The Impact of Idiosyncratic Uncertainty
When Investment Opportunities Are Endogenous

Junghoon Lee∗

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Abstract

This paper develops a general equilibrium model to study the impact of aggregate fluctuations in idiosyncratic volatility that incorporates the endogenous determination of investment opportunities. By making investment options more valuable, an increase in volatility encourages the creation of new investment options. I find the response of the economy to a volatility shock depends on how investment opportunities are obtained. If potential entrants are allowed to invest in new idiosyncratic technologies, thereby acquiring options for further investment, the volatility shock increases overall investment and results in an economic boom. On the other hand, if such an investment in option creation is precluded and investment opportunities are exogenously given, the volatility shock decreases aggregate investment.

∗Department of Economics, Emory University, 1602 Fishburne Drive, Atlanta, GA 30322.
Email: junghoon.lee@emory.edu
1 Introduction

Uncertainty has conflicting effects on investment. On the one hand, it makes firms hesitant to commit to irreversible investments. On the other hand, it encourages entrepreneurs to experiment with new investment ideas. I present a macroeconomic model that captures both of these effects of uncertainty in order to explore the consequences of allowing for the development of new investment options.

It is well known that an irreversible investment opportunity is like a financial call option. An investment opportunity gives its holder the right to delay an investment until its prospects look good. Making the investment corresponds to exercising the option and once the holder exercises that option, she gives up the possibility of waiting for new information to arrive. Therefore, the holder has to make an investment decision while considering the opportunity cost of losing the real option. The option is valuable since it gives its holder access to outcomes on the upside while it also confines exposure on the downside. Therefore, the values of real options, like financial option prices, increase with volatility.

Existing macroeconomic studies on the real options effect of uncertainty have focused on the exercise of given investment options (e.g., Bloom, 2009). The economy consists of many plants endowed with investment options. More volatile productivity at an individual plant raises the value of an investment option, thereby raising the opportunity cost of exercising the option and in turn delaying investment. Hence, those studies predict that an aggregate increase in idiosyncratic volatility across all plants will imply a synchronized delay of investment and cause a sharp, though temporary, reduction in aggregate investment.
This reasoning, however, is silent about how investment opportunities are obtained in the first place. Investment opportunities sometimes arise inadvertently, but generally they are discovered and created by deliberate activity such as research, innovation, management, and so on. By making investment options more valuable, an increase in volatility also encourages the creation of new options. Therefore, uncertainty enhances investment in new option acquisition.¹

The idea of making a foothold investment in a new uncertain area in order to acquire options for further expansion is empirically supported by Folta and O’Brien (2004), McGrath and Nerkar (2004), and Oriani and Sobrero (2008). They perceive entry into a new industry, taking out a patent, and R&D, respectively, as ways of creating growth options and find that high uncertainty encourages these activities. Stein and Stone (2013) also show that high uncertainty causes firms to increase R&D spending and they explain this result by arguing that uncertainty makes investment options created by R&D more valuable. This micro-level evidence supports the creation of an option as an additional propagation mechanism of uncertainty shocks.

I construct a general equilibrium model of entry and exit to study these two conflicting effects of uncertainty on investment. The economy consists of plants, investment option holders, potential entrants, and consumers. Plants produce aggregate output with their plant-specific productivity. Potential entrants decide optimally when to enter the market. The novel feature of the model is that entry takes place in two stages. At the first stage, a potential entrant pays a cost to discover his own specific technology and becomes a

¹Roberts and Weitzman (1981) study the funding criteria for research, development, and exploration projects and show that the decision maker should be more willing to fund those projects when uncertainty is high.
new holder of the investment option. Option holders and plants face persistent shocks to their specific productivity, so their future prospects are time-varying. At the second stage, the option holder can buy capital and build a plant, or delay his entry into the market until favorable shocks later hit his productivity. The first stage corresponds to the creation of a new investment option and the second stage to the exercise of the option. Once a plant is built, exiting the market can recover only a part of its capital. Hence, existing plants also optimally delay the exit decision.

I assume free entry at the first stage, in which a prospective entrant randomly draws his initial technology from a common pool. New investment opportunities are therefore created endogenously in equilibrium. To make a comparison, I also consider an additional case in which the number of new option holders is fixed in each period so that the creation margin cannot play a role. To emphasize the real options effect of uncertainty, I do not assume any market frictions and I examine the economy from the perspective of the social planner.

I find an aggregate increase in idiosyncratic volatility leads to a higher aggregate productivity and welfare in both cases. Since a higher volatility expands the range of productivity realization, the economy as a whole selects higher productivity units from a more dispersed pool of idiosyncratic productivities. In other words, a market selection mechanism featuring the entry of high-productivity units and the exit of low-productivity ones leads to a better

\[ \text{\footnotesize 2} \text{Hence, the aggregate state includes a distribution of options over their specific productivity in addition to a distribution of plants. Note that the standard firm dynamics models need only the latter in order to describe the economy.}\]

\[ \text{\footnotesize 3} \text{For example, a depressing effect of uncertainty comes from financial frictions, not from real options effects, in Arellano et al. (2012), Christiano et al. (2014), and Gilchrist et al. (2014).}\]
outcome when the selection pool is expanded.\textsuperscript{4} This result is natural since an individual’s objective is aligned with the social planner’s and the logic behind the increased option value of waiting at the individual level is the same: volatility increases the value of the option because the option holder can choose from among a larger set of payoffs.

I also show the economy’s responses to the volatility shock differ sharply in the two cases. High volatility makes investment in new technology lucrative: the productivity of such new technology is more likely to evolve to a high level. If such investment is allowed, the social planner takes advantage of more dispersed idiosyncratic productivity by heavily investing in obtaining productivity draws at the first stage and by adopting that productivity more selectively at the second stage. In other words, the option exercise at the second-stage entry becomes more cautious, but more options end up being exercised because of the increased stock of options. Hence, the volatility shock increases overall investment and leads to an economic boom.

On the other hand, if new investment opportunities are exogenously given, a more cautious exercise of investment options translates directly into a drop in aggregate investment. However, I find an increase in aggregate productivity more than compensates for the decline in aggregate capital: a drop in aggregate output occurs because aggregate productivity growth makes consumers wealthier, thereby reducing the labor supply. Hence the volatility shock in this case behaves like the news shock about future productivity in the standard real business cycle model (e.g., Cochrane, 1994).

The model’s prediction about idiosyncratic volatility and aggregate productivity is consistent with the empirical research of Chun et al. (2008), who

\textsuperscript{4}Schaal (2012) also shows that high idiosyncratic volatility increases the aggregate productivity in his search-and-matching model.
find that industries in the U.S. with higher idiosyncratic volatility achieve faster total factor productivity (TFP) growth over the subsequent five years. Similarly, Durnev et al. (2004) use country-level data to show that idiosyncratic volatility of stock returns correlates with national TFP growth. These results demonstrate that higher idiosyncratic volatility presents the economy with a better opportunity as long as market selection or creative destruction ensures that good productivity survives and bad productivity ceases.

The model also offers a different interpretation on the empirical findings of Chan et al. (2001), Comin and Philippon (2005), and Cao et al. (2008). Chan et al. (2001) and Comin and Philippon (2005) show that firm-level volatility correlates positively with R&D expenses. Cao et al. (2008) document the positive relation between aggregate idiosyncratic volatility and an aggregate level of growth options. Bekaert et al. (2012) confirm these findings. The interpretation by these authors is that R&D or a growth option causes a higher firm-level volatility. In contrast, my model proposes an alternative possibility that the causation is in the opposite direction: a higher volatility encourages option creation activity.

Recently, the depressing effect of uncertainty on the economy due to an increased option value of waiting has drawn a lot of attention. This idea dates back to Bernanke (1983). Bloom (2009) analyzes uncertainty shocks and quantifies their impact. He considers a simultaneous increase in aggregate and idiosyncratic volatility and find that it generates a rapid drop and rebound in investment and output. Bloom et al. (2012) and Bachmann and Bayer (2013) show these effects survive in general equilibrium. All of them assume a fixed number of plants and investment opportunities and do not consider the possibility of new entry.⁵ My model is consistent with these results

⁵Many different features in the models make difficult a direct comparison of this paper.
when investment opportunities are exogenous. My findings in the case of an endogenous investment opportunity, however, also calls for further investigation into the real option effects of uncertainty on aggregate economic activity; how big the depressing effect of uncertainty through a ‘wait-and-see’ mechanism is depends on how strongly the innovation or option-creating activity responds to heightened uncertainty.

There is a growing literature that documents varied measures of idiosyncratic volatility is countercyclical and various explanations have been proposed in the literature. Bloom (2009) features the real options effect and Arelano et al. (2012), Christiano et al. (2014), and Gilchrist et al. (2014) present models in which high idiosyncratic volatility aggravates financial frictions and generates economic downturns. In addition, Alessandria et al. (2015), Bachmann and Moscarini (2012), Berger and Vavra (2013), Decker et al. (2014), and Ilut et al. (2014) show that the countercyclical idiosyncratic volatility might be a symptom rather than a cause of aggregate fluctuations. The analysis of this paper is limited to the real option channel only and does not claim that the idiosyncratic volatility should be procyclical. The paper only suggests that it is less clear that the real option effect is the main channel to explain the countercyclical idiosyncratic volatility.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 outlines the computation of the equilibrium. Section 4 discusses the parameter choices. Section 5 presents and analyzes the results with Bloom (2009) and Bloom et al. (2012). This paper considers an entry and exit model without intensive margin of capital adjustment and the only adjustment frictions in the economy are due to irreversibility of entry and exit. In contrast, Bloom (2009) and Bloom et al. (2012) focus on the intensive margin and allow for the presence of various convex and non-convex adjustment costs in capital and labor.

6See, for example, Campbell et al. (2001) for stock returns, Storesletten et al. (2004) for labor earnings, Bloom et al. (2012) for TFP shock and sales growth, and Berger and Vavra (2010) for price changes.
and Section 6 concludes.

2 The Model

The economy consists of plants, investment option holders, potential entrants, and households. Plants hire labor and produce output. Potential entrants pay fixed costs to acquire the investment options. Investment option holders and plants have their specific, time-varying productivity, and optimally decide when to build plants and exit the market, respectively. Finally, households are identical and own all plants and investment options.

As explained later, idiosyncratic shocks are assumed to be more frequent than aggregate shocks. Figure 1 summarizes the timeline of the model. At the start of each period $t$, aggregate and idiosyncratic shocks are realized. Plants make employment and production decisions and households choose how much to work and consume. By paying fixed costs, potential entrants become option holders in the next period $t + 1$. At the middle of period $t'$, additional idiosyncratic shocks are realized. The existing option holders and plants decide whether to exercise entry and exit options or wait. By exercising options, option holders build plants and start production in the next period $t + 1$ and plants exit by selling off their capital. At the end of the period, destruction shocks hit the remaining plants and investment options, thus destroying $\delta$ share of plants and $\delta_h$ share of options.

2.1 Production

A continuum of production units exists. Capital is a fixed factor at the plant level and is normalized to one. A plant cannot adjust its capital over the life
Figure 1: Timeline. $\epsilon^\sigma$ are aggregate shocks; $\epsilon^\omega$ are idiosyncratic shocks.

cycle and all variation in aggregate capital comes from the extensive margin; that is, from the entry and exit of plants.\(^7\)

Each plant with a unit of capital behaves competitively and produces an aggregate good according to:

$$y_t = (e^{\omega_t} n_t)^\alpha.$$  

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\(^7\)The assumption of no intensive margin of capital adjustment is common in the firm dynamics literature (e.g., Hopenhayn, 1992) and makes it unnecessary to keep track of the distribution of the plant’s capital stock. Suppose instead that capital is not fixed and that the production function is given by $y_t = k_t^\gamma (e^{\omega t} n_t)^\alpha$, $\gamma + \alpha < 1$. Decreasing returns to scale is assumed to bound the plant’s size. If capital can also be adjusted freely, it is optimal for an option holder to always enter and for a plant to never exit—plants can always become competitive by reducing their size, no matter how bad their idiosyncratic productivity is. The option value and irreversibility are therefore irrelevant in this case. A lump-sum adjustment cost can prevent this outcome, but capital adjustment is then also irreversible and a plant includes a capital adjustment option, the value of which is increasing with volatility. Hence, the option holder’s decision becomes complicated as the second-stage entry (building a plant) involves acquiring a capital adjustment option as well as losing the option to delay entry. This situation is similar to Abel et al. (1996), who consider an investment project with future expandability so that the current investment leads to an acquisition of the expansion option; they show that uncertainty has an ambiguous effect on the incentive to invest. However, note that even if uncertainty increases the incentive of option holders to invest here, its impact on the total investment is still ambiguous because plants will delay adjusting their capital (exercise of the capital adjustment option). Investigating the robustness of my results to the presence of the intensive margin of capital adjustment would be of substantial interest, but is beyond the scope of this paper.
The plant’s output of the aggregate good is $y_t$ and its labor input is $n_t$. I assume that labor can be adjusted freely. The plant’s productivity consists of two components: $z_t$, which is aggregate productivity common across all plants, and $\omega_t$, which is idiosyncratic productivity specific to each plant.

Aggregate productivity grows deterministically:

$$z_{t+1} = \mu z + z_t,$$

and idiosyncratic productivity follows a random walk.\(^8\) I assume that idiosyncratic shocks are more frequent than aggregate shocks:

$$\omega_t = \omega_t + \sigma_t \varepsilon_t^\omega, \quad \varepsilon_t^\omega \sim \text{i.i.d. (across time and plants)} \, N(0,1/2) \quad (1)$$

$$\omega_{t+1} = \omega_t + \sigma_t \varepsilon_{t+1}^\omega, \quad \varepsilon_{t+1}^\omega \sim \text{i.i.d. (across time and plants)} \, N(0,1/2),$$

where $t'$ is the midpoint between $t$ and $t+1$. \(\sigma_t\) represents time-varying volatility, which evolves as an autoregressive process:

$$\log \sigma_{t+1} = (1 - \rho) \left( \log \varsigma_{\omega} - \frac{\varsigma_{\omega}^2}{2(1 - \rho^2)} \right) + \rho \log \sigma_t + \varsigma_{\sigma} \varepsilon_{t+1}^\sigma,$$

$$\varepsilon_{t+1}^\sigma \sim \text{i.i.d. (across time)} \, N(0,1).$$

\(\sigma_t\) is normalized so that its mean is equal to the mean value of idiosyncratic volatility, \(\varsigma_{\omega}\).

Note that no new aggregate information is revealed between $t$ and $t'$. I will assume later that entry and exit decisions are made at time $t', t'+1, t'+2, \ldots$

\(^8\)Oi (1961), Hartman (1972), and Abel (1983) show that a higher volatility can increase investment when the marginal revenue product of capital is convex in shocks. To remove this effect, I also experiment with the productivity process such that the expected marginal revenue product of capital is not increasing with volatility. I find that it does not change any qualitative predictions of my model.
so that each idiosyncratic productivity level at $t, t+1, t+2, \ldots$ has a *stochastic* entry or exit rate from $t$ to $t+1$.\textsuperscript{9} The merit of this assumption will become clear as the computation strategy is presented later (see Section 3.2).

In the above specifications, stochastic volatility only affects idiosyncratic productivity, not aggregate productivity. I focus on an aggregate change in idiosyncratic volatility for two reasons. First, most plant-level volatility is idiosyncratic (e.g., Davis and Haltiwanger, 1992) and the studies of irreversible investment rely on a large amount of idiosyncratic uncertainty in order to obtain a substantial role of investment irreversibility.\textsuperscript{10} Second, I utilize a perturbation method to solve the equilibrium. A third or higher order perturbation is required to capture the substantial effects of time-varying aggregate volatility (see Fernandez-Villaverde et al., 2010), but applying this perturbation to the current model with a large number of state variables requires large computational resources. In contrast, the effects of an aggregate variation of idiosyncratic volatility can be computed even by a first-order perturbation.

2.2 Investment: Entry and Exit

An unbounded mass of prospective entrants is present. Building a new plant means combining physical capital with new technology and occurs over two

\begin{itemize}
    \item \textsuperscript{9} Suppose I instead assume entry and exit decisions are made at time $t, t+1, t+2, \ldots$. Then the probability of entry or exit between $t$ and $t+1$ becomes a discontinuous indicator function of $\omega_t$. My timing assumption makes it a smooth function of $\omega_t$.
    \item \textsuperscript{10} The importance of idiosyncratic uncertainty in the model of irreversible investment is emphasized by Bertola and Caballero (1994): because the economy grows on average and capital depreciates, disinvestment is never optimal and irreversibility constraints become irrelevant unless a big enough negative shock hits the plant. The low volatility of aggregate variables makes such a large negative shock unlikely without idiosyncratic uncertainty. In the current context, this size difference between idiosyncratic and aggregate volatility is important for the following reason: because aggregate volatility is a small portion of the total plant-level volatility, fluctuations in aggregate volatility alone do not much affect the total plant-level uncertainty or the plant’s decision.
\end{itemize}
stages. At the first stage at time $t$, a prospective entrant pays a fixed cost and discovers a new technology or a new investment idea. In particular, by paying $\zeta$ units of an aggregate good at $t$, the entrant can draw an initial idiosyncratic productivity $\omega_{t+1}$ at the next period $t+1$ from a common pool of productivities:

$$\omega_{t+1} \sim N(0, \varsigma_e^2).$$

Note that $\varsigma_e$ is not affected by stochastic volatility $\sigma_t$. I assume a constant $\varsigma_e$ to focus on the effect of uncertainty about the future evolution of idiosyncratic productivity. Once obtained, the idiosyncratic productivity of new technology also evolves by (1). I call this idiosyncratic technology before adoption at the plant an *idea*.

Potential entrants enter the first stage until the expected profit net of the entry cost is zero.$^{11}$ Since the expected payoff of the first-stage entry is always zero, there is no gain to achieve by waiting. Hence, consideration of the option value of waiting is absent in the first-stage decision.

The possession of an idea gives its owner an investment option that characterizes the second stage. An idea owner decides when to exercise this option and build a plant at times $t', t' + 1, t' + 2, \ldots$. If she decides to build a plant, she has to purchase a unit of aggregate good (physical capital) to implement his idea (technology). Once a plant is built, exiting can recover only $1 - \eta$ unit of capital. This investment decision is therefore (partially) irreversible. If the productivity of the idea is not good enough, the idea owner can postpone the investment decision until later favorable shocks hit her productivity. The combination of irreversibility and postponability makes option value consider-

$^{11}$Note that the first stage entry is competitive (free entry), but competition is limited after that point. Idea owners and plants have exclusive rights to their technologies and plants earn strictly positive profits as their capacity (capital) is fixed.
ation central to the entry decision at the second stage. Because idiosyncratic productivity is persistent, the optimal decision rule is characterized by the productivity thresholds $\omega'_t, \omega'_{t+1}, \omega'_{t+2}, \ldots$ above which the idea owner builds a plant and begins operation.

Investments on the first and second stages are contrasted in terms of competitive interactions. On the first stage, all potential entrants can invest in innovation (i.e., free entry), which drives away any value of delay. In contrast, on the second stage, the idea owner has a monopolistic access to her idea and therefore can indefinitely delay investment without fear of preemption by competitors. Reality probably lies between the extremes of perfect competition and monopoly. I consider these two extremes to separate and compare the conflicting aspects of real option effects, option creation, and exercise.

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12The central role of the option value consideration in the model is in contrast to the existing literature on firm dynamics. The standard firm dynamics models following Hopenhayn (1992) typically assume a free-entry condition at the single-stage entry. A new entrant discovers her idiosyncratic productivity only after entry. Zero expected profit (net of the entry cost) makes the option value absent in the entry decision. Some models, such as Veracierto (2001)’s, dispense with a free-entry assumption and allow a prospective entrant to discover its idiosyncratic productivity before entry. However, these models also assume that the entry decision is a now-or-never type so that the productivity of a non-entering unit is abandoned. Lee and Mukoyama (2007) consider two-stage entry; however, their second-stage decision is also a now-or-never type. These features keep the option value consideration from playing a role in the entry decision. On the other hand, Jovanovic (2009) features the option value consideration in the entry decision. However, in his model, plants are homogenous and new investment options fall off the existing capital at an exogenous rate.

13Dixit and Pindyck (1994) states in their introduction, “Of course, firms do not always have the opportunity to delay their investments. For example, there can be occasions in which strategic considerations make it imperative for a firm to invest quickly and thereby preemp investment by existing or potential competitors. However, in most cases, delay is at least feasible. There may be a cost to delay—the risk of entry by other firms, or simply foregone cash flows—but this cost must be weighed against the benefits of waiting for new information.” Kulatilaka and Perotti (1998), Grenadier (2002), and Miller and Folta (2002) show how competition erodes the option value of waiting.

14Determining the right mix of monopoly and competition in each stage is beyond the scope of this paper and requires further investigation. I conjecture that explorative investment is more vulnerable to competition than later stage investment because exclusive rights to an investment project can be better protected when the project is well-defined. If I instead assume each entrant has a monopoly over the first stage investment, the first stage will
Once a plant is built, it also acquires a disinvestment option and decides when to sell off its capital and leave the economy at times $t', t' + 1, t' + 2, \ldots$.

If a plant decides to exit, it can recover a $1 - \eta$ unit of aggregate good. It can alternatively keep operating and wait, hoping its productivity improves in the future. Hence, the opportunity cost of losing the option to operate in the future is a consideration in the exit decision. Similarly, idiosyncratic productivity thresholds $\omega_{t'}, \omega_{t'+1}, \omega_{t'+2}, \ldots$ exist and determine whether the plant exits.

Note that new ideas are only occasionally ever adopted for building plants and that unused ideas keep accumulating. To keep a well-behaved distribution of the idea’s productivity, I assume each idea becomes obsolete with probability $\delta_h$. I also assume that plants die at probability $\delta$, which can be justified by capital depreciation.

2.3 Preferences

The economy is populated by a unit measure of identical households with the following utility function in consumption $c_t$ and labor $n_t$:

$$ v_t = \max_{c_t, n_t} \left( (1 - \beta) \left[ \log c_t - \kappa n_t \right] + \beta E_t[v_{t+1}] \right). $$

be characterized by the exercise decision of the compound option and the results will depend on how the supply of those compound options is determined; if it is exogenous, the results will be similar to the case of a fixed idea investment in Section 5.2. Also, note that even if the investor has exclusive rights to a project, investment lags can reverse the negative effect of uncertainty (see Bar-Ilan and Strange, 1996). This result can be interpreted in terms of option creation—the project can be abandoned at any point in time until completion of the investment; hence, the project investment creates an abandonment option, which is more valuable when uncertainty is high.

\footnote{By assumption, a plant cannot adjust its capital stock by selling only a part of it which would be better than exiting the market if the marginal product of capital goes to infinity as capital goes to zero and there is no lump-sum cost of capital adjustment. See footnote 7.}

\footnote{The log utility and infinite Frisch elasticity of the labor supply are also assumed in Bloom et al. (2012) and Bachmann and Bayer (2013).}
Households supply labor and finance the investments in plants and ideas so that the households’ wealth is held as shares in plants and ideas.

2.4 Equilibrium

Let $K_t(\cdot)$ and $H_t(\cdot)$ denote measures over the plants’ and ideas’ productivity, respectively. The aggregate state of the economy is described by $\Lambda_t = (K_t(\cdot), H_t(\cdot), \sigma_t, z_t)$. Denote the perceived law of motion for $K_t(\cdot)$ and $H_t(\cdot)$ by $\Gamma^k$ and $\Gamma^h$:

$$K_{t+1}(\cdot) = \Gamma^k(\Lambda_t), \quad H_{t+1}(\cdot) = \Gamma^h(\Lambda_t).$$

Let $M_{t,t+1}$ represent the stochastic discount factor between $t$ and $t+1^{17}$ and $W(\Lambda_t)$ represents the equilibrium wage rate. Also, let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the pdf and cdf of the standard normal distribution, respectively.

A recursive competitive equilibrium is defined by the following conditions:

- Plants decide how much labor to hire and whether to exit the market.

The plants’ problem is:

$$v^k(\omega_t, \Lambda_t) = \max_n \left[ (e^{zt+\omega_t n})^\alpha - W(\Lambda_t)n + \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \omega'_t - \omega_t \right) \left( 1 - \eta_t \right) \left( 1 - \delta_t \right) \Phi \left( \omega'_t - \omega_t \sigma_t/\sqrt{2} \right) \right] \times \max \left\{ 1 - \eta_t, (1 - \delta_t) E \left[ M_{t,t+1} v^k(\omega_{t+1}, \Lambda_{t+1}) | \omega_t, \Lambda_t \right] \right\} d\omega'_t$$

$$= \max_n \left[ (e^{zt+\omega_t n})^\alpha - W(\Lambda_t)n + (1 - \eta_t) \Phi \left( \omega'_t - \omega_t \sigma_t/\sqrt{2} \right) \right] \times \max \left\{ 1 - \eta_t, (1 - \delta_t) E \left[ M_{t,t+1} v^k(\omega_{t+1}, \Lambda_{t+1}) | \omega_t, \Lambda_t \right] \right\} d\omega'_t.$$ 

\footnote{$M_{t,t+1}(\Lambda_t, \Lambda_{t+1})$ depends on the aggregate state of the economy in the current and next periods. Its arguments are dropped for notational simplicity.}
Their policy functions are employment \( n^k(\omega_t, \Lambda_t) \) and exit threshold \( \omega(\Lambda_t) \).\(^{18}\) Also note that employment decision \( n \) is atemporal and can be solved separately. Let \( \pi(\omega_t, \Lambda_t) \) denote the period profit. Then we obtain

\[
\pi(\omega_t, \Lambda_t) = \max_n \left( e^{zt+\omega_t n} \right)^{\alpha} - W(\Lambda_t) n = \frac{(1 - \alpha)\alpha^{\frac{1}{1-\alpha}} e^{\frac{\alpha}{1-\alpha} (zt+\omega_t)}}{W(\Lambda_t)^{\frac{1}{1-\alpha}}}. \tag{2}
\]

- **Idea owners decide whether to build plants.** The idea owners’ problem is:

\[
v^h(\omega_t, \Lambda_t) = \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_t' - \omega_t}{\sigma_t/\sqrt{2}} \right) \max \left\{ -1, + (1 - \delta) E \left[ M_{t,t+1} v^k(\omega_{t+1}, \Lambda_{t+1}) | \omega_{t'}, \Lambda_t \right], \right. \\
&\left. + (1 - \delta_h) E \left[ M_{t,t+1} v^h(\omega_{t+1}, \Lambda_{t+1}) | \omega_{t'}, \Lambda_t \right] \right\} d\omega_{t'}
\]

\[
= \max \pi - \left[ 1 - \Phi \left( \frac{\omega - \omega_t}{\sigma_t/\sqrt{2}} \right) \right] + (1 - \delta) \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_t' - \omega_t}{\sigma_t/\sqrt{2}} \right) \\
&\times E \left[ M_{t,t+1} v^k(\omega_{t+1}, \Lambda_{t+1}) | \omega_{t'}, \Lambda_t \right] d\omega_{t'}
\]

\[
+ (1 - \delta_h) \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_t' - \omega_t}{\sigma_t/\sqrt{2}} \right) \\
&\times E \left[ M_{t,t+1} v^h(\omega_{t+1}, \Lambda_{t+1}) | \omega_{t'}, \Lambda_t \right] d\omega_{t'}.
\]

Their policy function is entry threshold \( \overline{\omega}(\Lambda_t) \).

- **An unbounded mass of prospective entrants is present and they invest in ideas until the expected profits net of the investment cost are zero.**

\(^{18}\)Note that \( \max \left\{ 1 - \eta, (1 - \delta) E \left[ M_{t,t+1} v^k(\omega_{t+1}, \Lambda_{t+1}) | \omega_{t'}, \Lambda_t \right] \right\} \) depends only on \( \omega_{t'} \) and \( \Lambda_t \), not on \( \omega_t \). Hence, the exit threshold \( \omega \) is a function of \( \Lambda_t \) only, independently of \( \omega_t \).
Therefore,

\[ E \left[ M_{t,t+1} v^h(\omega_{t+1}; \Lambda_{t+1}) | \Lambda_t \right] - \zeta = 0. \]

This determines \( I^h(\Lambda_t) \), which is the mass of potential entrants who pay \( \zeta \).

- Households’ state variables include their share holdings of plants and ideas, \( \tilde{K}_t(\cdot) \) and \( \tilde{H}_t(\cdot) \), in addition to aggregate state variables \( \Lambda_t \). They solve:

\[
v^c(\tilde{K}_t(\cdot), \tilde{H}_t(\cdot), \Lambda_t) = \max_{c,n^c,K^c(\cdot),H^c(\cdot)} \left( 1 - \beta \right) \left[ \log c - \kappa n^c \right] + \beta E\left[ v^c(K^c(\cdot), H^c(\cdot), \Lambda_{t+1}) \right],
\]

subject to

\[
c + \int_{-\infty}^{\infty} q^k(\omega_{t+1}; \Lambda_t) K^c(\omega_{t+1}) d\omega_{t+1} + \int_{-\infty}^{\infty} q^h(\omega_{t+1}; \Lambda_t) H^c(\omega_{t+1}) d\omega_{t+1} = W(\Lambda_t) n^c + \int_{-\infty}^{\infty} \tilde{q}^k(\omega_t; \Lambda_t) \tilde{K}_t(\omega_t) d\omega_t + \int_{-\infty}^{\infty} \tilde{q}^h(\omega_t; \Lambda_t) \tilde{H}_t(\omega_t) d\omega_t,
\]

where \( \tilde{q}^k(\omega_t; \Lambda_t) \) and \( \tilde{q}^h(\omega_t; \Lambda_t) \) are the share prices of plants and ideas with the current productivity \( \omega_t \) and \( q^k(\omega_{t+1}; \Lambda_t) \) and \( q^h(\omega_{t+1}; \Lambda_t) \) are the current period share prices of plants and ideas that begin the next period with productivity \( \omega_{t+1} \).

Their policy functions are consumption \( c(\tilde{K}_t(\cdot), \tilde{H}_t(\cdot), \Lambda_t) \), labor supply \( n^c(\tilde{K}_t(\cdot), \tilde{H}_t(\cdot), \Lambda_t) \), and shares of plants and ideas \( K^c(\omega_{t+1}; \tilde{K}_t(\cdot), \tilde{H}_t(\cdot), \Lambda_t) \), \( H^c(\omega_{t+1}; \tilde{K}_t(\cdot), \tilde{H}_t(\cdot), \Lambda_t) \).

\[ ^{19}\text{In equilibrium, } \tilde{q}^k(\omega_t; \Lambda_t) = v^k(\omega_t; \Lambda_t), \tilde{q}^h(\omega_t; \Lambda_t) = v^h(\omega_t; \Lambda_t), q^k(\omega_{t+1}; \Lambda_t) = E\left[ M_{t,t+1} v^k(\omega_{t+1}; \Lambda_{t+1}) | \Lambda_t \right], q^h(\omega_{t+1}; \Lambda_t) = E\left[ M_{t,t+1} v^h(\omega_{t+1}; \Lambda_{t+1}) | \Lambda_t \right]. \]
• The asset market clears:

\[ K^c(\omega_{t+1}; K_t(\cdot), H_t(\cdot), \Lambda_t) = K_{t+1}(\omega_{t+1}) \text{ for all } \omega_{t+1} \]

\[ H^c(\omega_{t+1}; K_t(\cdot), H_t(\cdot), \Lambda_t) = H_{t+1}(\omega_{t+1}) \text{ for all } \omega_{t+1}. \]

• The labor market clears:

\[ n^c(K_t(\cdot), H_t(\cdot), \Lambda_t) = \int_{-\infty}^{\infty} n^k(\omega_t, \Lambda_t) K_t(\omega_t) d\omega_t. \]

• The aggregate good market clears:

\[ c(K_t(\cdot), H_t(\cdot), \Lambda_t) = \int_{-\infty}^{\infty} (e^{z_t+\omega_t} n^k(\omega_t, \Lambda_t))^\alpha K_t(\omega_t) d\omega_t \]

\[ + (1 - \eta) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - \omega_t}{\sigma_t/\sqrt{2}} \right) K_t(\omega_t) d\omega_t \]

\[ - \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_t - \omega_t}{\sigma_t/\sqrt{2}} \right) \right] H_t(\omega_t) d\omega_t - \zeta I^h(\Lambda_t). \]

• Rational expectations: the economy indeed evolves as \( K_{t+1}(\cdot) = \Gamma^k(\Lambda_t) \), \( H_{t+1}(\cdot) = \Gamma^h(\Lambda_t) \); that is, for all \( \omega_{t+1} \),
\[\Gamma^k(\Lambda_t)(\omega_{t+1}) = (1 - \delta) \int_{\omega(\Lambda_t)}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t/\sqrt{2}} \right) K_t(\omega_t) d\omega_t \] 
\[\times \left[ \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_t - \omega_t'}{\sigma_t/\sqrt{2}} \right) K_t(\omega_t') d\omega_t' \right] d\omega_t' \]
\[+ (1 - \delta) \int_{\pi(\Lambda_t)}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_{t+1} - \omega_t'}{\sigma_t/\sqrt{2}} \right) H_t(\omega_t) d\omega_t \]
\[\times \left[ \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_t' - \omega_t}{\sigma_t/\sqrt{2}} \right) H_t(\omega_t') d\omega_t' \right] d\omega_t'.\]

\[\Gamma^h(\Lambda_t)(\omega_{t+1}) = (1 - \delta_h) \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_{t+1} - \omega_t'}{\sigma_t/\sqrt{2}} \right) \]
\[\times \left[ \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_t' - \omega_t}{\sigma_t/\sqrt{2}} \right) H_t(\omega_t') d\omega_t' \right] d\omega_t'
\[+ \frac{1}{\zeta_e} \phi \left( \frac{\omega_{t+1}}{\zeta_e} \right) I^h(\Lambda_t).\]

3 Solving Equilibrium

3.1 The Social Planner’s Problem

I characterize the equilibrium allocations by solving the social planner's problem. The social planner's problem is:

\[V(K_t(\cdot), H_t(\cdot), \sigma_t, z_t) = \max_{C_t, H_t^+, N_t, n_t, \omega_t'} (1 - \beta) \left[ \log C_t - \kappa N_t \right] \]
\[+ \beta E_t \left[ V(K_{t+1}(\cdot), H_{t+1}(\cdot), \sigma_{t+1}, z_{t+1}) \right],\]
subject to

\[ Y_t = C_t + \zeta I^h_t + \int_{-\infty}^{\infty} H_{t'}(\omega_{t'}) d\omega_{t'} - (1 - \eta) \int_{-\infty}^{\infty} K_{t'}(\omega_{t'}) d\omega_{t'} \quad (4) \]

\[ Y_t = \int_{-\infty}^{\infty} (e^{z_t + \omega_t} n_t(\omega_t))^\alpha K_t(\omega_t) d\omega_t \quad (5) \]

\[ N_t = \int_{-\infty}^{\infty} n_t(\omega_t) K_t(\omega_t) d\omega_t \quad (6) \]

\[ K_{t'}(\omega_{t'}) = \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_{t'} - \omega_t}{\sigma_t/\sqrt{2}} \right) K_t(\omega_t) d\omega_t \quad (7) \]

\[ K_{t+1}(\omega_{t+1}) = \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t/\sqrt{2}} \right) K_t(\omega_t) d\omega_t \quad (8) \]

\[ H_{t'}(\omega_{t'}) = \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_{t'} - \omega_t}{\sigma_t/\sqrt{2}} \right) H_t(\omega_t) d\omega_t \quad (9) \]

\[ H_{t+1}(\omega_{t+1}) = \int_{-\infty}^{\infty} \frac{1}{\sigma_t/\sqrt{2}} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t/\sqrt{2}} \right) H_t(\omega_t) d\omega_t \quad (10) \]

The social planner optimally chooses aggregate consumption \( C_t \), the creation of new ideas \( I^h_t \), aggregate labor \( N_t \), the labor allocation at each plant \( n_t(\cdot) \), and entry and exit thresholds \( (\omega_t' \text{ and } \omega_{t'}') \). Equation (4) is the aggregate resource constraint; output equals the sum of consumption, investment cost in new ideas, and the capital purchase by new plants minus the capital released by exiting plants. Equations (5) and (6) compute the aggregate output and labor as the weighted sum of plant-level output and labor.

Equations (7) and (9) describe the transition of measures over plant and idea productivity from \( t \) to \( t' \)—since entry and exit is not allowed between \( t \) and \( t' \), the transition is dictated purely by exogenous normal shocks. On the
other hand, the transition from $t'$ to $t+1$ [equations (8) and (10)] are more complicated. Plants with productivity below a certain level exit and highly productive ideas become new plants. Hence, the measure over plant productivity at $t+1$ in equation (8) is constructed from lower-truncated measures over plant and idea productivity at $t'$. Similarly, the measure over idea productivity at $t+1$ in equation (10) derives from an upper-truncated measure over idea productivity at $t'$ and the measure over initial productivity draws by new idea owners.

This original problem is transformed in the following ways. First, by the law of large numbers, $K_{t'}(\cdot)$ and $H_{t'}(\cdot)$ are known at $t$; only identities of plants and idea owners of each productivity $\omega_{t'}$ are revealed at $t'$, but their measures for each $\omega_{t'}$ are deterministic between $t$ and $t'$. Hence, the social planner can choose the entry and exit thresholds, $\overline{\omega}_{t'}$ and $\underline{\omega}_{t'}$, at $t$. By replacing $\overline{\omega}_{t'}$ and $\underline{\omega}_{t'}$ with $\overline{\omega}_{t}$ and $\underline{\omega}_{t}$, then plugging equations (7) and (9) into equations (8) and (10), and integrating $\omega_{t'}$ out, the social planner’s problem can be described only by variables at $t$.

Second, note that the labor allocation decision is atemporal so it can be solved separately:

$$\max_{n_t} Y_t = \int_{-\infty}^{\infty} (e^{z_t+\omega_t} n_t(\omega_t))^\alpha K_t(\omega_t)d\omega_t \quad \text{subject to} \quad N_t = \int_{-\infty}^{\infty} n_t(\omega_t)K_t(\omega_t)d\omega_t.$$

The solution is

$$n_t(\omega_t) = \frac{e^{z_t+\omega_t}}{\hat{K}_t} N_t, \quad Y_t = \hat{K}_t^{1-\alpha}(e^{z_t}N_t)^\alpha, \quad \text{where} \quad \hat{K}_t = \int_{-\infty}^{\infty} e^{\frac{1}{1-\alpha} \omega_t} K_t(\omega_t)d\omega_t.$$

$\hat{K}_t$ is the productivity-weighted capital stock.

Third, the economy grows because of labor-augmenting technological progress.
To solve the social planner’s problem, it must be transformed into a stationary one. Dividing all variables except labor and entry/exit thresholds by the level of labor-augmenting productivity accomplishes this transformation. By abuse of notation, I hereafter use the same symbols to denote the deflated variables.

Variations in aggregate productivity are important for understanding this economy. Note that the aggregate total factor productivity $A_t$ in this economy is given by:

$$A_t = \frac{Y_t}{K_t^{1-\alpha}} N_t^{\alpha} = e^{\alpha z_t} \frac{K_t^{1-\alpha}}{K_t^{1-\alpha}}$$

where $\overline{K}_t = \int_{-\infty}^{\infty} K_t(\omega_t) d\omega_t$ denote the aggregate stock of capital.

### 3.2 Computation Strategy

The equilibrium conditions of the model are functional equations that require solving for the distribution of plant and idea productivity. To deal with this infinite dimensional problem, I adopt an approach developed by Campbell (1998) of approximating the distribution functions by their values at a large but finite set of grid points and then applying a perturbation method that can handle many state variables relatively easily. Since those distribution functions appear as integrands in equilibrium conditions, this approximation amounts to applying numerical integration.

This computation approach is different from Bloom et al. (2012) and Bachmann and Bayer (2013), who tackle the high dimensionality of heterogeneous

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20 See Appendix A for the step-by-step transformation of the social planner’s problem into a stationary one. Appendix B describes the first-order conditions of the social planner’s problem.

21 This method has not been widely used. Michelacci and Lopez-Salido (2007), Reiter (2009), and McKay and Reis (2013) are a few exceptions.
plant models by adopting the method by Krusell and Smith (1998) of approximating the distribution by a finite set of its moments. I adopt the Campbell (1998) approach because it can more easily capture a rich structure of the distribution.\footnote{The models with an irreversible investment imply an inaction region for individual units. The position of each unit relative to the thresholds of the inaction regions determines whether this unit reacts to shocks or not. Hence, keeping track of the distribution of relative positions is central in determining the aggregate responses of the economy.}

More specifically, I fix a large set of grid points $\omega^1 < \omega^2 < \cdots < \omega^L$ along with the corresponding weights $w^1, w^2, \ldots, w^L$ and transform equations of type

$$f(\omega') = \int_{-\infty}^{\infty} g(\omega) d\omega, \quad \forall \omega'$$

into

$$f(\omega_i) = \sum_{j=1}^{L} w^j g(\omega_j), \quad \forall i.$$  

Hence, functions $f(\omega)$ and $g(\omega)$ are approximated by a finite set of variables, $f(\omega^1), \ldots, f(\omega^L)$ and $g(\omega^1), \ldots, g(\omega^L)$. More efficient numerical integration methods usually exploit the property of a particular integrand and adjust its points of evaluation and weights accordingly. However, the integrand is unknown here before approximating and solving for it. I therefore use the predetermined grid points and weights, which do not depend on the integrand. The approximation error is expected to diminish as $L$ increases with finer grids and a wider span.

Note that my timing assumption removes entry and exit thresholds from the interval of integration. Without this assumption, I would have to approximate

$$f(\omega) = \int_{-\infty}^{\infty} g(\omega) d\omega$$

by

$$f(\omega^i) = \sum_{j=1}^{k} w^j g(\omega^j),$$

where $k$ is the grid interval to which $\omega$ belongs. In this case, if $\omega$ increases enough to cross the grid point, the equation is no longer differentiable because an abrupt change occurs from

$$f(\omega^i) = \sum_{j=1}^{k} w^j g(\omega^j)$$

to

$$f(\omega^i) = \sum_{j=1}^{k+1} w^j g(\omega^j).$$

Therefore, to apply a perturbation method, the driving shocks would have to be restricted to a small enough size that the thresholds never leave their original grid intervals. The small shock restriction would mean that as the grid becomes finer in order to achieve a better approximation, the size of the shocks would have
to be decreased accordingly. Moreover, because these thresholds are sensitive to the level of volatility, only very small volatility shocks could be considered. The assumption of more frequent idiosyncratic shocks randomizes the entry and exit rate and ensures all equilibrium conditions are differentiable, even without a small shock restriction.\textsuperscript{23}

Though finite, the dimension of the approximated problem is large. To avoid the curse of dimensionality, I apply a perturbation method: finding a Taylor approximation of the decision rules around the steady state of the model where all aggregate shocks are removed. More specifically, a perturbation parameter $\lambda$ is introduced to aggregate shocks, such as $\lambda \sigma_{t+1}$. This parameter controls the sensitivity of the dynamic system to aggregate shocks: $\lambda = 1$ represents the original system, and $\lambda = 0$ corresponds to eliminating all aggregate shocks. Then the steady state of the economy without aggregate shocks is computed by the use of standard techniques, and a certain order of Taylor approximation of the decision rules is taken around $\lambda = 0$. Finally, the approximated decision rules are substituted into the equilibrium conditions and the unknown coefficients of the decision rules are solved so that they satisfy the equilibrium conditions.\textsuperscript{24} I obtain the solution by using Dynare++

\textsuperscript{23}Note that the entry and exit thresholds $\overline{\omega}, \underline{\omega}$ no longer appear as limits of integration in the transformed social planner’s problem (11) in Appendix A. One might think simply of approximating the step function with a function $h$ that is smooth but has a steep slope around $\overline{\omega}$: \[ \int_{-\infty}^{\overline{\omega}} g(\omega) d\omega \approx \int_{-\infty}^{\infty} h(\omega; \overline{\omega}) g(\omega) d\omega \approx \sum_{j=1}^{L} w_j b(\omega^j; \overline{\omega}) g(\omega^j). \] The problem with this approach is that as $h$ becomes steeper, the first approximation improves, while the second one deteriorates; the grid should be very fine in order to make a good approximation of a steep function. My timing assumption provides a rationale for not using a very steep function to approximate the step function. This strategy—of introducing randomization to smooth out a lumpy decision rule—is similar to those of Dotsey et al. (1999) and Thomas (2002). Since I have two randomization devices (continuous idiosyncratic productivity and timing) whereas they have only one (random adjustment cost), I can dispense with the need of a small shock assumption.

\textsuperscript{24}When applying a second or higher order perturbation, I take Lombardo (2010)’s perspective—the solution is represented as the sum of terms that have different orders of approximation with respect to $\lambda$ and terms of different orders are computed recursively. This
4 Parameter Choices

The model period is a quarter. The mean growth rate of the aggregate labor-augmenting technology $\mu_z$ is set to 0.37 percent in order to imply a 1.5 percent annual growth rate of output per capita. The time discount factor $\beta$ is set to match an annual interest rate of 4 percent. The disutility of labor $\kappa = 2.76$ is chosen so that the steady-state level of labor is $\frac{1}{3}$. I set $\alpha = 2/3$ for labor’s share of income. The capital depreciation rate $\delta = 0.025$ is typical in the literature.\footnote{This value is usually chosen to match an average investment-to-capital ratio. However, in my model, capital is lost by resale losses incurred by an exit as well as by capital depreciation (or destruction). Hence, my model implies a slightly higher investment-to-capital ratio than the standard model without a capital reallocation for the same value of $\delta$.}

I set the remaining parameters based on the considerations stated below; however, the following parameter values are for benchmark computations only. I experiment with a wide range of values for these parameters and the main results remain robust.

I use the cross-sectional interquartile range of establishment TFP shocks from Nicholas Bloom’s website\footnote{Bloom et al. (2012) use the same time series to calibrate their uncertainty process modeled as a two-state Markov chain.} to estimate an AR(1) process for the time-varying idiosyncratic volatility. This leads to $\rho = 0.94$ and $\varsigma_\sigma = 0.043$. The quarterly persistence of the volatility process $\rho = 0.94$ is in line with Bloom et al. (2012), who calibrate a two-state Markov chain with a quarterly 92 percent probability of remaining in the high-uncertainty state. I use the volatility of volatility parameter $\varsigma_\sigma = 0.043$ to simulate the economy in normal times. I approach guarantees that the approximated system is stable. The resulting computation is similar to the pruning method by Kim et al. (2008).
then set $\sigma = 0.5$ when the uncertainty shock occurs in order to consider a sudden and large volatility shock comparable to the experiments by Bloom (2009) and Bloom et al. (2012). The remaining shocks in my model are idiosyncratic and are governed by $\omega$. I take the value of the average idiosyncratic volatility $\omega$ from Bloom et al. (2012) and set it to 0.039.

$\eta$ is the fraction of the physical capital that is lost during reallocation. Ramey and Shapiro (2001) find that this loss can be more than one half based on the study of used aerospace equipment. However, a large resale loss makes the exit and capital reallocation unlikely to occur in the model. Eisfeldt and Rampini (2006) report that, depending on the measure of the capital stock, between 1.4 and 5.5 percent of the capital stock turns over each year. In my model, capital recovered from exiting plants is interpreted as being reallocated to the entering plants. I set $\eta = 0.14$ so that the annual turnover rate at the steady state is 1.6 percent of the capital stock.

The remaining parameters are the idea depreciation rate $\delta_h$, idea creation cost $\zeta$, and dispersion parameter for distribution of the initial productivity draw $\sigma$. Ideas in the model represent new investment opportunities or technology that are not yet implemented. Hence, finding counterparts in the data to the parameters related to ideas is difficult. I set $\delta_h$ to 0.05 (the quarterly rate) as most empirical estimates of the depreciation rate of R&D capital ranges from 15 to 20 percent per year.

I choose idea creation cost $\zeta$ to imply that more than half of newly-created ideas are immediately adopted in plants; if the entry threshold is too high

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27Bloom (2009) considers a two-fold increase in volatility over a month, whereas Bloom et al. (2012) considers a three-fold increase over a quarter.

28See Mead (2007) and the references therein. Corrado et al. (2009) argue that other types of intangible capital such as computerized information, advertising, strategic planning, and reorganization depreciate faster than R&D capital.
and only a few ideas can pass through it, an increase in volatility is directly beneficial. This parametrization thus works against my results. \( \zeta \) directly affects the entry and exit thresholds because a high idea-creation cost implies that more ideas develop into plants or plants produce for a longer time in order to recoup the cost. The variation in \( \zeta \) therefore changes the relative size of entry and exit responses, but the qualitative results remain intact. The resulting spending on idea creation is 6.6% of output at the steady state. This is much larger than R&D spending in the U.S. national income and product account (NIPA) data (less than 2 percent of GDP); however, R&D is only a part of innovative activity.\(^{29}\) Corrado et al. (2005) estimate total business investment in intangible capital ranges from 9 to 16 percent of GDP in the 1988–2002 period.\(^{30}\) The ratio of idea investment to physical capital investment in the model is 32.5 percent, which is in line with the NIPA measure if investment in intellectual property products is considered.\(^{31}\)

Finally, I assume a dispersion parameter for initial productivity distribution \( \varsigma_e \) is equal to 0.039, the value of the average idiosyncratic volatility. The effect on the impulse responses of different values of \( \varsigma_e \) is small.

\(^{29}\)“Innovation results from a range of complementary assets that go beyond R&D, such as software, human capital, and new organizational structures.” (OECD, 2010)


\(^{31}\)The ratio of private fixed investment in intellectual property products to investment in nonresidential structures and equipment has been rising: 21 percent in the 1980s, 33 percent in 1990s, and 42 percent in the 2000s. The ratio considering R&D only is 12 percent, 16 percent, and 17 percent for the same decades.
5 Results

5.1 Steady State

I first consider the economy in which all aggregate shocks are removed. Idiosyncratic shocks still hit the economy; however, by the law of large numbers, all aggregate variables, including the distribution of plants and ideas, follow a deterministic path. I compute the steady state of this economy without aggregate shocks by approximating the distribution by its values at various grid points.

Table 1 shows the steady-state calculation. Note that the entry and exit thresholds reflect the option value. At the exit threshold \( \omega \), the present value of future profit is only 0.70, whereas the scrap value is \( 1 - \eta = 0.86 \). The difference represents the value of the exit option. Similarly, at the entry threshold \( \overline{\omega} \), an idea produces a zero present value of future profit, whereas building a plant earns a net present value of 0.18. The difference reflects the option value of waiting as an idea.

The inaction region where either the entry or exit is delayed is widened as the idiosyncratic volatility \( \varsigma_\omega \) is increased: the entry threshold is increased and the exit threshold is decreased. This widened region, however, does not necessarily mean that the size of the entry and exit declines; investment by

\[ \sum_{t=1}^{\infty} \left( 1 - \frac{\delta}{R_f} \right)^t E \left[ \pi_{ss}(\omega_t) | \omega_0 = \omega \right] = (1 - \alpha) \left( \frac{\alpha}{W_{ss}} \right)^{1-\alpha} \sum_{t=1}^{\infty} \left( 1 - \frac{\delta}{R_f} \right)^{t} e^{t \frac{\varsigma \omega}{\omega_0}} = 0.70, \text{ and } -1 + \sum_{t=1}^{\infty} \left( 1 - \frac{\delta}{R_f} \right)^t E \left[ \pi_{ss}(\omega_t) | \omega_0 = \overline{\omega} \right] = 0.18, \text{ respectively. Recall the plant’s profit function is given by (2).} \]

\[ 33 \sum_{t=1}^{\infty} \left( 1 - \frac{\delta}{R_f} \right)^t E \left[ \pi_{ss}(\omega_t) | \omega_0 = \omega \right] = (1 - \alpha) \left( \frac{\alpha}{W_{ss}} \right)^{1-\alpha} \sum_{t=1}^{\infty} \left( 1 - \frac{\delta}{R_f} \right)^{t} e^{t \frac{\varsigma \omega}{\omega_0}} = 0.70, \text{ and } -1 + \sum_{t=1}^{\infty} \left( 1 - \frac{\delta}{R_f} \right)^t E \left[ \pi_{ss}(\omega_t) | \omega_0 = \overline{\omega} \right] = 0.18, \text{ respectively. Recall the plant’s profit function is given by (2).} \]
Table 1: Steady State. Risk-free Rate $R_{fs}^f = 1/(\beta e^{-\mu_s}) = 1.01$.

<table>
<thead>
<tr>
<th>Idiosyncratic Volatility $\varsigma_\omega$</th>
<th>0.026</th>
<th>0.039</th>
<th>0.059</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{ss}$ output</td>
<td>0.934</td>
<td>0.970</td>
<td>1.055</td>
</tr>
<tr>
<td>$C_{ss}$ consumption</td>
<td>0.685</td>
<td>0.710</td>
<td>0.778</td>
</tr>
<tr>
<td>$N_{ss}$ labor</td>
<td>0.335</td>
<td>0.333</td>
<td>0.331</td>
</tr>
<tr>
<td>$V_{ss} = \log C_{ss} - \kappa N_{ss}$ welfare</td>
<td>-1.294</td>
<td>-1.254</td>
<td>-1.155</td>
</tr>
<tr>
<td>$I_{ss}^k = I_{ss}^e - I_{ss}^x$ net investment</td>
<td>0.200</td>
<td>0.196</td>
<td>0.185</td>
</tr>
<tr>
<td>$I_{ss}^e = \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_t - \omega_0}{\sigma_t} \right) \right] \times H_t(\omega_t) d\omega_t$ investment by entry</td>
<td>0.216</td>
<td>0.223</td>
<td>0.218</td>
</tr>
<tr>
<td>$I_{ss}^x = (1 - \eta) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - \omega_0}{\sigma_t} \right) \times K_t(\omega_t) d\omega_t$ disinvestment by exit</td>
<td>0.015</td>
<td>0.026</td>
<td>0.032</td>
</tr>
<tr>
<td>$I_{ss}^h$ investment in ideas</td>
<td>0.228</td>
<td>0.267</td>
<td>0.384</td>
</tr>
<tr>
<td>$\zeta I_{ss}^h$ ideas investment cost</td>
<td>0.055</td>
<td>0.064</td>
<td>0.092</td>
</tr>
<tr>
<td>$\omega_{ss}$ exit threshold</td>
<td>-0.048</td>
<td>-0.010</td>
<td>0.073</td>
</tr>
<tr>
<td>$\omega_{ss}$ entry threshold</td>
<td>-0.251</td>
<td>-0.273</td>
<td>-0.286</td>
</tr>
<tr>
<td>$\hat{K}<em>{ss} = \int</em>{-\infty}^{\infty} e^{\alpha \omega} K_{ss}(\omega) d\omega$ productivity-weighted capital</td>
<td>7.409</td>
<td>8.206</td>
<td>10.728</td>
</tr>
<tr>
<td>$K_{ss}$ aggregate capital stock</td>
<td>6.712</td>
<td>6.516</td>
<td>6.105</td>
</tr>
<tr>
<td>$H_{ss}$ aggregate idea stock</td>
<td>0.433</td>
<td>1.038</td>
<td>3.303</td>
</tr>
<tr>
<td>$A_{ss} = \left( \frac{\hat{K}<em>{ss}}{N</em>{ss}} \right)^{1-\alpha}$ aggregate TFP</td>
<td>1.034</td>
<td>1.080</td>
<td>1.207</td>
</tr>
<tr>
<td>$W_{ss} = \alpha \left( \frac{\hat{K}<em>{ss}}{N</em>{ss}} \right)^{1-\alpha}$ wage</td>
<td>1.872</td>
<td>1.939</td>
<td>2.126</td>
</tr>
</tbody>
</table>

entry and disinvestment by exit do not decrease with volatility. All else being equal, a higher volatility makes the idiosyncratic productivity hit the thresholds more often and the size of the entry and exit also depends on the stock of existing plants and ideas. Hence, although the entry threshold is monotone in volatility, the investment by entry is not.

Figure 2 displays the steady-state measures over plant and idea productivity.\footnote{A little bump occurs in the idea measure with $\varsigma_\omega = 0.026$, because the initial infusion of new ideas with volatility $\varsigma_c = 0.039$ is less smoothed out by the subsequent volatility} Note that the stock of ideas is very large in the high volatility case,
Figure 2: Steady-state measures over plant (left) and idea (right) productivity. $\varsigma_\omega$ denotes the mean value of idiosyncratic volatility.

which we can expect from a large value of idea investment $I_{ss}^h$. Higher volatility makes the tails of the productivity distribution fatter and the social planner exploits the increased chance of high productivity by investing heavily in ideas and selectively building plants. This substitution of ideas for capital leads to a large gain in aggregate productivity, which makes the economy enjoy more output with less labor and capital.

5.2 Impact of Volatility Shock

I study the properties of the model by computing impulse responses. I apply a second-order perturbation to the equilibrium conditions, although I find the difference between the first- and the second-order impulse responses does not $\varsigma_\omega = 0.026$ when the latter is relatively smaller than the former.
affect the results.\textsuperscript{36}

In the following results, the impulse responses are computed by a simulation. The economy is simulated for 100 quarters starting from the steady-state values. This simulation is then repeated 2,000 times. In each simulation, two identical economies are considered from the 101st quarter on; a large volatility shock additionally hits only one of them in the 101st quarter and no additional shocks are assumed to later occur.\textsuperscript{37} The difference represents the impulse response starting from the state of the economy in the 100th quarter.

Figure 3 shows the impulse responses to a volatility shock $\epsilon$. At the outset of the shock, both entry and exit are delayed; a higher entry threshold and a lower exit threshold leads to a decrease in both investment by entry and disinvestment by exit. As the steady-state level of investment is much larger than disinvestment, the net capital investment drops.

It is important to note that the next period more than recovers all investments even though the entry threshold stays at a higher level; this is because idea investment $I^h$ also increases during the first period, which leads to a larger stock of ideas. Although entry decisions are more cautious, more ideas pass these criteria. High idiosyncratic volatility makes investment in ideas lucrative as the productivity of ideas is more likely to evolve to a high level. The econ-

\textsuperscript{36}I take a Taylor expansion in the logs of variables except for the entry and exit thresholds $\omega_t$ and $\Xi_t$ as well as welfare $V_t$, since most variables should be positive. However, the results from the approximation in levels are similar to those in logs.

\textsuperscript{37}The second or higher order impulse responses depend on the state of the economy when the shock occurs and the expectation of future shocks, and they are typically computed as follows: first, the state of the economy on impact is taken from a long simulated sample for the state variables; and second, the expectations of future shocks are evaluated by a simulation for each of those possible states of the economy on impact. Because my approximated solution has over 300 state variables, the full simulation would take a very long time. Hence, I skipped the second step by assuming that there are no additional shocks after the considered volatility shock occurs. In other words, I compute $E(y_{t+1}|x_t, \epsilon_{t+1}^y = 1, \epsilon_{t+k}^y = 0, k = 2, \ldots, l) - E(y_{t+1}|x_t, \epsilon_{t+1}^y = 0, k = 2, \ldots, l)$ instead of $E(y_{t+1}|x_t, \epsilon_{t+1}^y = 1) - E(y_{t+1}|x_t, \epsilon_{t+1}^y = 0)$, where $x$ denotes the state variables.
Figure 3: Idea investment is freely chosen. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds and welfare.
omy takes advantage of this opportunity by working more and consuming less in order to finance investment in ideas. For the same reason, the exit threshold returns to the normal level quickly and leads to an increase in disinvestment.

This story is opposite to that of a recession caused by uncertainty as highlighted in Bernanke (1983) and Bloom (2009). An increase in volatility brings about an investment boom in the early periods and the economy enjoys the larger output. Note that aggregate capital is substituted for ideas and diminishes, but the rise in aggregate TFP makes output remain at a higher than normal level even after labor declines.

These results raise the question of what will happen if the creation of new investment ideas is not possible. To answer this question, I remove the freedom of the social planner to choose an idea investment and I fix it to its steady-state value. Figure 4 displays the responses in this case.

The impact of a shock expands both the entry and exit thresholds, which return to the normal level only gradually. Since the stock of ideas is not increased, the higher entry threshold translates directly into a lower level of investment by entry. A sudden spike in consumption and a drop in output and labor accompany the sharp decrease in investment. These responses look similar to the story of Bloom et al. (2012), who predict that upon impact the volatility shock causes a fall in output, labor, and investment and a rise in consumption.

However, my model importantly differs from theirs in the response of the aggregate TFP. Bloom et al. (2012) argue that a volatility shock decreases

More precisely, I fix $I^h_t$ (the value deflated by labor-augmenting technology) to its steady-state value so that its pre-deflated value grows exogenously at a rate of $e^{\mu z}$, which is the economy’s trend growth rate.

The magnitude of the impact responses is also comparable; Bloom et al. (2012) display a fall in output (by just over 3 percent), labor (by about 7 percent), and investment (by about 18 percent) and a rise in consumption (by about 0.7 percent).
Figure 4: Idea investment is fixed. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds and welfare.
the aggregate productivity by reducing the reallocation of resources toward more productive units. In contrast, the aggregate TFP in figure 4 grows because higher idiosyncratic productivities are selected from the expanded set of productivity realization.\footnote{The aggregate productivity growth in my model is driven by a market selection mechanism: the entry of high-productivity units and the exit of low-productivity ones. This mechanism is missing in Bloom et al. (2012), who do not consider the possibility of entry and exit and instead assume a fixed number of plants. The potential for enhanced productivity originating from a more dispersed productivity distribution can only be realized by the expansion (contraction) of high- (low-) productivity units. This reallocation between the existing plants is further restricted as, in addition to the investment irreversibility considered by both Bloom et al. (2012) and my model, labor adjustment costs and decreasing returns to scale are also present in their model. This difference—the number of investment opportunities is fixed and the good investment opportunities are not easily exploited—explains the fall in aggregate productivity in their model.} In fact, a drop in output and labor in figure 4 is the economy’s response to the aggregate productivity growth—the same amount of capital and labor produces more than before; hence, wealthier consumers (see the increase in welfare\footnote{Although Bloom et al. (2012) do not report the response of welfare in their model, it may decrease—as opposed to what occurs in my model. The reason for this difference is due to the various types of investment costs featured in the two models (also see footnote 40). In my model, each idea (plant) pays a (nonconvex) capital adjustment cost once in its lifetime when it enters (exits). In contrast, in Bloom et al. (2012), each plant pays an adjustment cost repeatedly in its lifetime whenever it adjusts its capital stock in response to a change in its productivity. Hence, a higher idiosyncratic volatility implies more frequent capital adjustments, thereby incurring larger costs. This negative effect can dominate the beneficial effect featured in my model, depending on the size of adjustment cost and the persistence of the idiosyncratic productivity.} in figure 4) are less willing to save and work. This is exactly what happens in a standard business cycle model when news shocks hit the economy—good news about future productivity makes consumers wealthier and so they increase their consumption as well as their leisure, thereby reducing the labor supply, which in turn causes output to fall. The rise in wage accompanied by a fall in labor also supports this labor-supply-side scenario.

Figure 5, where labor is additionally fixed to its steady-state value as well as idea investment, further confirms this supposition. Even though the entry
Figure 5: Idea investment and labor are fixed. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds and welfare.
and exit are delayed and investment and aggregate capital shrink, output does not fall; this is because the aggregate productivity growth more than offsets the decrease in capital stock.

Aggregate productivity growth is a natural consequence of market selection. Entry and exit ensure that only the upper tail of productivity distribution is adopted into the economy; a wider dispersion therefore enhances the average level of surviving productivities. This mechanism is empirically documented by Chun et al. (2008), who find that U.S. industries with a higher idiosyncratic volatility exhibit faster productivity growth. The authors argue that adoption of information technology (IT) causes an increase in both idiosyncratic volatility and productivity growth, but their regression results show that adoption of technology affects productivity growth primarily through its effect on the idiosyncratic volatility. Moreover, Durnev et al. (2004) use country-level data to show that a higher idiosyncratic volatility of stock returns positively correlates with national TFP growth and they link a greater volatility to sound property rights, corporate transparency, and capital market openness. Their more robust findings are the positive relation between idiosyncratic volatility and productivity growth, which the authors also interpret as a result of creative destruction.

Figures 6 and 7 displays the responses of the mean value of plants and ideas, along with the stock of capital and ideas. The mean plant (idea) value is computed as time $t$-value of total capitalization of plants (ideas) existing at $t + 1$ (recall that time $t + 1$ distribution of plants and ideas is predetermined.
Figure 6: Idea investment is freely chosen. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations.

Figure 7: Idea investment is fixed. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations.

at $t$, divided by the total stock of capital (idea) at $t + 1$.\footnote{More specifically, the mean value of plants and ideas, $P^k_t$ and $P^h_t$, is computed by}

\[ P^k_t = \frac{\int_{-\infty}^{\infty} Q_t(\omega_{t+1}) K_{t+1}(\omega_{t+1}) d\omega_{t+1}}{\int_{-\infty}^{\infty} K_{t+1}(\omega_{t+1}) d\omega_{t+1}} \]

\[ P^h_t = \frac{\int_{-\infty}^{\infty} Q^h_t(\omega_{t+1}) H_{t+1}(\omega_{t+1}) d\omega_{t+1}}{\int_{-\infty}^{\infty} H_{t+1}(\omega_{t+1}) d\omega_{t+1}}, \]

where $Q_t(\omega_{t+1})$ ($Q^h_t(\omega_{t+1})$) is the time $t$-price of plant (idea) available at $t + 1$ with idiosyncratic productivity $\omega_{t+1}$. See Appendix B for more details.
investment (figure 6) and the idea stock rises sharply.

This is reminiscent of the dot-com boom in the late 1990s. The boom saw a dramatic increase in the number of initial public offerings (IPOs) and aggregate idiosyncratic volatility spiked nearly fivefold (Fink et al., 2010). My model suggests the highly volatile prospect of internet technology made the growth option very valuable and led to a frenzy of option acquisition. Pastor and Veronesi (2005, 2006) also argue that the dot-com boom was a response to an increase in uncertainty, but they do not consider option values. They note that the present value of future cash flows is a convex function of the cash-flow growth rate and describe the dot-com boom as a period of high uncertainty about internet firms’ growth rate, which raises the valuation of those firms. In contrast, Hall (2000) and McGrattan and Prescott (2010) agree that high investment in intangible capital such as R&D led to an economic boom in the 1990s. However, they attribute increases in intangible investment to technical progress in producing intangible capital and not to heightened uncertainty.

On the other hand, we may think of the value of plants as measured Tobin’s Q—intangible assets such as ideas and technology are not often accounted for and the replacement cost of a plant is then measured by its capital stock, 1. The (market) value of plants, however, reflects the value of ideas since building a plant requires an idea as well as a unit of capital. Therefore, a rise in the value of investment options at the outset of the volatility shock translates into an immediate rise in Tobin’s Q in the case of a fixed idea investment (figure 43). Pastor and Veronesi (2006) illustrate this point using the Gordon growth model, \( \frac{P}{D} = \frac{1}{(r - g)} \), where \( P \) is the stock price, \( D \) is the next period’s dividend, \( r \) is the discount rate, and \( g \) is the mean dividend growth rate.

McGrattan and Prescott (2010) focus on a puzzling fact; that in the 1990s, measured factor incomes were low when output and hours were booming. They do not discuss their model’s implication on the stock market. However, McGrattan and Prescott (2004) use similar arguments to explain high stock prices before the stock market crash of 1929.

39
In contrast, free entry ensures that the value of ideas readily adopted in plants is stable and Tobin’s Q does not instantly rise in the case of a free idea investment (figure 6). The mean value of plants of course also reflects the average plant productivity (aggregate productivity), displaying a hump-shaped pattern in both cases.

This relation between the value of ideas and Tobin’s Q is similar to Jovanovic (2009). In his model, all plants and investment options are homogeneous and new investment options fall off of the existing capital at a constant rate. A prolonged boom depletes the stock of investment options, thereby increasing the value of investment options and leading to a high Tobin’s Q.

### 5.3 Sensitivity Analysis

This section discusses the robustness of my finding to different parameterizations on idea creation. The benchmark value of the idea creation cost implies about 60 percent of newly-created ideas are immediately adopted. Table 2 and figure 8 consider two alternative values while keeping the other parameters fixed. Table 2 shows that a lower cost for idea generation not only increases overall investment, but also tilts its composition towards ideas, relative to capital. Ideas are therefore more selectively adopted from a larger pool, as can be seen by a higher entry threshold.

---

45 More than half of newly-created ideas are immediately adopted in plants under a baseline parameterization, and free entry guarantees that the mean value of newly-created ideas always equals a time-invariant idea-creation cost. Since old ideas also comprise the stock of ideas and the productivity (hence the value) of old ideas is lower than that of newly-created ideas, a large influx of new ideas increases the mean value of all ideas even in the case of a free idea investment.

46 The initial drop in the mean value of plants in the case of a free idea investment is due to the fact that high volatility makes high productivity plants less scarce; a plant’s productivity is more likely to evolve to a high level. This effect is dominated by the effect of an increased value of ideas in the case of a fixed idea investment.
Table 2: Steady State. Risk-free Rate $R^f_{ss} = 1.01$.

<table>
<thead>
<tr>
<th>different parametrization</th>
<th>$\zeta = 0.203$</th>
<th>$\zeta = 0.279$</th>
<th>$\zeta = 0.208$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>innovation lags</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_{ss}$ output</td>
<td>0.989</td>
<td>0.951</td>
<td>0.963</td>
</tr>
<tr>
<td>$C_{ss}$ consumption</td>
<td>0.724</td>
<td>0.696</td>
<td>0.710</td>
</tr>
<tr>
<td>$N_{ss}$ labor</td>
<td>0.333</td>
<td>0.333</td>
<td>0.331</td>
</tr>
<tr>
<td>$V_{ss}$ welfare</td>
<td>-1.235</td>
<td>-1.274</td>
<td>-1.248</td>
</tr>
<tr>
<td>$I_k^{ss}$ net investment</td>
<td>0.203</td>
<td>0.189</td>
<td>0.195</td>
</tr>
<tr>
<td>$I_e^{ss}$ investment by entry</td>
<td>0.231</td>
<td>0.213</td>
<td>0.221</td>
</tr>
<tr>
<td>$I_x^{ss}$ disinvestment by exit</td>
<td>0.028</td>
<td>0.024</td>
<td>0.026</td>
</tr>
<tr>
<td>$I_h^{ss}$ investment in ideas</td>
<td>0.307</td>
<td>0.237</td>
<td>0.280</td>
</tr>
<tr>
<td>$\zeta I_h^{ss}$ ideas investment cost</td>
<td>0.062</td>
<td>0.066</td>
<td>0.058</td>
</tr>
<tr>
<td>$\omega_{ss}^{e}$ entry threshold</td>
<td>0.010</td>
<td>-0.029</td>
<td>-0.010</td>
</tr>
<tr>
<td>$\omega_{ss}^{x}$ exit threshold</td>
<td>-0.254</td>
<td>-0.293</td>
<td>-0.273</td>
</tr>
<tr>
<td>$K_{ss}$ productivity-weighted capital</td>
<td>8.698</td>
<td>7.742</td>
<td>8.147</td>
</tr>
<tr>
<td>$K_{ss}$ aggregate capital stock</td>
<td>6.722</td>
<td>6.284</td>
<td>6.469</td>
</tr>
<tr>
<td>$H_{ss}$ aggregate idea stock</td>
<td>1.629</td>
<td>0.651</td>
<td>1.030</td>
</tr>
<tr>
<td>$A_{ss}$ aggregate TFP</td>
<td>1.090</td>
<td>1.072</td>
<td>1.080</td>
</tr>
<tr>
<td>$W_{ss}$ wage</td>
<td>1.978</td>
<td>1.902</td>
<td>1.940</td>
</tr>
</tbody>
</table>

Benchmark value is $\zeta = 0.239$ for idea creation cost.

Figure 8 shows that the responses of the economy to an increase in idiosyncratic volatility are stronger in the case of a lower idea creation cost. A more selective adoption implies a focus on the upper tail of distribution, and a high volatility is especially good news. The economy takes advantage of this opportunity by increasing investment in idea creation—the exit threshold rises even upon impact in order to help financing a new investment by releasing resources. Although the responses are relatively weaker in the case of a higher idea creation cost, as in figure 9, the main results are the same—an aggregate increase in idiosyncratic volatility makes the economy expand when investment in option creation is allowed.
Figure 8: $\zeta = 0.203$. Idea investment is freely chosen. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds.

Figure 9: $\zeta = 0.279$. Idea investment is freely chosen. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds.
It is assumed so far that idea creation takes one quarter to conclude, which is unrealistic. The innovation literature has generally found long lags, particularly for basic research. Figure 10 instead assumes that a prospect entrant discovers a new idea with a constant probability $\frac{1}{16}$ in any quarter subsequent to paying a cost $\zeta$. A lower value of $\zeta$ is chosen to compensate for a lag and to allow for an easier comparison with the benchmark case (see table 2). Figure 10 shows economic activity again rises at the outset of the shock. The economy quickly builds up the idea stock in the first few quarters and subsequently increases consumption and leisure by depleting the idea stock.\(^{47}\)

Why does the positive effect of uncertainty on an option creation dominate the negative effect on an option exercise under all parameterizations considered? One might think of a case in which the option creation cost is high and there already exists a large stock of options. In such a case, a drop in investment due to the delay of exercising the option would be larger than an increase in investment in option creation, thus leading to an economic contraction. The results of this paper show that such a case is unlikely to occur because the economy accumulates a small (large) stock of options when the creation cost is high (low). Of course, this would no longer be true if the economy is subject to a rich set of shocks either increasing option creation costs or generating a windfall of investment opportunities.\(^{48}\) Moreover, the curvature of the idea creation cost can make a difference. If the cost is low at the steady state but rises fast with an idea creation over the steady-state level, the delay

\(^{47}\)The average lag of 2 years might still be too short; for example, Fraumeni and Okubo (2005) report a range from 1 to 7 years. However, note that a two-year lag of idea creation is eight times longer than one quarter, the time assumed for capital. Taking one year to build for capital is common in the literature; for example, Tsoukalas (2011) states a lag of 3 to 4 quarters for equipment and 2 to 3 years for nonresidential structures.

\(^{48}\)For example, Jovanovic (2009) considers the consequence of a prolonged war: defense oriented research leads to a pile-up of investment options.
Figure 10: Average 2-year lag of idea creation. Idea investment is freely chosen. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds and welfare.
of implementing a large stock of ideas may lead to a large drop in investment while the option creation is dampened by the rising cost. The dominance of the positive option effect will not always hold in general and the results of this paper call for further investigation into which of the two option effects dominates under what conditions in a richer model.

6 Concluding Remarks

The existing studies on the real options effect of uncertainty suggest that an aggregate increase in idiosyncratic volatility leads to a synchronized delay of investment across plants and causes a recession. This paper is motivated by the observation that volatility also has other positive aspects. A wider dispersion in idiosyncratic productivity is beneficial to the economy as a whole as long as market selection ensures only good productivity survives. Moreover, an uncertain future prospect promotes exploratory research and innovation, which creates more growth options.

To incorporate these aspects, I have developed a general equilibrium model with plant-specific productivity in which investment options are endogenously created. I find an aggregate increase in idiosyncratic volatility causes an economic boom. High volatility increases the value of investment options, thereby inducing the creation of new options. Individual units still display the wait-and-see behavior, but the increased stock of investment opportunities outweighs more selective implementation and leads to a rise in investment and output. On the other hand, if new investment opportunities are exogenously given, the volatility shock contracts economic activity because future productivity growth discourages the labor supply through the wealth effect.

This paper demonstrates that assumptions about the flow of investment
opportunities are important for the real option effects of uncertainty. This, however, serves only as the first step towards incorporating the option creation margin and assessing both the positive and negative effects of uncertainty on the macroeconomy. Future research should explore the role of the intensive margin of capital adjustment and of interaction between the right to delay and the competitive pressure. In addition, this paper considers two polar cases of option creation with a linear cost and no creation at all. The reality resides in the middle, and modeling the creation margin more realistically with convex entry costs will enable the model to be quantitatively more serious.

The aspect of option creation also demands further empirical research. The difficulty in empirically investigating a real option exercise decision is well known (Dixit and Pindyck 1994, Carruth et al. 2000)—real options reasoning does not describe the level of investment per se, but only the timing of the exercise decision. As my model demonstrates, a high level of investment can coexist with the delay of an individual investment decision. Empirical work on option creation decisions is more scarce, and measuring innovative or option-creating activity is challenging. However, many activities such as R&D, patent filing, foothold entry into a new market, venture capital seed funding, and so on, can be considered as option creation or acquisition. With micro-level data on these activities, I hope to make a more detailed quantitative assessment of the option creation margin in the future.

Acknowledgments

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Alvarez, Nicholas Bloom, Jaroslav Borovicka, Rui Cui, Thorsten Drautzburg, Ali Hortacsu, Samuel S. Kortum, Albrecht Ritschl, and Tao Zha for their comments. Earlier versions were entitled “Does an Aggregate Increase in Idiosyncratic Volatility Cause a Recession?”
Appendix A  Transforming the Social Planner’s Problem into a Stationary One

After eliminating variables at \( t' \) and solving labor allocation problems separately, the social planner’s problem (3) becomes:

\[
V(K_t(\cdot), H_t(\cdot), \sigma_t, z_t) = \max_{C_t, I_t, N_t} \left( 1 - \beta \right) \left[ \log C_t - \kappa N_t \right] + \beta E_t \left[ V(K_{t+1}(\cdot), H_{t+1}(\cdot), \sigma_{t+1}, z_{t+1}) \right],
\]

subject to

\[
\hat{K}_t^{1-a}(e^{z_t} N_t)^a = C_t + \zeta I^h_t + \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_t - \omega_t}{\sigma_t/\sqrt{2}} \right) \right] H_t(\omega_t) d\omega_t
\]

\[
-(1 - \eta) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - \omega_t}{\sigma_t/\sqrt{2}} \right) K_t(\omega_t) d\omega_t
\]

\[
\hat{K}_t = \int_{-\infty}^{\infty} e^{\frac{\omega_t}{\sigma_t}} K_t(\omega_t) d\omega_t
\]

\[
K_{t+1}(\omega_{t+1}) = (1 - \delta) \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t)/2}{\sigma_t/2} \right) \right]
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) K_t(\omega_t) d\omega_t
\]

\[
+(1 - \delta) \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t)/2}{\sigma_t/2} \right) \right]
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) H_t(\omega_t) d\omega_t
\]

\[
H_{t+1}(\omega_{t+1}) = (1 - \delta) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t)/2}{\sigma_t/2} \right) \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) H_t(\omega_t) d\omega_t
\]

\[
+ \frac{1}{\varsigma_e} \phi \left( \frac{\omega_{t+1}}{\varsigma_e} \right) I^h_t.
\]

Divide all variables by the level of the labor-augmenting productivity \( e^{z_t} \)

and with some abuse of notation, hereafter denote the deflated variables by
the same symbols: \( C_t / e^{z_t} \) by \( C_t \), \( I_t^h / e^{z_t} \) by \( I_t^h \), \( K_t(\cdot) / e^{z_t} \) by \( K_t(\cdot) \), \( H_t(\cdot) / e^{z_t} \) by \( H_t(\cdot) \) and \( V(K_t(\cdot) / e^{z_t}, H_t(\cdot) / e^{z_t}, \sigma_t, z_t / z_t) - \beta \mu z / (1 - \beta) \) by \( V(K_t(\cdot), H_t(\cdot), \sigma_t) \).

The following stationary social planner’s problem is then obtained:

\[
V(K_t(\cdot), H_t(\cdot), \sigma_t) = \max_{C_t, I_t, N_t, \omega_t, \omega_t} (1 - \beta) [\log C_t - \kappa N_t] \\
+ \beta E_t[V(K_{t+1}(\cdot), H_{t+1}(\cdot), \sigma_{t+1})],
\]

subject to

\[
(\hat{K}_t)^{1-\alpha} N_t^\alpha = C_t + \zeta I_t^h + \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\overline{\omega}_t - \omega_t}{\sigma_t / \sqrt{2}} \right) \right] H_t(\omega_t) d\omega_t \\
-(1 - \eta) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - \omega_t}{\sigma_t / \sqrt{2}} \right) K_t(\omega_t) d\omega_t \\
\hat{K}_t = \int_{-\infty}^{\infty} e^{\tau - \alpha \omega_t} K_t(\omega_t) d\omega_t \\
e^{\mu z} K_{t+1}(\omega_{t+1}) = (1 - \delta) \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t) / 2}{\sigma_t / 2} \right) \right] \\
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) K_t(\omega_t) d\omega_t \\
+(1 - \delta) \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t) / 2}{\sigma_t / 2} \right) \right] \\
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) H_t(\omega_t) d\omega_t \\
e^{\mu z} H_{t+1}(\omega_{t+1}) = (1 - \delta_h) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t) / 2}{\sigma_t / 2} \right) \\
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) H_t(\omega_t) d\omega_t + \frac{1}{\varepsilon e} \phi \left( \frac{\omega_{t+1}}{\varepsilon e} \right) I_t^h.
\]
Appendix B  First-order Conditions of the Social Planner’s Problem

The Lagrangian for the problem (11) is:

\[
\mathcal{L} = \max (1 - \beta) \log C_t - \kappa N_t + \beta E_t [V (K_{t+1} (\cdot), H_{t+1} (\cdot), \sigma_{t+1})]
+ M_t \left[ (\hat{K}_t)^{1-\alpha} N_t^\alpha - C_t - \zeta I^h_t - \int_{-\infty}^{\infty} [1 - \Phi \left( \frac{\omega_t - \omega_i}{\sigma_t} \right)] \right. 
H_t(\omega_t) d\omega_t 
+ (1 - \eta) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - \omega_i}{\sigma_t} \right) K_t(\omega_t) d\omega_t 
+ M_t D_t \left[ \int_{-\infty}^{\infty} e^{\frac{\alpha}{\alpha} \omega_i \omega_t} K_t(\omega_t) d\omega_t - \hat{K}_t \right] 
+ M_t \int_{-\infty}^{\infty} Q_t(\omega_{t+1}) \left[ (1 - \delta) \int_{-\infty}^{\infty} \left( 1 - \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t)/2}{\sigma_t} \right) \right) 
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) K_t(\omega_t) d\omega_t 
+ (1 - \delta) \int_{-\infty}^{\infty} \left( 1 - \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t)/2}{\sigma_t} \right) \right) 
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) H_t(\omega_t) d\omega_t - e^{\mu_z} K_{t+1}(\omega_{t+1}) \right] d\omega_{t+1} 
\left. + M_t \int_{-\infty}^{\infty} Q^h_t(\omega_{t+1}) \left[ (1 - \delta_h) \int_{-\infty}^{\infty} \Phi \left( \frac{\omega_t - (\omega_{t+1} + \omega_t)/2}{\sigma_t} \right) 
\times \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) H_t(\omega_t) d\omega_t + \frac{1}{\zeta_e} \phi \left( \frac{\omega_{t+1}}{\zeta_e} \right) I^h_t - e^{\mu_z} H_{t+1}(\omega_{t+1}) \right] d\omega_{t+1}. \]

Note that \( Q_t(\omega_{t+1}) \) and \( Q^h_t(\omega_{t+1}) \) represent the prices of a unit of \( e^{\mu_z} K_{t+1}(\omega_{t+1}) \) and \( e^{\mu_z} H_{t+1}(\omega) \), respectively. The first-order conditions are as follows:
• Optimal labor:

\[ \kappa = \frac{1}{C_t} \times \alpha \left( \frac{\hat{K}_t}{N_t} \right)^{1-\alpha}. \]

The marginal disutility of labor equals the product of the marginal utility of consumption and the marginal product of labor.

• Optimal investment in ideas:

\[ \zeta = \int_{-\infty}^{\infty} \frac{1}{\varsigma_e} \phi \left( \frac{\omega_{t+1}}{\varsigma_e} \right) \times Q_t(\omega_{t+1})d\omega_{t+1}. \]

The cost of idea creation equals the expected value of a new idea.

• Plant asset pricing:

\[ Q_t(\omega_{t+1}) = E_t \left[ \beta e^{-\mu_2} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( (1-\alpha) \left( \frac{\hat{K}_{t+1}}{N_{t+1}} \right)^{-\alpha} e^{\frac{\alpha}{1-\alpha} \omega_{t+1}} \right. \right. \]

\[ \left. + \Phi \left( \frac{\omega_{t+1} - \omega_{t+1}}{\sigma_{t+1}/\sqrt{2}} \right) \times (1-\eta) \right. \]

\[ \left. + (1-\delta) \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\omega_{t+1} - (\omega_{t+2} + \omega_{t+1})/2}{\sigma_{t+1}/2} \right) \right] \right. \]

\[ \left. \times \frac{1}{\sigma_{t+1}} \phi \left( \frac{\omega_{t+2} - \omega_{t+1}}{\sigma_{t+1}} \right) \times Q_{t+1}(\omega_{t+2})d\omega_{t+2} \right]. \]

The price of a plant with the idiosyncratic productivity \( \omega_{t+1} \) is the expected discounted value of the following terms: marginal product of the plant with \( \omega_{t+1} \); exit probability of \( \omega_{t+1} \) multiplied by the resale value of capital; and the transition probability from \( \omega_{t+1} \) to \( \omega_{t+2} \) without experiencing an exit, multiplied by the price of a plant with \( \omega_{t+2} \).
• Idea asset pricing:

\[
Q_t^h(\omega_{t+1}) = E_t \left[ \beta e^{-\mu z} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( -1 - \Phi \left( \frac{\overline{\omega}_{t+1} - \omega_{t+1}}{\sigma_{t+1}/\sqrt{2}} \right) \right) \times 1 \\
+ (1 - \delta) \int_{-\infty}^{\infty} \left[ 1 - \Phi \left( \frac{\overline{\omega}_{t+1} - (\omega_{t+2} + \omega_{t+1})/2}{\sigma_{t+1}/2} \right) \right] \\
\times \frac{1}{\sigma_{t+1}} \phi \left( \frac{\omega_{t+2} - \omega_{t+1}}{\sigma_{t+1}} \right) \times Q_{t+1}(\omega_{t+2}) \, d\omega_{t+2} \\
+ (1 - \delta_h) \int_{-\infty}^{\infty} \Phi \left( \frac{\overline{\omega}_{t+1} - (\omega_{t+2} + \omega_{t+1})/2}{\sigma_{t+1}/2} \right) \\
\times \frac{1}{\sigma_{t+1}} \phi \left( \frac{\omega_{t+2} - \omega_{t+1}}{\sigma_{t+1}} \right) \times Q_{t+1}^h(\omega_{t+2}) \, d\omega_{t+2} \right].
\]

The price of an idea with the idiosyncratic productivity \( \omega_{t+1} \) is the expected discounted value of the following terms: the entry probability of \( \omega_{t+1} \) multiplied by expenditure on capital; the transition probability from \( \omega_{t+1} \) to \( \omega_{t+2} \) with experiencing entry, multiplied by the price of the plant with \( \omega_{t+2} \); and the transition probability from \( \omega_{t+1} \) to \( \omega_{t+2} \) without entry, multiplied by the price of an idea with \( \omega_{t+2} \).
• Optimal entry:

\[
\int_{-\infty}^{\infty} \left[ \frac{1}{\sigma_t} \phi \left( \frac{ct - \omega_t}{\sigma_t} \right) \times 1 
+ (1 - \delta_h) \int_{-\infty}^{\infty} \frac{1}{\sigma_t/2} \phi \left( \frac{ct - (\omega_{t+1} + \omega_t)/2}{\sigma_t/2} \right) 
\cdot \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) \times Q_t(\omega_{t+1}) \right] H_t(\omega_t) d\omega_t 
= \int_{-\infty}^{\infty} \left[ (1 - \delta) \int_{-\infty}^{\infty} \frac{1}{\sigma_t/2} \phi \left( \frac{ct - (\omega_{t+1} + \omega_t)/2}{\sigma_t/2} \right) 
\cdot \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) \times Q_t(\omega_{t+1}) \right] H_t(\omega_t) d\omega_t.
\]

The marginal benefit of increasing the entry threshold \( \overline{\omega}_t \) is saving the following values for each \( \omega_t \): a change in the entry probability of \( \omega_t \) multiplied by the price of a unit of capital and a change in the transition probability from \( \omega_t \) to \( \omega_{t+1} \) without entry, multiplied by the price of an idea with \( \omega_{t+1} \). The marginal cost is losing the following values for each \( \omega_t \): a change in the transition probability from \( \omega_t \) to \( \omega_{t+1} \) with entry, multiplied by the price of a plant with \( \omega_{t+1} \).

• Optimal exit:

\[
\int_{-\infty}^{\infty} \left[ \frac{1}{\sigma_t} \phi \left( \frac{ct - \omega_t}{\sigma_t} \right) \times (1 - \eta) \right] \times K_t(\omega_t) d\omega_t 
= \int_{-\infty}^{\infty} \left[ (1 - \delta) \int_{-\infty}^{\infty} \frac{1}{\sigma_t/2} \phi \left( \frac{ct - (\omega_{t+1} + \omega_t)/2}{\sigma_t/2} \right) 
\cdot \frac{1}{\sigma_t} \phi \left( \frac{\omega_{t+1} - \omega_t}{\sigma_t} \right) \times Q_t(\omega_{t+1}) \right] K_t(\omega_t) d\omega_t.
\]

The marginal benefit of increasing the exit threshold \( \omega_t \) is earning the
following values for each $\omega_t$: a change in the exit probability of $\omega_t$ multiplied by the resale value of capital. The marginal cost is losing the following values for each $\omega_t$: a change in the transition probability from $\omega_t$ to $\omega_{t+1}$ without exit, multiplied by the price of a plant with $\omega_{t+1}$.

The risk-free rate and wage implied by the optimal allocation can be computed by

$$R^f_t = \frac{1}{E_t} \left[ \beta e^{-\mu z} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \right]$$

$$W_t = \alpha \left( \frac{K_t}{N_t} \right)^{1-\alpha}.$$

### Appendix C Additional Sensitivity Analysis

The additional robust checks are considered. The first case assumes a higher dispersion for the initial productivity draws: $\varsigma_e = 0.078$ instead of the benchmark value of $\varsigma_e = 0.039$. Figure 11 shows the impulse responses are almost the same. Figures 12 and 13 consider the case in which idiosyncratic productivity is mean-reverting with the persistence of 0.98.\footnote{The mean-reverting productivity makes the idea investment less profitable because of a lower probability of staying at a high level of productivity. Hence, a lower cost of idea creation $\zeta = 0.064$ is considered.} Although the impact is smaller, especially for the first period, the overall patterns are similar to the benchmark case of the random walk productivity.
Table 3: Steady State. Risk-free Rate $R_{ss}^f = 1.01$.

<table>
<thead>
<tr>
<th>different parametrization</th>
<th>$\zeta_e = 0.078$</th>
<th>$\zeta = 0.064$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean-reverting productivity</td>
<td></td>
</tr>
<tr>
<td>$Y_{ss}$ output</td>
<td>0.982</td>
<td>0.837</td>
</tr>
<tr>
<td>$C_{ss}$ consumption</td>
<td>0.718</td>
<td>0.590</td>
</tr>
<tr>
<td>$N_{ss}$ labor</td>
<td>0.333</td>
<td>0.346</td>
</tr>
<tr>
<td>$V_{ss}$ welfare</td>
<td>-1.243</td>
<td>-1.472</td>
</tr>
<tr>
<td>$I_{ss}^k$ net investment</td>
<td>0.193</td>
<td>0.225</td>
</tr>
<tr>
<td>$I_{ss}^e$ investment by entry</td>
<td>0.217</td>
<td>0.265</td>
</tr>
<tr>
<td>$I_{ss}^x$ disinvestment by exit</td>
<td>0.024</td>
<td>0.040</td>
</tr>
<tr>
<td>$I_{ss}^h$ investment in ideas</td>
<td>0.295</td>
<td>0.327</td>
</tr>
<tr>
<td>$\zeta I_{ss}^h$ ideas investment cost</td>
<td>0.070</td>
<td>0.021</td>
</tr>
<tr>
<td>$\omega_{ss}$ entry threshold</td>
<td>0.002</td>
<td>-0.005</td>
</tr>
<tr>
<td>$\omega_{ss}$ exit threshold</td>
<td>-0.261</td>
<td>-0.166</td>
</tr>
<tr>
<td>$K_{ss}$ productivity-weighted capital</td>
<td>8.509</td>
<td>4.903</td>
</tr>
<tr>
<td>$\bar{K}_{ss}$ aggregate capital stock</td>
<td>6.421</td>
<td>4.465</td>
</tr>
<tr>
<td>$\bar{H}_{ss}$ aggregate idea stock</td>
<td>1.637</td>
<td>1.088</td>
</tr>
<tr>
<td>$A_{ss}$ aggregate TFP</td>
<td>1.098</td>
<td>1.032</td>
</tr>
<tr>
<td>$W_{ss}$ wage</td>
<td>1.963</td>
<td>1.614</td>
</tr>
</tbody>
</table>

Benchmark values are $\zeta_e = 0.039$ for initial productivity dispersion and $\zeta = 0.239$ for idea creation cost.
Figure 11: $\zeta_e = 0.078$. Idea investment is freely chosen. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds and welfare.
Figure 12: Mean-reverting idiosyncratic productivity with persistence 0.98. Idea investment is freely chosen. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds and welfare.
Figure 13: Mean-reverting idiosyncratic productivity with persistence 0.98. Idea investment is fixed. Impulse responses to a 50 percent increase in idiosyncratic volatility with 90 percent error bands. All deviations are in percent deviations except for the entry and exit thresholds and welfare.
References


