Holdup and repeated interaction:
the case of complementary monopoly

Kaz Miyagiwa

this version
February 19, 2007

Abstract

Suppose consumers buy complementary goods sequentially from several monopolists. If prices cannot be contracted on, there may be no sale in a one-shot game due to the holdup problem. Dynamic interaction of agents attenuates the problem. In equilibrium, the first and the last monopolist capture the entire monopoly profit while the other monopolists break even. Vertical integrations that exclude the last monopolist neither lower the price nor increase social welfare. On the other hand, partial integrations that include the last monopoly can reduce the combined profit and hence may never occur despite the welfare-improving potential.

JEL classification codes: D4, L13
Keywords: Holdup, complementary monopoly, vertical integration, repeated games

Correspondence: Kaz Miyagiwa, Department of Economics, Emory University, Atlanta, GA 30322, U.S.A.; Telephone: (404) 727-6363, E-mail: kmiyagi@emory.edu
1. Introduction

Suppose that a consumer buys complementary goods sequentially from two monopolists. If prices are not contracted on, the price the consumer paid to the first monopolist becomes sunk. In such a case, the second monopolist can charge the price that maximizes his profit in total disregard of the price the consumer paid to the first monopolist. Thus the consumer is held up once he has bought the good from the first monopolist. The forward-looking consumer anticipates the second monopolist’ pricing behavior and refuses to purchase from the first monopolist.

This type of holdup problem, first examined by Feinberg and Kamien (2001), may occur in many instances. For example, a vacationer flying with a major airline company to the local city can potentially be held up by the local provider of connecting services to the remote vacation spot, and farmers who have harvested their produce are likely to be held up by wholesale buyers. The study of tolls on the medieval Rhine by Gardner, Gaston and Masson (2002) conjures up a more vivid image of this type of holdup problem.\(^1\)

In the literature the holdup problem is usually analyzed in a one-shot game, and vertical integration is often proposed as a solution. The objective of this paper is to show that repeated interaction attenuates the type of holdup problem described above, implying that the holdup problem may be less serious than is portrayed in one-shot game settings (see Che and Sakovics 2004 for a similar view).

\(^1\) Gardner, Gaston and Masson (2002) however analyze the simultaneous-move one-shot game Nash equilibrium.
In addition to showing that sales occur in equilibrium in a dynamic setting, our model yields some surprising results regarding complementary monopoly. The key result is that the entire monopoly rent is captured by the first and the last monopoly while the other "intermediate" monopolists just break even. This has the consequence that a vertical integration of monopolies that excludes the last neither lowers the price nor increases social welfare, which contrasts with the standard result. Further, although a partial integration involving the last monopoly can lower the price and improve social welfare as in the standard case, such an integration may reduce the combined profits of the integrating monopolists, and hence may not be initiated despite its welfare-improving potential. The remainder of the paper demonstrates these results.

2. Analysis

Suppose that consumers buy the goods from three monopolists successively (three is enough to demonstrate the results mentioned in the Introduction). The goods are complementary in the sense that only when they obtain all three goods do consumers enjoy the benefit from them. The demand for (the enjoyment of) the three goods is denoted by \( p(x) \), where \( x \) is quantity. We assume \( x \) is a real number and each consumer buys one unit from each monopolist. The demand function is continuously differentiable with derivatives \( p' < 0 \) and \( p'' \leq 0 \) at \( x > 0 \). Let \( p(0) < \infty \).

Begin with the analysis of the one-shot game, which later serves as the stage game of the dynamic model. The stage game proceeds as follows. Monopolist 1 sets the price \( p_1 \). Consumers observe \( p_1 \) and decide whether to buy the good (good 1). After they bought good 1, monopolist 2 sets the price \( p_2 \), and then consumers decide whether to buy
good 2 from monopolist 2. After consumers buy good 2, monopolist 3 sets the price $p_3$, and then consumers decide whether to buy from monopolist 3. Monopolist $i = 1, 2, 3$, faces the constant unit cost $c_i > 0$.

The model is solved backward. Suppose that, having bought goods 1 and 2, a consumer faces monopolist 3, who demand $p_3$. Then, the consumer’s net benefit is $p(x) - p_1 - p_2 - p_3$ if he buys good 3, and $-(p_1 + p_2)$ if he does not. Thus, a consumer buys from monopolist 3 as long as $p(x) \geq p_3$. That is, monopolist 3 faces the standard monopoly demand, and hence solves the problem: $\max_{x} \{p(x) - c_3\}x$ subject to the constraint that $x \leq x_2$, where $x_2$ is the number of consumers who bought goods 1 and 2.

Let $\tilde{x}_3$ denote the solution to the maximization problem. As is easily checked, if $x_2 > \tilde{x}_3$, the optimal price for monopolist 3 is $p(\tilde{x}_3)$, and if $x_2 \leq \tilde{x}_3$ consumers bought goods 1 and 2, the optimal price is $p(x_2)$. In either case, the consumers at the margin (those who have the valuation equal to $p_3$) have the negative surplus of $-(p_1 + p_2)$. Anticipating monopolist 3’s behavior, the marginal consumers would not buy goods 1 and 2 unless $p_1 = p_2 = 0$, which is impossible given the unit costs being positive. Thus, as the marginal consumers drop out, $x_2$ decreases, which prompts monopolist 3 to raise $p_3$, which causes more consumers to drop out. In the end sales disappear.\(^2\)

When interaction is repeated over time, however, there is an equilibrium in which there are positive sales. Let $x^* > 0$ be the equilibrium number of consumers who purchase

\(^2\) There is sale but it is of measure zero.
the goods from three monopolists every period. Assume that $x^* \leq \tilde{x}_3$. Assume also that consumers repeat purchases as long as they did not get a negative net surplus in any of the previous purchases. The monopolists discount the profit at the common discount factor $\delta \in (0, 1)$.

We first claim that in equilibrium monopolist 3 sets the price equal to

$$p_3 = p(x^*) - p_2 - p_1$$

so that the marginal consumer, with valuation $p(x^*)$, gets zero net benefit. To see this, observe that $p_3 < p(x^*) - p_2 - p_1$ cannot hold in equilibrium because given $x^*$ monopolist 3 can raise the price to increase profit. On the other hand, if $p_3 > p(x^*) - p_2 - p_1$, the marginal consumers do not make future purchases, as their current-period surplus is negative, and hence $x^*$ cannot be an equilibrium quantity.

Thus, monopolist 3’s equilibrium profit is $[p(x^*) - p_1 - p_2 - c_3]x^*$ per period. Summing up, we have $[p(x^*) - p_1 - p_2 - c_3]x^*/(1 - \delta)$. In equilibrium, monopolist 3 has no incentive to act myopically by raising the price to the optimum $p(x^*)$ for the one-shot game. The no-deviation condition is expressed as

$$[p(x^*) - p_1 - p_2 - c_3]x^*/(1 - \delta) \geq [p(x^*) - c_3]x^*.$$

Simplifying it to

$$p_1 + p_2 \leq \delta[p(x^*) - c_3]$$

we obtain monopolist 3’s best responses as follows:

---

This is justified shortly.
Suppose that monopolist 2 sets the price equal to 
\[ p_2 > \delta[p(x^*) - c_3] - p_1. \] 
Then, the consumer at the margin infers from (1b) that, once he buys the good from monopolist 2, monopolist 3 will act myopically, giving him the negative surplus – (\( p_1 + p_2 \)). Since he can secure the loss of only – \( p_1 \) by not buying good 2, this consumer will not buy the good from monopolist 2. Then the cascading effect sets in, and no consumers will buy from monopolist 2. Thus, the optimal price for monopolist 2 is
\[ p_2^* = \delta[p(x^*) - c_3] - p_1 \] 
(lowering the price reduces the profit, given \( x^* \)). Monopolist 2 must not make negative profit in equilibrium, so \( p_2^* \geq c_2 \), which is expressed, using (2), as
\[ p_1 \leq \delta[p(x^*) - c_3] - c_2. \]

Monopolist 1 maximizes the profit \((p_1 - c_1)x^*\) subject to (3). Again, should he violate the constraint (3), consumers would refuse to buy from him. Since the objective function is increasing in \( p_1 \), given the optimum \( x^* \), the optimal \( p_1 \) must make (3) hold with equality:
\[ p_1 = \delta[p(x^*) - c_3] - c_2. \]
Substituting from (4) into the profit function and maximizing it, we obtain the following first-order condition
\[ \delta[p(x^*) + x^*p'(x^*) - c_3] - c_2 - c_1 = 0. \]
which defines the equilibrium $x^*$. The left-hand side of (5) is negative at $\tilde{x}_3$, implying $x^* < \tilde{x}_3$ as we assumed.

We can now substitute the $x^*$ into (4), (2), and (1a) successively to find the equilibrium prices given below:

**Proposition:** The model with three monopolists has a stationary equilibrium, in which the equilibrium number of merchants is $x^*$ given by (5), and the equilibrium prices are

\[
p_1^* = \delta[p(x^*) - c_3] - c_2 > c_1
\]

\[
p_2^* = c_2
\]

\[
p_3^* = (1 - \delta)p(x^*) + \delta c_3 > c_3.
\]

The most striking result is that monopolist 2 makes zero profit. Intuitively, what prevents monopolist 3 from acting myopically is the sum of $p_1$ and $p_2$ not exceeding the threshold [see the condition (1a)]. Then, monopolist 1 can take advantage of the first-mover advantage vis-à-vis monopolist 2 to set the price $p_1$ as high as possible without violating the threshold. That means that monopolist 2 has no choice but set its price equal to the unit cost. This intuition generalizes straightforwardly to the case of more than 3 monopolists: with $N \geq 3$ monopolists only the first and the last monopoly capture the monopoly profit.

An immediate consequence of the proposition is that a vertical integration of monopolists excluding the last has no effect on the equilibrium price or social welfare.
By contrast, a partial integration that includes the last monopolist yields the expected result that the price is lower and welfare greater. However, the next example shows that such an integration need not be profitable for the integrating monopolists and hence may not occur despite its welfare-improving potential.

**Example:** Partial integration that reduces the combined profits

Consider the vertical integration of monopolists 2 and 3. Let the inverse demand be linear \( p = a - x \), where \( a \) is the demand intercept and \( a - c_1 - c_2 - c_3 > 0 \). It is straightforward to compute, using the results in the proposition, the equilibrium output

\[
x^* = \frac{1}{2}(a - c_2 - c_3/\delta - c_1/\delta)
\]

before the integration and

\[
x^{**} = \frac{1}{2}(a - c_2 - c_3 - c_1/\delta)
\]

after the integration. Before the integration, since monopolist 2 makes zero profit, the combined profits of monopolists 2 and 3 just equals the profit of monopolist 3:

\[
\pi^* \equiv (1 - \delta)(a - x^* - c_3)x^*.
\]

After the integration, the integrated monopolist has the unit cost \( c_2 + c_3 \), and hence the combined equilibrium profit is

\[
\pi^{**} \equiv (1 - \delta)(a - x^{**} - c_3 - c_2)x^{**}.
\]

Taking the difference,

\[
4(\pi^* - \pi^{**})/(1 - \delta) = (a - x^* - c_3)x^* - (a - x^{**} - c_3 - c_2)x^{**}.
\]

Substituting the equilibrium outputs, the right-hand side becomes
RHS = \((a - c_3 - c_2/\delta - c_1/\delta)(a - c_3 + c_2/\delta + c_1/\delta)\)

\[-(a - c_3 - c_2 - c_1/\delta)(a - c_3 - c_2 + c_1/\delta)\]

\[= (a - c_3)^2 - (a - c_3 - c_2)^2 - (1/\delta)^2(c_2 + c_1)^2 + (1/\delta)^2c_1^2\]

\[= c_2[2a - 2c_3 - (1 + 1/\delta^2)c_2 - 2c_1/\delta^2]\]

If the demand intercept, \(a\), is sufficiently large or \(\delta\) is not too small, the final expression is positive, meaning \(\pi^* > \pi^{**}\). Thus, the integration of the last two monopolists reduces the combined profit.

3. Concluding remarks

Many products are sold through a chain of intermediaries. If the intermediaries have monopoly power and if the prices cannot be contracted on, sales do not occur due to the holdup problem in a one-shot game. In this paper we show that the opportunity to interact over time can attenuate the type of holdup problem considered here. The model has a few surprising results. First, the monopoly profit is captured entirely by the first and the last monopolist while the other monopolists break even. Further, a vertical integration that does not involve the last monopoly has no effect on the equilibrium price and social welfare, whereas a vertical integration that includes the last monopoly may decrease the combined profit for the integrating monopolies, and hence may not be initiated despite its potential to improve social welfare.
References

