**Collusion and research joint ventures**

Kaz Miyagiwa*

**Abstract:** We examine whether cooperation in R&D leads to product market collusion. Suppose firms compete in a stochastic R&D race while maintaining the collusive equilibrium in a repeated-game framework. Innovation creates a cost asymmetry and destabilizes the collusive equilibrium. Firms forming an R&D joint venture can maintain cost symmetries through technology sharing agreement, thereby stabilizing collusion. The stability of post-discovery collusion makes collusion stable in pre-discovery periods. However, formation of R&D cooperatives may increase social welfare because firms share an efficient technology. Interestingly, a welfare improvement is less likely if innovation leads to a large cost reduction.

**JEL Classifications System Numbers:** L12, L13

**Keywords:** Oligopoly, Collusion, Research Joint Ventures, Innovation, R&D

Correspondence: Kaz Miyagiwa, Department of Economics, Emory University, Atlanta, GA 30322, U.S.A.; e-mail: kmiyagi@emory.edu; telephone: 404-727-6363, fax: 404-727-4639

---

* I am grateful to an anonymous referee and Yeon-koo Che, the Editor of This Journal, for their valuable comments and suggestions that led to substantial improvements. Thanks also go to Sue Mialon and Yuka Ohno for their comments on earlier versions. Errors are my own responsibility.
1. Introduction

This paper evaluates the age-old suspicion that cooperation in R&D leads to product market collusion. Prior to the 1960s this suspicion was so strong in the U.S. that antitrust authorities there threatened to punish any form of research joint ventures (RJVs) with full forces of antitrust laws. The sentiment abated during the 1960s and early 1970s, when key American industries were losing the competitive edge to foreign rivals that had made considerable technological progress through formation of RJVs.\(^1\) Although joint R&D activities among firms are encouraged everywhere today, the same old suspicion lingers: does cooperation in R&D facilitate product market collusion?\(^2\)

To investigate this question analytically, suppose that a group of ex ante symmetric firms manage implicitly to maintain a collusive equilibrium in an infinitely repeated-game framework. In such an environment, a firm that discovers a cost-cutting technology has a strong incentive to lower the price to increase its market share, thereby destabilizing the collusive arrangement. Further, the prospect that the collusion breaks down with a discovery of new technology destabilizes the collusion in pre-discovery periods as well. In contrast, if firms are allowed to form an RJV and share innovations, no firm has a cost advantage over others in post-discovery periods, which stabilizes collusion in pre-discovery periods. In short, cooperation in R&D leads to more collusion.

However, social welfare need not be lower under cooperative R&D, because all firms involved benefit from innovation as opposed to just one innovator as under non-cooperative R&D. The net welfare impact of R&D cooperation depends on the balance of

---

\(^1\) See Caloghirou, Ioannides and Vonortas (2003).
such efficiency gains against the welfare losses due to the collusive pricing. Although efficiency gains drive welfare improvements, surprisingly, social welfare is unlikely if innovation leads to a large cost reduction. The intuition is that, the smaller a cost reduction, the smaller a cost asymmetry, and hence the easier it is to maintain collusion. If firms can collude without cooperation in R&D, formation of an RJV does not exacerbate the price distortion, and hence the welfare change is dominated by the efficiency gains.

The present paper is of course not the first to address the question regarding cooperation in R&D and product market collusion. However, the literature is still scanty compared with a plethora of studies on relative effects of competitive and cooperative R&D. Martin (1995) uses a continuous-time version of a repeated-game framework to show, as in this paper, that cooperative R&D facilitates collusion. Contrary to our result, however, he argues that formation of RJVs reduces social welfare. The difference in welfare assessments lies with his assumption that collusion ends with a discovery. If innovation is non-drastic, collusion need not end with a discovery of new technology, in which case welfare can be increased by formation of RJVs.

While Martin (1995) focuses exclusively on the stability of collusion before innovation, the stability of collusion after innovation takes center stage in the work of Lambertini, Poddar and Sasaki (2002, 2003). In the 2002 article, which is more related to

---

2 For example, see the Federal Trade Commission’s Comment and Hearings on Joint Venture Project to witness its continuing ambivalence towards RJVs (http://www.ftc.gov/os/1997/jointven.htm).
3 Most analyses in this literature use atemporal models; e.g., D’Aspremont and Jacquemin (1988), and Kamien, Muller and Zang (1992). An intertemporal model is developed in Miyagiwa and Ohno (2002).
4 Cabral (2000) considers a similar model under the assumption that firms cannot observe each other’s effort, and shows that firms may set the price below the monopoly price to sustain collusion under cooperative R&D.
the present paper, the authors consider a three-stage game, in which two firms first decide whether to form a joint venture, then choose horizontal locations in the Hotelling-style product space, and finally choose to compete or collude in prices over time. In the second or R&D stage of the game, firms can select locations freely when acting competitively in R&D but are constrained to choose a single location when coordinating R&D activities as an R&D cooperative. Having to produce an identical product intensifies price competition and can make collusion more difficult to maintain, a result that contrasts with ours and Martin’s. Their model however assumes that firms make investment decisions simultaneously in a deterministic R&D environment, and offers no analysis of collusion in pre-discovery periods.5

A major difficulty that arises in the analysis of collusion in post-discovery periods is that there is no natural focal equilibrium due to cost asymmetries.6 A small but growing literature on collusion under cost heterogeneity typically assumes that firms maximize joint profits, and determines the unique equilibrium price and market-sharing rule by an appeal to the notion of balanced temptation equilibrium of Friedman (1971)7. Bae (1987) initiates this approach in his analysis of Bertrand duopoly, and Verboven (1997), Rothschild (1999) and Collie (2004) examine Cournot cases. This approach however is not without criticisms. Harrington (1991) argues that the hypotheses of joint profits

5 Lambertini, Poddar and Sasaki (2003) devlope a non-spatial model of product differentiation, where formation of the RJV is assumed and focus is on the firms’ (costly) choice of product substitutability for the maintenance of collusion in post-discovery periods.
6 This difficulty is absent in Martin (1995) because collusion ends with a discovery in his model, and in Lambertini, Poddar and Sasaki (2002) because R&D is non-stochastic and R&D decisions are made simultaneously.
7 This requires that the firms’ ratios of the per-period losses due to breakdown of the collusion over the maximum one-period gains from deviations be the same among firms.
maximization and balanced temptation equilibrium are both ad hoc, and develops an alternative approach based on Nash bargaining.

However, the Harrington (1991) approach may also be subject to a subtler criticism that it does not model the negotiation process explicitly. If it takes long and hard negotiations to come to an agreement, such a process is likely to raise suspicion in the watchful eyes of antitrust authorities and affect the equilibrium outcome. Furthermore, when applied to the current situation, both the Nash bargaining and the joint-profit maximization approach turn out intractable because of ambiguous comparative statics results with respect to cost changes. Therefore, in this paper we propose another approach, which may be call the price leadership hypothesis. Under this hypothesis, an innovator chooses a price and a market-share rule to maximizes his individual profit and makes a take-it-or-leave-it offer to the non-innovator. The price leadership hypothesis satisfies Harrington (1991) criticism, and is more tractable than his or the joint-profit maximization approach.

The remainder of this paper is organized in five sections. The next section gives an overview of the model and discusses the non-cooperative equilibrium. Section 3 establishes the conditions for the collusive equilibrium under competitive R&D. Section 4 is devoted to the analysis of the RJVs on firms’ incentive to collude. In Section 5 we relax some assumptions of the model and conclude in section 6.

2. Model

2.1 Setup
We consider repeated interactions between two a priori symmetric firms over an infinite time horizon. Time is discrete and indexed by $t \geq 1$. At $t = 1$ firms possess the common technology that enables them to produce homogeneous goods at the constant unit cost of $\bar{c}$. In any period $t \geq 1$, each firm decides whether to invest in R&D for the discovery of a new technology that reduces the unit cost to $c - (\bar{c})$. A firm investing in R&D incurs a fixed cost $k$ per period. R&D investment is risky in the sense that it fails with probability $\phi < 1$ per period. If both firms invest in R&D, a discovery occurs to either firm with probability $2\phi(1 - \phi)$ and to both with probability $(1 - \phi)^2$. In the case of simultaneous discoveries each firm has an equal chance of obtaining patent protection. Call a firm with the patent an innovator, and the other firm a non-innovator. Assume permanent patent protection for simplicity.

Firms are price-setters. Consumers buy from a firm offering a lower price. In case of ties, they buy from both firms equally so each firm captures half the market. Demand is stationary and is written $D(p)$, where $p$ is price. $D(p)$ is differentiable, with first and second derivatives denoted by $D'(p) < 0$ and $D''(p) \leq 0$. Let $p^m(c)$ be the unconstrained monopoly price when the unit cost is $c$, i.e., $p^m(c) = \text{argmax } D(p)(p - c)$. The conditions on demand make industry profit strictly concave so $p^m(c)$ is unique. It is easy to check that $p^m(c) < p^m(\bar{c})$.

---

8 The assumption of fixed-intensity R&D, adopted in Bloch and Markowitz (1996) and others, simplifies the analysis.

9 This assumption is often adopted in the literature; see Cardon and Sasaki (1998), for example.
Assume lastly that $\overline{c} < p^m(c)$, i.e., non-drastic innovation. If innovation is drastic, an innovator becomes a monopoly unthreatened by the non-innovator, and hence has no incentive to collude in post-discovery periods.

2.2 Non-collusive equilibrium

In the non-collusive game, each firm setting prices equal to $\overline{c}$ in every period regardless of histories is a subgame-perfect Nash equilibrium (SPNE). Denote this strategy profile by $\lambda$. Adopting this strategy, firms earn zero profit (excluding the investment cost $k$) before a discovery. After a discovery the innovator limit-prices the non-innovator and earns the per-period profit of

$$\pi_L = D(\overline{c})(\overline{c} - c) > 0$$

while the non-innovator receives zero profits.\textsuperscript{10}

Assuming that both firms invest in R&D, with probability $(1 - \phi^2)/2$ each firm has the chance of obtaining the exclusive right to use the new technology and earning the per-period profit $\pi_L$. With probability $\phi^2$ investments flop for both firms, putting them in exactly the same state the next period as they are currently. If we let $V_\lambda$ denote the present discounted sum of equilibrium profits, the recursive structure of the model leads to:

$$V_\lambda = -k + \delta(\pi_L/2)(1 - \phi^2)/(1 - \delta) + \delta^2 V_\lambda,$$

(1)
where $\delta (< 1)$ denotes a common discount factor. Collecting terms,

$$V_{\lambda} = \frac{-k + \delta(1 - \phi^2)(\pi_L / 2) / (1 - \delta)}{1 - \delta \phi^2}.$$  

Assume $V_{\lambda} > 0$ so investing in R&D is worthwhile for each firm.

3. Collusion with non-cooperative R&D

We now consider a class of trigger strategies with Nash threats that induce implicit collusion in pre-discovery and post-discovery periods. We begin with non-cooperative R&D.

3.1. Collusion in post-discovery periods

In post-discovery periods firms have asymmetric costs. As stated in Section 1 we focus on the equilibrium based on the price leadership hypothesis, under which the innovator chooses a price and a market-share rule to maximize his profit and makes a take-it-or-leave-it offer to the non-innovator. Let $(p^c, s)$ denote such an offer, where $s (0 < s < 1)$ is the fraction of market served by the innovator.

The price leadership hypothesis amounts to saying the following. As soon as he acquires new technology, the innovator communicates his optimal price and market-share rule to the non-innovator and executes this decision in the first post-discovery period. If the non-innovator responds with the same price and the limited sales, then the collusion is on. Otherwise, the innovator believes that the latter is uninterested in colluding and starts

---

10 Strictly speaking, the innovator sets a price slightly below $c$ to capture the entire market.
competing. Formally, we consider the following strategy profile. Given that there is a discovery in period $\tau \geq 1$, in period $\tau + 1$ firms set a price equal to $p^c$ and split the market according to the market-sharing rule $s$. In $t \geq \tau + 2$, they choose $(p^c, s)$ if no other outcomes than $(p^c, s)$ have been observed since $\tau + 1$; otherwise they adopt the non-collusive strategy $\lambda$ forever. Denote this strategy profile by $a$ (a mnemonic for “after” a discovery).

We look for the condition that makes $a$ subgame-perfect in post-discovery games. If firms adopt $a$, every (post-discovery) subgame belongs to one of the two classes; one in which all past outcomes have been $(p^c, s)$, and one in which another outcome has been observed at least in one post-discovery period. In the latter, $\lambda$ is subgame-perfect, so we need only to show that $a$ is subgame-perfect in the first class of subgames.

Along this collusive equilibrium path, the innovator earns the per-period profit of

$$\pi_i = sD(p^c)(p^c - c)$$

while the non-innovator earns

$$\pi_n = (1 - s)D(p^c)(p^c - c).$$

Since $p^c$ maximizes $\pi_i$, put $p^c = m(c)$. Then, we can write the above profits as

$$\pi_i = sm, \text{ and } \pi_n = (1 - s)D[m(c)][p^m(c) - c],$$

where $m$ denotes the monopoly profit:

$$m = D[p^m(c)][p^m(c) - c].$$
Let $v_i$ and $v_n$ denote the sums of equilibrium profits for the innovator and the non-innovator, respectively. The recursive structure implies

$$v_i = \pi_i + \delta v_i, \quad \text{and} \quad v_n = \pi_n + \delta v_n,$$

and hence:

$$v_i = \pi_i/(1 - \delta) \quad \text{and} \quad v_n = \pi_n/(1 - \delta).$$

Now consider a one-period deviation. A non-innovator earns $D[p^m(c)][p^m(c) - \overline{c}]$ one period but loses all future profits as it gets limit-priced. He thus has no incentive to deviate if $v_n \geq D[p^m(c)][p^m(c) - \overline{c}]$, which simplifies to

$$\delta \geq s. \quad (2)$$

On the other hand, a devious innovator can earn $m$ one period and $\pi_L$ in all subsequent periods. Therefore, the innovator has no incentive to deviate if

$$v_i \geq m + s\pi_L/(1 - \delta),$$

or

$$\delta \geq (1 - s)m/(m - \pi_L).$$

Thus, a is subgame-perfect in post-discovery games if

$$\delta \geq \max \{(1 - s)m/(m - \pi_L), s\}. \quad (3)$$

Now, the innovator chooses $s$ to maximize $sm$ subject to the non-innovator's incentive compatibility condition (2). This puts $s = \delta$. Substituting $s = \delta$ in (3) and rearranging yields
(4) \[ \delta \geq \frac{m}{2m - \pi_L} \equiv \delta^A(\epsilon) > 1/2. \]

The strategy profile \( \mathbf{a} \) is a SPNE if and only if (4) is satisfied. Observe that \( \delta^A(\epsilon) > 1/2 \), implying that a cost symmetry makes collusion more difficult to maintain.

Define \( c^d \) by \( \bar{c} = p^m(c^d) \). Then \( \epsilon > c^d \) implies innovation is non-drastic. This next proposition summarizes what we have found so far.

**Proposition 1**: Assume \( c^d < \epsilon < \bar{c} \).

(i) The strategy profile \( \mathbf{a} \) is a SPNE if and only \( \delta \geq \frac{m}{2m - \pi_L} \equiv \delta^A(\epsilon) > 1/2. \)

(ii) The collusive equilibrium price and market-sharing rule are

\[ p^e = p^m(\epsilon) \text{ and } s = \delta > 1/2. \]

Since \( s > 1/2 \), the innovator has a greater market share than the non-innovator, a result that is consistent with the findings of Bae (1987) and Harrington (1991).

The per-period equilibrium profits are

\[ \pi_i = \delta m \text{ and } \pi_n = (1 - \delta)D[p^m(\epsilon)][p^m(\epsilon) - \epsilon]. \]

Observe that the equilibrium profits to each firm are sensitive to the prevailing discount factor, whereas in Bae (1987) and Harrington (1991) they are independent of it as long as the discount factor exceeds the threshold level.
The locus AA in Figure 1 plots $\delta^A(c)$ against $c$. The depiction reflects the fact that:

$$\frac{\partial \delta^A(c)}{\partial c} = D(\bar{c})D[p^m(c)][\bar{c} - p^m(c)]/(2\bar{m}_L^2 - \pi_L^2) < 0$$

for $p^m(c) > \bar{c}$ (i.e., non-drastic innovation). It is easy to check that $\delta^A(c)$ increases towards unity as $c$ falls towards $c^d$, while in the other direction it approaches $1/2$ as $c$ approaches $\bar{c}$.

Proposition 1 says that if $\delta < \delta^A(c)$ firms cannot collude at the monopoly price $p^m(c)$, but it leaves open the question whether they can collude at another price. However, such partial collusion is impossible here. Intuitively, because the market-sharing rule is sensitive to the discount factor in the present model, the innovator cannot commit credibly to his offer when the discount factor falls below the critical level $\delta^A(c)$.

(The proof in Appendix A.)

**Lemma 1.** If $\delta < \delta^A(c)$, partial collusion is impossible in post-discovery subgames.

3.2. Collusion in pre-discovery periods

---

11 We limit analysis to $\delta \geq 1/2$. 
We now turn to the stability of collusion in pre-discovery periods. Consider the following symmetric strategy: In \( t = 1 \), set a price equal to \( p^m(\bar{c}) \), the monopoly price under the old technology. In any pre-discovery period \( t \geq 2 \), there are four possible states of nature.

(i) No other prices than \( p^m(\bar{c}) \) have been observed and there was a discovery in \( t - 1 \).

(ii) No other prices than \( p^m(\bar{c}) \) have been observed and there has been no discovery to date.

(iii) Prices other than \( p^m(\bar{c}) \) have been observed at least once in the past and there was a discovery in \( t - 1 \).

(iv) Prices other than \( p^m(\bar{c}) \) have been observed at least once in the past and there has been no discovery to date.

In state (i), adopt \( a \). In state (ii) set the price equal to \( p^m(\bar{c}) \). In states (iii) and (iv) adopt \( \lambda \). Call this strategy profile \( b \) (a mnemonic for “before” a discovery).

Since \( b \) is subgame-perfect in states (i), (iii) and (iv), we need only to check state (ii). In state (ii) the equilibrium payoff, denoted by \( V_c \), satisfies this recursive equation

\[
V_c = \frac{\bar{m}}{2} - k + \delta(1-\phi^2)(v_i + v_n)/2 + \delta\phi^2 V_c,
\]

where \( \bar{m} \) denotes the monopoly profit under the old technology; i.e.,

\[
\bar{m} = D[p^m(\bar{c})][p^m(\bar{c}) - \bar{c})].
\]

Collecting terms in (5) yields
\[ V_c = \frac{m/2 - k + \delta(1 - \phi^2)(v_i + v_n)/2}{1 - \delta\phi^2}. \]

Consider now a one-period deviation. A deviating firm earns \((\bar{m} - k)\) but finds itself in state (iii) or state (iv) the next period, with a switch to \(\lambda\), thereby receiving the following expected profits:

\[ \bar{m} - k + \delta(1 - \phi^2)(\pi_L/2)/(1 - \delta) + \delta\phi^2 V_\lambda = \bar{m} + V_\lambda \]

where (1) is used to get to the right-hand side. A deviation is unprofitable if this profit is less than \(V_c\), i.e.,

(6) \[ V_c - V_\lambda \geq \bar{m}, \]

where

(7) \[ V_c - V_\lambda = \frac{\delta(1 - \phi^2)[v_i + v_n - \pi_L/(1 - \delta)]/2 + \bar{m}/2}{1 - \delta\phi^2}. \]

The difference in profits, \(V_c - V_\lambda\), between the collusive and the competitive paths increases without bounds as \(\delta\) goes to unity, so (6) holds with strict inequality for a high enough \(\delta < 1\). On the other hand, when \(\delta\) is sufficiently close to 1/2, (6) fails, as shown in Appendix B. Therefore, there exists a unique \(\delta \in (1/2, 1)\) at (6) holds with strict equality, which we denote by \(\delta^B_c\).

The locus BB (comprising the thick and dotted segments) in Figure 1 plots \(\delta^B_c(c)\) against \(c\), assuming that firms maintain the collusive equilibrium in post discovery
periods and that $\phi^2 \geq 1/2$. The depiction is based on the following lemma (see Appendix C for a proof):

**Lemma 2:** Given that $c^d < \underline{c} < \bar{c}$

(i) $\partial \delta^B(c)/\partial \underline{c} < 0$

(ii) $1/2 < \delta^B(c) < 1/(2\phi^2)$.

(iii) If $\phi^2 \geq 1/2$, there is a point $(\underline{c}, \delta(c))$ at which the loci AA and BB intersect.

Thus, the locus BB curves upward as $c$ falls but stays strictly between $1/2$ and $1/(2\phi^2)$. If $\phi^2 < 1/2$, however, the upper bound exceeds unity, implying that the locus BB may stay above the locus AA for all $c > c^d$.

Finally, the analysis of this subsection is predicated on there being collusion in post-discovery periods, i.e., $\delta \geq \delta^A(c)$. Thus, we have

**Proposition 2:** If $\delta \geq \max \{\delta^A(c), \delta^B(c)\} > 1/2$, the strategy profile $b$ is subgame-perfect and entails collusion in pre-discovery and post-discovery periods.

---

12 This establishes the existence. There may be more than one such point. However, the results we show below do not depend on the uniqueness.
The prospect that collusion is less stable in post-discovery periods due to a cost asymmetry makes collusion more difficult to maintain in pre-discovery periods, during which costs are still symmetric. In terms of Figure 1, collusion is sustainable before and after a discovery if and only the \((\delta, c)\) pair is in Region 1 defined by the set \(\{(\delta, c) | \delta \geq \max \{\delta^A(c), \delta^B(c)\}\}\). Outside Region 1, the strategy profile \(b\) cannot support collusion. In Region 2 defined by the set \(\{(\delta, c) | \delta^A(c) \leq \delta < \delta^B(c)\}\), for example, firms can maintain full collusion after a discovery but not before because the monopoly price \(p^m(c)\) cannot satisfy the no-deviation condition (6). The question is: can firms collude partially, that is, at a price different from the monopoly price \(p^m(c)\) until there is a discovery? As in the post-discovery game, the next lemma shows they cannot (the proof in Appendix D).

**Lemma 3.** If \(\delta^A(c) \leq \delta < \delta^B(c)\) there is no partial collusion in pre-discovery periods.

Thus, in Region 2, although they cannot collude in pre-discovery periods, firms can fully collude in post-discovery periods by playing a competitive one-shot game until there is a discovery and then switch to playing \(a\). This is a SPNE, yielding zero profits (minus investment cost \(k\)) until a discovery and \(m/2\) per period afterwards.

\[\text{It is always possible to maintain collusion at prices below the monopoly price outside Region 1. To keep the analysis compact such partial collusion is ruled out, so firms choose either full collusion (at the monopoly prices) or competition.}\]
Outside of Regions 1 and 2, firms cannot collude partially after a discovery. Given that they play a limit-price game after a discovery, however, firms may still be able to collude up to a discovery by playing (the pre-discovery components of) $b$ until a discovery and then switching to $\lambda$. Since firms adopting this strategy play the limit-pricing game after a discovery, the first term in the numerator of (7) vanishes, and the condition to support collusion in pre-discovery period is given by

$$\frac{(\bar{m}/2)/(1 - \delta\phi^2)}{1 - \delta\phi^2} > \bar{m},$$

instead of by (6). This inequality holds if $\delta \geq 1/(2\phi^2)$. This condition is satisfied if the $(\delta, c)$ pair is in Region 3 of Figure 3 defined by \{$(\delta, c)$| $1/(2\phi^2) \leq \delta < \delta^A(c)$\}, which is non-empty if $\phi$ is greater than $\sqrt{2}/2$. Outside all these regions firms cannot collude at all.

4. Research joint ventures

We interpret the RJV broadly to encompass any technology-sharing arrangement including a royalty-free cross-licensing agreement, under which each firm runs its own research lab, incurs own R&D costs and gains free access to any innovations made by partners. We thus assume that each firm retains its R&D facility and shares technology with each other, as is commonly assumed in the RVJ literature.\textsuperscript{14}

4.1 Collusion in post-discovery periods

\textsuperscript{14} See Kamien, Muller and Zang (1992) and Miyagiwa and Ohno (2002), for example.
Suppose there is a discovery in period $\tau \geq 1$ and the firms adopt the following post-discovery strategy, denoted by $\alpha$. In $\tau + 1$, set a price equal to the monopoly price $p^m(c)$ under the new technology. In all $t + \tau$, $(t \geq 2)$, choose $p^m(c)$ if no other prices than $p^m(c)$ have been observed since $\tau + 1$; otherwise set a price to $c$. Thus, $\alpha$ is a standard collusive strategy profile for symmetric price-setting duopoly and is a SPNE for $\delta \geq 1/2$. Compared with Proposition 1, this result shows that formation of an RJV facilitates collusion in post-discovery periods by preventing a cost symmetry from arising, which tends to stabilize collusion before a discovery. The question is how low the threshold discount factor falls. We turn to this question next.

4.2 Collusion in pre-discovery periods

Consider the following collusive strategy denote by $\beta$:

In $t = 1$, set a price equal to the monopoly price $p^m(\bar{c})$. In any pre-discovery period $t \geq 2$, there are four possible states of nature.

(i) No other prices than $p^m(\bar{c})$ have been observed, and there was a discovery in $t - 1$.

(ii) No other prices than $p^m(\bar{c})$ have been observed and there has been no discovery to date.
(iii) Prices other than $p^m(c)$ have been observed at least in one period in the past and 
there was a discovery in $t - 1$.\textsuperscript{15}

(iv) Prices other than $p^m(c)$ have been observed at least once in the past and there has 
been no discovery to date.

In state (i) adopt $\alpha$. In state (ii) set a price equal to $p^m(c)$. In state (iii) set a price equal to 
c in every period. In state (iv), withdraw from the RJV and switch to playing $\lambda$ forever.

The strategy profile $\beta$ is subgame-perfect in states (i), (iii) and (iv), so we only 
need to check state (ii). In that state, $V_J$, the equilibrium profit per firm satisfies the 
following recursive equation:

$$V_J = \frac{m}{2} - k + \delta(1 - \phi^2)(m/2)/(1 - \delta) + \delta\phi^2 V_J.$$

Collecting terms, we obtain

$$V_J = \frac{m/2 - k + \delta(1 - \phi^2)(m/2)/(1 - \delta)}{1 - \delta\phi^2}.$$

A one-period deviation raises a devious firm’s profit to $\bar{m}$ but puts firms in states 
(iii) or (iv) the next period, depending on whether there is a discovery during the period 
in question. In state (iii), which occurs with probability $(1 - \phi^2)$, the innovation is shared 
but a switch to playing the one-shot symmetric Bertrand game forever wipes out all 
future profits. In state (iv), which arises with probability $\phi^2$, firms switch to the non-

\textsuperscript{15} Here, a subtle question arises: who owns the innovation when a deviation occurs. We assume that 
innovation is shared since it has occurred before the breakup of the RJV.
collusive strategy \( \lambda \), which has the prevent value of \( V_\lambda \). Thus, a deviation yields the profit of

\[
\bar{m} - k + \delta \phi^2 V_\lambda
\]

so a firm has no incentive to deviate if

\[
V_J \geq \bar{m} - k + \delta \phi^2 V_\lambda
\]

After arranging terms, this condition can be rewritten

\[
(8) \quad \delta (1 - \phi^2) (m/2)/(1 - \delta) + \delta \phi^2 (V_J - V_\lambda) \geq \bar{m}/2
\]

where

\[
V_J - V_\lambda = \frac{\delta (1 - \phi^2)(m/2 - \pi_L/2)/(1 - \delta) + \bar{m}/2}{1 - \delta \phi^2}
\]

Differentiation shows that the left-hand side of (8) is increasing in \( \delta \). Further, we prove, in Appendix E, that (8) holds with strict inequality at \( \delta = 1/2 \). Thus, (8) holds with strict inequality for all \( \delta \geq 1/2 \). Hence,

**Proposition 3.** The strategy \( \beta \) is subgame perfect for \( \delta \geq 1/2 \)

Thus, formation of an RJV lowers the threshold discount factors both in pre-discovery and in post-discovery periods to \( 1/2 \) for all \( c > c^d \). A comparison with the non-cooperative case, in which the threshold discount factor exceeds \( 1/2 \), indicates that
cooperative R&D leads to collusion everywhere outside Region 1. It is in this sense that formation of an RJV facilitates collusion.

**Proposition 4.** Formation of an RJV facilitates collusion for all $c > c^d$ if

$$1/2 \leq \delta < \max \{\delta^A(\cdot), \delta^B(\cdot)\}.$$

4.3 Welfare and policy implications

Collusion lowers social welfare as firms set the monopoly price. However, it should not be inferred from Proposition 4 that cooperation in R&D should be banned or penalized as in earlier days, for it can increase social welfare by making all firms involved more efficient instead of just one innovator as under competitive R&D. To find a net welfare impact of cooperation in R&D, both these factors must be taken into consideration.

In Region 1 of Figure 1, firms collude before and after a discovery without cooperation in R&D. Therefore, formation of an RJV does not exacerbate the market distortions, and leads to an welfare improvement through the technology-sharing effect. In Region 2, firms manage to maintain collusion only after a discovery without cooperation in R&D. Then, by the preceding argument, formation of an RJV must improve social welfare in post-discovery periods. However, welfare falls in pre-discovery periods falls when firms learn to collude. The net welfare impact of cooperative R&D is in general ambiguous, therefore. In Region 3, firms can collude only before a discovery under non-cooperative R&D. Cooperation in R&D therefore generates
no welfare change in pre-discovery periods, but a welfare loss in post-discovery periods as the market gets monopolized. Thus cooperation in R&D lowers social welfare. Similarly, welfare falls unambiguously outside the three regions.

To sum, formation of an RJV can increase social welfare only if the collusive equilibrium is maintained in post-discovery periods in the absence of cooperation in R&D. Thus we have a counterintuitive result: a cost-cutting technology drives welfare improvements under cooperative R&D, but welfare is likely to fall when there is too sharp a cost reduction under new technology. This has the obvious policy implication for antitrust authorities: firms should be more closely monitored for anticompetitive behavior when they form an RJV aiming for a major technological breakthrough.

5. Extensions

In section we relax some of the assumptions of the model. They are (i) non-drastic innovation, (ii) permanent patent protection, and (iii) Nash threats (punishment by reversions to repeated play of a one-shot game).

First, suppose innovation is drastic as in Martin (1995). With drastic innovation an innovator becomes a monopoly so there is no collusion in post-discovery periods. Thus, the analysis is similar to the one associated with Region 3; namely, formation of an RJV facilitates collusion in post-discovery periods and lowers social welfare, which is exactly what Martin (1995) has argued.

Second, suppose that patent life is finite. If patent life is, say, T periods, the value of new technology to the innovator falls from \( \frac{\pi_L}{(1 - \delta)} \) under permanent patent
protection to $\pi_L (1 - \delta^T)/(1 - \delta)$. In the collusive equilibrium finite patent life thus decreases the value of a deviation, thereby making collusion easier to maintain both before and after a discovery.

Third, it is well known in the implicit collusion literature that collusion can be sustained for a lower range of discount factors if firms can commit to a severer punishment scheme than Nash reversions. In a recent paper Thal (2006) considers such a scheme for Bertrand duopoly with asymmetric costs and finds that a credible punishment strategy with an Abreu (1986, 1988) stick-and-carrot structure reduces the payoff to the firm with lowest cost to zero. Although not concerned with the uniqueness of equilibrium selection, when applied to our model, her analysis implies that there is an optimal punishment scheme that can reduce the threshold discount factor after a discovery to 1/2 without formation of an RJV. However, it is not clear whether the threshold discount factor also falls to 1/2 in pre-discovery periods. We show in Appendix F, however, that that is the case under the assumption of linear demand. In that case, firms can collude before and after a discovery at any discount factor greater or equal to 1/2 without forming an RJV, meaning cooperation in R&D always improve social welfare.

6. Summary

We examine whether cooperation in R&D leads to product market collusion. Our basic model has firms managing implicitly to maintain the collusive equilibrium while engaged in a stochastic R&D race. Under competitive R&D innovation gives rise to an

---

16 In period $T + 1$ the technology becomes public and competition wipes out profits for the innovator.
inter-firm cost asymmetry and can destabilizes collusion when the discount factor is low or the cost reduction under new technology is large. The prospect that collusion ends with innovation further destabilizes collusion in pre-discovery periods. Cooperation in R&D preserves a cost symmetry through innovation sharing, and leads to more collusion in product markets. However, cooperation in R&D does not necessarily decrease social welfare, as sharing of new technology improves efficiency in production. Although new technology is the driving force for a welfare improvement, cooperation in R&D is more likely to decrease welfare if the cost falls too much under new technology.
Appendices

Appendix A: Proof of Lemma 1. Let $\delta < \delta^A(c)$ be given. Suppose there is a pair $(p^*, s^*)$, where $p^* \neq p^m(c)$, such that $p^*$ maximizes the innovator’s profit $s^*m^* = s^*D(p^*)(p^* - c)$ and satisfies the no-deviation constraints for both the innovator and the non-innovator.

Case 1. $p^* < p^m(c)$

Partial collusion is stable if

$$\delta \geq \max \left\{ \frac{(1 - s^*)m^*/(m^* - \pi_L)}{s^*} \right\}$$

If $\delta \geq (1 - s^*)m^*/(m^* - \pi_L) > s^*$, $s^*$ can be increased up to $\delta$, increasing the profit to the innovator, without violating the no-deviation constraint. So, at the optimum the innovator sets $s^* = \delta$. Therefore, we have

$$\delta \geq (1 - \delta)m^*/(m^* - \pi_L)$$

or

$$\delta \geq m^*/(2m^* - \pi_L).$$

But the right-hand side is decreasing in $m^*$ for $0 \leq m^* \leq m$, and hence

$$\delta \geq m^*/(2m^* - \pi_L) > m/(2m - \pi_L) = A^A(c),$$

which contradicts the assumption $\delta < \delta^A(c)$.

Case: $p^m(c) < p^* < p^m(c)$

There is no deviation if
\[ \delta \geq \max \{ (m - s*m^*)/(m - \pi_L), s^* \}. \]

Again, \( s^* \) can be increased up to \( \delta \) without violating this constraint so \( \delta = s^* \). We can write the above as

\[ \delta \geq (m - \delta m^*)/(m - \pi_L). \]

that is,

\[ \delta \geq m/(m - \pi_L + m^*). \]

But since \( m^* < m \)

\[ \delta \geq m/(m - \pi_L + m^*) > m/(2m - \pi_L) = \delta^A(c), \]

a contradiction.

Case 3: \( p^* \geq p^m(c) \).

The innovator does not deviate if

\[ \delta \geq \bar{m} - (1 - s^*)\bar{m}^*/\bar{m}, \]

and the non-innovator does not if

\[ \delta \geq (m - s^*m^*)/(m - \pi_L). \]

Partial collusion is sustained if both conditions hold. The first must hold with equality, for otherwise the innovator can increase profit by raising \( s^* \). Therefore, \( s^* \) satisfies

\[ (1 - \delta)\bar{m} = (1 - s^*)\bar{m}^*. \]

Using this the second condition can be writte

\[ \delta(m - \pi_L) \geq (m - s^*m^*) = m + (1 - \delta)\bar{m} - \bar{m}^*. \]
Collecting terms,

$$\delta \geq [(m + \bar{m} - \bar{m}^\ast )/(m - \pi_L + \bar{m}) > m/(2m - \pi_L) = \delta^A(c),$$

a contradiction. □

Appendix B: We show that (6) fails at $\delta = 1/2$. Assume the contrary, and evaluate (6) at $\delta = 1/2$ to obtain this equivalent condition:

$$(A1) \quad m + D[p^m(c)][p^m(c) - \bar{c}] - \pi_L - 2\bar{m} \geq 0.$$ However, we can express the left-hand side of (A1) as

$$D[p^m(c)][2p^m(c) - \bar{c} - \bar{c}] - D(\bar{c})(\bar{c} - c) - 2D[p^m(\bar{c})][p^m(\bar{c}) - \bar{c})$$

$$< 2D(\bar{c})[p^m(c) - \bar{c}] - 2D[p^m(\bar{c})][p^m(\bar{c}) - \bar{c})$$

$$< 0,$$

where the first inequality is obtained from substitution of $D(\bar{c})$ for $D[p^m(c)]$ while the second follows the fact that $p^m(\bar{c})$ is the profit-maximizing price at cost $\bar{c}$. This contradiction is what we wanted. □

Appendix C: We prove Lemma 2. Proof of Result (i) $d\delta^B(c)/d\bar{c} < 0$. Write $\delta^B(c) = \delta^B$ to save space, $\delta^B$ is implicitly defined by (6) or satisfies this implicit function

$$g(\delta^B, c) \equiv V_J - V_\lambda - \bar{m} = 0.$$
where \( V_J - V_\lambda \) is given by (7). Write \( g(\delta^B, \varepsilon) = g \). Straightforward differentiation of (7) shows that \( \partial g/\partial \delta^B > 0 \). On the other hand,
\[
\text{sgn } \{\partial g/\partial \varepsilon\} = \text{sgn } \{\partial(V_J - V_\lambda)/\partial \varepsilon\} = \text{sgn } \{\partial(\pi_i + \pi_n - \pi_L)/\partial \varepsilon\}
\]
The last derivative is
\[
- \delta^B D[p_m^m(\varepsilon)] + D(\bar{c}) > 0
\]
since \( p_m^m(\varepsilon) > \bar{c} \). Therefore, by the implicit-function theorem we conclude that
\[
\frac{d\delta^B(\varepsilon)}{d\varepsilon} = - \frac{(\partial g/\partial \varepsilon)}{(\partial g/\partial \delta^B)} < 0.
\]

Proof of Result (ii): As \( \varepsilon \) approaches \( \varepsilon^d \), \( V_c - V_\lambda \) approaches \((\bar{m}/2)(1 - \delta \phi^2)\). Therefore, if \( \delta > 1/(2\phi^2) \), (6) holds with strict inequality at the limit \( \varepsilon = \varepsilon^d \), that is,
\[
(\bar{m}/2)(1 - \delta \phi^2) > \bar{m}.
\]
This implies that \( \delta^B(\varepsilon) \) is bounded from above by \( 1/(2\phi^2) \).

Proof of Result (iii). Suppose that \( \phi^2 \geq 1/2 \). Then \( 1/(2\phi^2) \leq 1 \). Hence, \( 1 > \delta^B(\varepsilon) > 1/2 \) by Result (ii) of this lemma. On the other hand, \( \delta^A(\varepsilon) \) approaches unity as \( \varepsilon \) approaches \( \varepsilon^d \), and approaches \( 1/2 \) as \( \varepsilon \) nears \( \bar{c} \). Thus, the two loci cross each other.

Appendix D. We prove Lemma 3: Let \( \delta \) be given such that \( \delta^A(\varepsilon) \leq \delta < \delta^B(\varepsilon) \). Suppose there is a price \( p^* \neq p_m^m(\bar{c}) \) and the total profit \( \bar{m}^* = D(p^*)(p^* - \bar{c}) < \bar{m} \) satisfying.
\[
\delta(1 - \phi^2)[v_i + v_n - \pi_L/(1 - \delta)]/2 + \bar{m}*/2 \\
1 - \delta\phi^2 
\geq \bar{m}*
\]

Byt firms cannot collude fully so

\[
\bar{m} > \frac{\delta(1 - \phi^2)[v_i + v_n - \pi_L/(1 - \delta)]/2 + \bar{m}/2}{1 - \delta\phi^2}.
\]

Adding and simplifying

\[
[\bar{m} - \bar{m}*/2(1 - \delta\phi^2)] \geq \bar{m} - \bar{m}^*
\]

which holds only if

\[
\delta > 1/(2\phi^2).
\]

By Lemma 2,

\[
1/(2\phi^2) > B^\delta(\zeta).
\]

These two imply \(\delta > B^\delta(\zeta)\), a contradiction. \(\Box\)

**Appendix E:** We show that (8) holds with strict inequality at \(\delta = 1/2\). First we show:

\((D1)\)

\[
m - \pi^L - \bar{m}
\]

\[
= D[p^m(\zeta)][p^m(\zeta) - \zeta] - D(\bar{c})(\bar{c} - \zeta) - D[p^m(\bar{c})][p^m(\bar{c}) - \bar{c})
\]

\[
> D[p^m(\zeta)][p^m(\zeta) - \zeta] - D(\bar{c})(\bar{c} - \zeta) - D(\bar{c})[p^m(\bar{c}) - \bar{c})
\]

\[
= D[p^m(\zeta)][p^m(\zeta) - \zeta] - D(\bar{c})[p^m(\bar{c}) - \zeta) > 0.
\]

Second, evaluate \(V_J - V_\lambda\) at \(\delta = 1/2\) to obtain
\[ V_J - V_\lambda = \frac{(1 - \phi^2)(m - \pi_L) + \bar{m}}{2 - \phi^2}. \]

Subtract \( \bar{m} \) from it.

(D2) \[ V_J - V_\lambda - \bar{m} = (1 - \phi^2)(m - \pi_L - \bar{m})/(2 - \phi^2) > 0 \]

by (D1). Finally, evaluate the left-hand side of (8) at \( \delta = 1/2 \) to obtain

(D3) \[ (1 - \phi^2)m/2 + \phi^2(V_J - V_\lambda)/2 \]

\[ > (1 - \phi^2)m/2 + \phi^2\bar{m}/2 \]

\[ > \bar{m}/2, \]

where the first inequality uses (D2). But (D3) shows that (8) holds with strict inequality at \( \delta = 1/2. \]

Appendix F: Since he receives zero profits following a deviation, the innovator has no incentive to deviate if \( sm/(1 - \delta) \geq \bar{m} \), or \( s \geq 1 - \delta \). Likewise, the non-innovator does not deviate if \( s \leq \delta \). Thus, \( 1 - \delta \leq s \leq \delta \). At \( \delta = 1/2 \), this means \( s = 1/2 \). Now, modify the pre-discovery components of \( b \) as follows. In state (iv) firms set price equal to \( \bar{c} \) until a discovery, after which they switch to the stick-and-carrot strategy described by Thal (2006). Then, a one-period deviation before a discovery yields zero profits (excluding \( k \)) past the period in which a deviation occurs, regardless of histories. It follows that a
deviant firm chooses not to invest in R&D investment. With this modification, (6) becomes

\[ V_c \geq \bar{m}. \]

Substituting \( \delta = 1/2 \) and canceling terms, we can write this condition as

\[ m + D[p^m(c)][p^m(c) - \bar{c}] - 2\bar{m} \geq 0. \]

This condition may or may not hold in general but it does hold under the assumption of linear demand as can easily be confirmed by directly evaluation of the profits. If it does, then firms can maintain collusion before and after innovation at any \( \delta \geq 1/2 \) as under cooperative R&D. \( \Box \)
References


Thal, J., Optimal collusion under cost asymmetry, unpublished paper.

Figure 1