inflation and output from their respective targets, i.e.:

\[ (y_t - y^T_t)^* = \frac{\lambda \gamma}{\lambda \gamma^2 + (1 - \lambda)} s_t \]
\[ (\pi_t - \pi^T_t)^* = \frac{-(1 - \lambda)}{\lambda \gamma^2 + (1 - \lambda)} s_t \]

Since both \( (y_t - y^T_t)^* \) and \( (\pi_t - \pi^T_t)^* \) are linear functions of the supply shock only, it follows directly that optimal policy implies a perfectly negative correlation between inflation and output, as pointed out by Cecchetti and Krause (2001). In the next section we show
how this latter result can also be obtained using a New Keynesian, forward-looking model.

3 Equivalency under Optimal Policy and Evaluation of Policy Efficiency: Comparison with a New Keynesian Model

We are interested in showing that under optimal policy the one-period AD-AS model is equivalent to the dynamic forward-looking model used by CGG (1999). To simplify notation, let us rewrite their model with the parameters we employ in our characterization of the economy. Optimal policy without commitment in CGG (1999) implies minimizing:

$$\mathcal{L} = E_t \sum_{i=0}^{\infty} \beta^i [\lambda \pi_t^2 + (1 - \lambda) x_t^2] ; \ 0 \leq \lambda \leq 1 ,$$

subject to the following forward-looking conditions for inflation and the output gap:

$$x_t = -\phi (i_t - E_t \pi_{t+1}) + E_t x_{t+1} + \phi d_t ; \ d_t = \mu d_{t-1} + \tilde{d}_t ; \ \phi > 0 ,$$

$$\pi_t = \gamma x_t + \beta E_t \pi_{t+1} - s_t ; \ s_t = \rho s_{t-1} + \tilde{s}_t ; \ \gamma > 0 , \ 0 \leq \beta, \rho \leq 1 ,$$

where $x_t \equiv y_t - y_t^P$ is the output gap.$^3$ The resulting reduced form representations for $x_t$ and $\pi_t$ under optimal policy are given by:$^4$

$$x_t^* = \frac{\lambda \gamma}{\lambda \gamma^2 + (1 - \lambda)(1 - \beta \rho)} s_t ,$$

$$\pi_t^* = \frac{- (1 - \lambda)}{\lambda \gamma^2 + (1 - \lambda)(1 - \beta \rho)} s_t .$$

$^3$Note that we have redefined their original residuals $u_t$ and $g_t$ with $s_t = -u_t$ and $d_t = \frac{1}{\phi} g_t (\phi \neq 0)$, to achieve an easier comparison between the outcomes of the two models. Clearly, these two normalizations do not affect the key properties of their residuals.

$^4$Equations (14) and (15) are equivalent to equations (3.4) and (3.5) in CGG (1999).
It is straightforward to verify that if we set $\pi_t^T = 0$ and choose potential output as the target level so that $y_t - y_t^T = y_t - y_t^P \equiv x_t$, (9) and (10) will be identical to (14) and (15) whenever $\rho = 0$ (i.e., no serial correlation in the supply shock). Still, even if we are not willing to make these assumptions, the fact remains that, under optimal policy, the New Keynesian model also yields a perfect negative correlation between inflation and the output gap.

Both models imply that policy makers’ performance can be evaluated empirically by examining how close the actual data for inflation and output comes to achieving the perfect correlation that exists in the ideal case of optimal policy. Nevertheless, the AD-AS specification has the virtue of allowing us to directly derive the reduced form representations for inflation and output under a general, non-optimal, policy rule (equations (4) and (5)). In contrast, the forward-looking conditions for inflation and the output gap in the New Keynesian model cannot yield these non-optimal reduced form representations without invoking further assumptions (for example, on the behavior of expectations) or engaging into exercises of estimation or calibration. If we are solely interested in assessing past performance of monetary policy makers, and, hence, we are not concerned about the formation of expectations, this constitutes a key advantage of the AD-AS model.

To obtain a measure for policy efficiency using the AD-AS model we first employ equations (4) and (5) to obtain the actual variances of output and inflation around their respective targets and the covariance of these variables, under the assumption that the second moments of the aggregate shocks are orthogonal:

$$\text{var} (y_t - y_t^T) \equiv \sigma_y^2 = \alpha_{ys}^2 \sigma_s^2 + \alpha_{yd}^2 (a - 1)^2 \sigma_d^2,$$

$$\text{var} (\pi_t - \pi_t^T) \equiv \sigma_\pi^2 = \alpha_{\pi s}^2 \sigma_s^2 + \alpha_{\pi d}^2 (a - 1)^2 \sigma_d^2,$$

$$\text{cov} (y_t - y_t^T, \pi_t - \pi_t^T) \equiv \sigma_{\pi y} = -\alpha_{ys} \alpha_{\pi s} \sigma_s^2 + \alpha_{yd} \alpha_{\pi d} (a - 1)^2 \sigma_d^2.$$
where:
\[ \alpha_{ys} = \frac{\omega - \phi b}{1 + \omega \gamma} ; \quad \alpha_{yd} = \frac{\phi}{1 + \omega \gamma} ; \quad \alpha_{\pi s} = \frac{1 + \gamma \phi b}{1 + \omega \gamma} ; \quad \alpha_{\pi d} = \frac{\gamma \phi}{1 + \omega \gamma}. \]

Substituting the expression of parameter \( b \) in equation (8) into the expressions for \( \alpha_{ys} \) and \( \alpha_{\pi s} \) yields:
\[ \alpha_{ys} = \frac{\lambda \gamma}{\lambda \gamma^2 + (1 - \lambda)} ; \quad \alpha_{\pi s} = \frac{(1 - \lambda)}{\lambda \gamma^2 + (1 - \lambda)}. \]

Equations (9) and (10), on the other hand, allow us to derive the following conditions for the optimal variances and covariances:

\[ \begin{align*}
\sigma_{2y}^* &= \alpha_{ys}^2 \sigma_s^2, \\
\sigma_{2\pi}^* &= \alpha_{\pi s}^2 \sigma_s^2, \\
\sigma_{\pi y}^* &= -\alpha_{ys} \alpha_{\pi s} \sigma_s^2.
\end{align*} \tag{19} \tag{20} \tag{21} \]

which imply:
\[ \sigma_{2y}^* \sigma_{2\pi}^* - \sigma_{\pi y}^* 2 = 0 \tag{22} \]

Consistent with the analysis by Cecchetti and Krause (2001) we employ equations (16)-(18) to define the following efficiency measure:
\[ h \equiv \sigma_y^2 \sigma_{\pi}^2 - (\sigma_{\pi y})^2 = (a - 1)^2 (\alpha_{ys} \alpha_{\pi d} + \alpha_{yd} \alpha_{\pi s})^2 \sigma_s^2 \sigma_d^2 \geq 0 \tag{23} \]

Given that optimal rule policy \( h^* = 0 \), the success of monetary policy in enhancing macroeconomic stability will be greater the closer \( h \) is to zero, controlling for the variance of aggregate demand and aggregate supply shocks. The particular estimation of the shocks and the parameters of interest is an empirical question and, therefore, beyond the scope of the present study.\(^5\)

\(^5\)Cecchetti and Krause (2001) use the measure given in equation (23) to examine changes in monetary policy maker’s performance for a cross-section of 23 countries between the 1980s and the 1990s. Eichengreen (2004) employs this measure to study the reorientation of monetary policy in 11 countries after the Great Depression, by comparing performance between the periods 1919-1930 and 1931-1939.
4 Conclusions

We compare a one-period AD-AS model to a New Keynesian specification and find that both models yield equivalent implications as to how inflation and output behave in the presence of optimal policy. Furthermore, under certain conditions, we are able to show that the reduced form representations of inflation and output resulting from each specification are identical.

We also argue that the AD-AS representation of the economy is better suited than the forward-looking New Keynesian model to evaluate monetary policy performance directly, since the former does not require any further assumptions on the behavior of the variables, which allows us to derive a measure of policy efficiency from a simple model that can be utilized in empirical analysis.

References


