Employers’ Preference for Discrimination

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Abstract

This paper models employers’ preference for discrimination toward ex ante identical groups of workers when the workers must compete for limited positions. Employers benefit from discrimination against minority workers because it can reduce the overall risk from workers’ noisy signals as it increases the expected quality of "majority" workers’ signals and their chance to win the competition for the limited positions. We show that employers can influence the selection of a discriminatory equilibrium by choosing the set of finalists in competition primarily from a majority group. We discuss the implications of Equal Opportunity Laws in this context.

Keywords and Phrases: Statistical Discrimination, Competitive Signaling, Group inequality, Asymmetric information, Cross-Group Risks

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1 Introduction

This paper models discrimination in an economy in which job opportunities are limited for workers, as there are fewer available positions than job candidates. In this case, the workers are not always guaranteed a return on their investment in their human capital. To each worker, the possibility of receiving a return depends on the decision of other competing workers to make the qualifying efforts.\textsuperscript{1} Thus, this feature makes the workers interact strategically in their decision to invest, based on their expected competitive advantage over the other workers.

We find that competition among workers promotes employers’ preference for discrimination. The reason for this is that with discrimination, employers can enhance the expected quality of signals from selected workers when the signals about workers’ ability are noisy. In the case of competition, what matters is the quality of winning workers’ signals. Discrimination generally increases the signaling incentive of the workers in an advantaged group and thus enhances the quality of their signals, whereas it lowers the incentive of disadvantaged group workers. This implies that discrimination will enhance the overall quality of signals as long as the effect on the advantaged group is significant and their probability of winning increases as a result. This indicates when discrimination is more likely to occur and, if it does occur, who should be the advantaged group. If one group’s presence is dominant in the population, workers from the majority group are more likely to be present in any given competition. Then, discrimination in favor of them is likely to enhance their probability of winning and the overall quality of signaling in any competition. Therefore, employers are likely to prefer discrimination that favors the majority group.

The novelty of this paper is to show that discrimination of \textit{ex ante} identical workers arises as a result of employers’ preference for discrimination and not merely as a result of one of many possible equilibria based on a self-fulfilling prophesy. To our knowledge, this paper is the first to show how employers can actively select a path that leads to a discriminatory equilibrium out of \textit{ex ante} identical workers and why discrimination needs to be tied to a group’s minority standing. As it is competition among workers that induces employers’ preference for discrimination toward \textit{ex ante} identical workers in our paper, discrimination is likely to be intrinsically prevalent as a result of competition even if there is no exogenous difference between groups.

In the literature on statistical discrimination that originated from seminal papers by Phelps (1972) and Arrow (1972), discrimination is understood to be an outcome of either intrinsic exogenous differences between groups of workers, or endogenously-derived, average differences

\textsuperscript{1}This nature of competition extends beyond the example of job market that we focus on in our modeling. In this context, see the case of the lawsuit against the University of Michigan’s law school. (\textit{New York Times}, May 11, 1999).
between groups in equilibrium.\footnote{The literature on discrimination, a subject first studied formally by Becker (1971) in the field of economics, suggests three broad causes of economic discrimination: discrimination driven by demand-side traits, such as employers’ or co-workers’ tastes; discrimination driven by supply-side traits, such as different turnover rates for men and women; and statistical discrimination based on self-fulfilling beliefs \cite{Cain1986,England1992}. Fang and Moro \cite{FangMoro2011} provides an excellent survey of statistical discrimination literature.}

In the case of discrimination caused by exogenous differences between groups of workers, the study is naturally focused on an explanation of what causes the groups to intrinsically differ. Lang \cite{Lang1986} offers language barriers between employers and minorities as a source of the difference that engenders group inequality. Phelps \cite{Phelps1972} and Aigner and Cain \cite{AignerCain1977} attribute discrimination to the difference in the variance of signals in different groups, although these papers do not explain what causes the variance to differ among groups. In this aspect, Cornell and Welch \cite{CornellWelch1996} show that discrimination based on different variances can arise in a model of tournament in which screening is important and employers who are from one group can more easily screen applicants from the same group than from another group. Klumpp and Su \cite{KlumppSu2013} also show that the second-moment differences of \textit{ex ante} identical groups of workers can arise in a statistical discrimination model, using within-group heterogeneity in workers’ ability and three levels of tasks.

In contrast, in the case of discrimination toward \textit{ex ante} identical groups of workers that we analyze, the discrimination can be explained only in terms of endogenously-determined, average group differences that arise as a result of self-fulfilling prophesy. The present paper also builds on a framework of statistical discrimination and the equilibrium is partially driven by self-fulfilling beliefs. The information structure in this paper resembles that of Coate and Loury \cite{CoateLoury1993}. In Coate and Loury \cite{CoateLoury1993}, discrimination of \textit{ex ante} identical workers occurs because the disadvantaged group fails to coordinate its investments into a better equilibrium. Such discrimination is inefficient as removing it only makes the disadvantaged group better off without affecting the advantaged group. However, in the present paper, removing discrimination always involves a welfare loss for the advantaged group.

Moro and Norman \cite{MoroNorman2004} was the first to demonstrate such a nature of conflicting interests between the advantaged and the disadvantaged. In their general equilibrium model, the production technology requires two complementary inputs. When too many workers invest in a high-skill job, the marginal product decreases, generating greater incentives to invest in a low-skill job. Asymmetric equilibrium occurs as a result of two groups’ coordination to specialize in different tasks. In contrast, in the present paper, two groups becomes specialized in different tasks because the availability of (high-skill) positions is limited. Therefore, discrimination does not require complementarity between jobs as it is imposed in the framework of Moro and Norman \cite{MoroNorman2004}. Given the prevalence of a shortage of positions, the result in our paper implies
that there likely will be discrimination.

More importantly, we show that the employers can actively influence the selection of a discriminatory equilibrium path by choosing a set of competing finalists primarily from a majority group at the pre-selection stage, and by implementing an impartial tie-breaking rule. These findings advocate the importance of equal opportunity laws. We also show the importance of a group’s majority standing in determining the benefit of discrimination. We explain why discrimination normally results in an advantage for the majority, but not the converse. Overall, these results show that discrimination occurs more systematically than is predicted by the literature of self-fulfilling equilibrium.

The welfare implication of discrimination in this framework, however, is ambiguous. Given that only one position is available for two candidates, encouraging both workers to invest in human capital can be wasteful, thereby indicating the inefficiency of symmetric equilibrium. However, discrimination may not benefit the majority workers either because of enhanced competition among the majority workers.

Our findings indicate that a shortage of available positions is the main source of the inefficiency because it creates competition among workers for positions and employers’ preference for discrimination. Because the shortage problem is independent of workers’ qualification, discrimination will not solve the problem of intrinsically wasteful investment in human capital. Instead, we advocate the importance of job creation efforts to reduce discrimination. Mialon (2013) considers the impact of shortage levels in modeling prejudice as employers decide to ignore some potentially informative signals from workers. Mialon (2013) finds that prejudice is less likely to occur as the level of shortage declines.

This paper is organized as follows. Section 2 describes the basic model and derives symmetric and asymmetric equilibria. Section 3 shows how discrimination alters the overall risk that an employer faces and derives his preference for discrimination. In Section 4, we discuss welfare implications of discrimination and the roles of equal opportunity laws. In Section 5, we conclude.

2 Model

Consider a market in which there are many identical employers and workers. Workers belong to one of two groups, $A$ and $B$, and the population share of group $i$ is $\lambda_i \in (0, 1)$, $i = A, B$. Each worker from group $i$ decides whether or not to make an investment in human capital to become qualified. The investment cost $c_i$ of each worker is drawn from a continuous CDF $F$ which has a density $f > 0$ on a support $[0, \tau]$, $0 < \tau$, for both groups. The group identity is

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3 The employers can be managers or admissions officers, and the workers can be job candidates, or college applicants. Hence, the selection decision can be interpreted broadly in the context of admission, as well as employment.
publicly observable at zero cost, but each worker’s qualification is private information to that worker.\(^4\)

Workers signal their qualification through a test. Signals are noisy in the sense that the signals from a qualified worker may take either \(H\) with a probability of \((1 - q)\) or \(M\) with a probability of \(q\), and the signals from an unqualified worker may be either \(M\) with a probability of \(u\) or \(L\) with \((1 - u)\), where \(q, u \in (0, 1)\).\(^5\) We assume that \(q > u\) so that the signaling distribution of qualified workers first-order stochastically dominates that of unqualified workers. Under this assumption, the signal \(M\) is more likely to be from qualified workers than from unqualified workers (MLRP).

Each employer is randomly matched with two workers from the whole population. After observing workers’ test results, the employer selects at most one of them for a position and pays \(v\) for the selected worker. Each employer gains a return of \(R > 0\) from hiring a qualified worker, and \(0\) otherwise. We assume that \(v \in (0, R)\), \(v < \pi\). Following Coate and Loury (1993) and Blume (2005), \(v\) is fixed and group-independent. We call \(\rho = v/R\) a wage/output ratio. The payoff for an unselected worker is normalized at \(0\).

2.1 Beliefs

When an employer is randomly matched with two workers from the population, each worker of group \(A\) has \(\lambda_A\) chance to compete with a worker of group \(A\) and \(1 - \lambda_A\) chance to compete with a worker of group \(B\), and vice versa. In the basic model, we assume that the employer has no control over the composition of \(\lambda_i\) in the pool of competing candidates. This assumption will be relaxed in Section 3 when we analyze the employer’s preference for discrimination.

Let \(i\) and \(j\) be the two randomly matched workers. The two workers may be from the same group, \(i = j\), or from different groups, \(i \neq j\). Let \(\alpha_i\) (\(\alpha_j\), respectively) be the fraction of group \(i\) (group \(j\), \(i = j\), or \(i \neq j\)) of workers whom the employer expects to be qualified by an investment in human capital. Then, the probability that a worker from group \(i\) has a signal \(S\), \(p_S(\alpha_i)\), for \(S \in \{H, M, L\}\), is derived as follows:

\[
\begin{align*}
    p_H(\alpha_i) &\equiv \alpha_i (1 - q) \\
    p_M(\alpha_i) &\equiv \alpha_i q + (1 - \alpha_i) u \\
    p_L(\alpha_i) &\equiv (1 - \alpha_i) (1 - u).
\end{align*}
\]

\(^4\)We only consider group characteristics that are not subject to individual choices such as race or sex.

\(^5\)We use a discrete distribution of signals for simplicity. It does not affect the qualitative results of our paper. Various types of simplified signaling distributions, resembling the present one, have been adopted in Blume (2005), Fryer (2007) and Chaudhauri and Sethi (2008). Our distribution is similar to that in Fryer (2007). Such a set-up is reasonable because, even when test scores are continuous, we often classify them for evaluation into discrete measures, such as typical grades at universities and qualifying examinations in doctoral programs.
Each worker’s strategy is a mapping \( Q_i : [0, \Theta] \rightarrow \{0, 1\} \), where 1 denotes qualified, and \( \Pr(Q_i = 1|S_i) \) denotes the conditional probability that a worker \( i \), if chosen, is qualified for a given \( S_i \).

From the specified signaling structure, it is clear that for a given signal \( H \), \( \Pr(Q_i = 1|S_i = H) = 1 \), and for a given signal \( L \), \( \Pr(Q_i = 1|S_i = L) = 0 \). For a given signal \( M \), the posterior probability for a worker from group \( i \) to be qualified is denoted by \( \mu : [0, 1] \rightarrow [0, 1] \) and

\[
\mu(\alpha_i) \equiv \frac{\alpha_i q}{\alpha_i q + (1 - \alpha_i)u}.
\]

Let \( \theta = (S_i, S_j) \in \Theta \), \( \Theta \equiv \{H, M, L\}^2 \), be a vector of observed signals from \( i \) and \( j \), i.e., \( S_i \) and \( S_j \). For a given \( \theta \), the employer’s payoff for hiring a worker is

\[
\mathbb{E}(u_E|\theta) = \max\{\Pr(Q_i = 1|S_i), \Pr(Q_j = 1|S_j)\}R - v. \quad (1)
\]

Each employer’s strategy is a mapping \( E : \Theta \rightarrow \{i, j, \phi\} \), where \( i \) or \( j \) means hiring either \( i \) or \( j \), whereas \( \phi \) means hiring no one.

The employer strictly prefers a worker with a signal \( H \) to one with \( M \), and \( M \) to \( L \). To determine the employer’s hiring strategy, let us establish that “\( \theta \)” is favorable to \( i \)” whenever \( S_i \) is strictly preferred to \( S_j \). For example, if \( \theta = (H, M) \), \( \theta \) is favorable to \( i \). With Bayesian inferences based on the observed signals from the two workers, we can determine that it is optimal for the employer to hire \( i \) whenever \( \theta \) is favorable to \( i \) and \( \mathbb{E}(u_E|\theta) \geq 0 \).

If one of the signals \( S_i \) is \( H \), \( \mathbb{E}(u_E|\theta) > 0 \) always. However, when the best available signal is \( M \), the condition \( \mathbb{E}(u_E|\theta) \geq 0 \) holds if and only if \( \mu(\alpha_i) \geq v/R = \rho \). For several cases of such a \( \theta \), the condition is written as

\[
\frac{\alpha_i q}{\alpha_i q + (1 - \alpha_i)u} \geq \rho, \text{ or } \alpha_i \geq \alpha_s \equiv \frac{u\rho}{u\rho + q(1 - \rho)} \quad (2)
\]

When \( \theta = (M, M) \), if \( \alpha_i \geq \alpha_s \), the probability that an employer will choose a worker from group \( i \) is summarized by a function \( \varphi : [0, 1]^2 \rightarrow [0, 1] \),

\[
\varphi(\alpha_i, \alpha_j) = \begin{cases} 1 & \text{if } \alpha_i > \alpha_j \\ [0, 1] & \text{if } \alpha_i = \alpha_j \\ 0 & \text{if } \alpha_i < \alpha_j \end{cases} \quad (3)
\]

and a worker \( j \) is selected with a probability of \( 1 - \varphi \) if \( \alpha_j \geq \alpha_s \); 0 otherwise. If the two workers are from the same group \( i \), they will be indistinguishable in their group characteristic and thus systematically differentiating them is not possible. Thus, \( \varphi(\alpha_i, \alpha_i) = 1/2 \).

This reveals that the employer’s choice depends not only on how likely it is for each group to be qualified, but also on how likely it is for one group to be more qualified than the other. As a
result, each group workers’ qualifying effort becomes interdependent in the sense that each group must take into account the other group’s qualifying effort in choosing its own. This differentiates our model from other models without a competition.

Now consider each worker’s decision to invest. For a given \( c_i \), group \( i \) workers invest as long as the expected return on their investment is greater than \( c_i \). Expecting to compete against another worker from group \( j \) (\( j = i \) or \( j \neq i \)), group \( i \) workers’ incentive to invest in qualification is a function of the net increase in the expected payoff from the investment, which is summarized by the following function \( \beta : [0, 1]^2 \rightarrow [0, 1] \)

\[
\beta(\alpha_i, \alpha_j) = (1 - q) \left[ p_H(\alpha_j) \frac{1}{2} + p_M(\alpha_j) + p_L(\alpha_j) \right]_{S_i=H} + \mathbf{1}_{\{\alpha_i \geq \alpha_j\}} (q - u) \left[ p_M(\alpha_j) \phi(\alpha_i, \alpha_j) + p_L(\alpha_j) \right]_{S_i=M}.
\]

Equation (4) shows that \( \beta \) is a decreasing function of \( \alpha_j \), implying a negative externality of each worker’s investment on other workers who compete with them.\(^6\) If \( \alpha_i < \alpha_s \), the term for \( S_i = M \) disappears and there is no effect of direct competition with the other worker. However, if \( \alpha_i \geq \alpha_s \), \( i \neq j \), any difference in investment incentives between the two groups matters (directly) to a worker in group \( i \). In particular, if \( \alpha_i > \alpha_j \), the competitive advantage over group \( j \) substantially enhances a worker \( i \)’s investment incentive in comparison to the case when \( \alpha_i \leq \alpha_j \). For this reason, an advantaged group can achieve a far greater level of human capital investment under asymmetric equilibrium than under symmetric equilibrium. In the next section, our analysis will show how this term for \( S_i = M \) appears differently under symmetric and asymmetric equilibria.

Since a worker of group \( i \) is matched to another worker from the same group \( i \) with a probability of \( \lambda_i \) or a worker from a different group \( j \neq i \) with a probability of \( 1 - \lambda_i \), the worker’s expected return from investment is described by \( \pi_i(\alpha_i, \alpha_j)v \), where

\[
\pi_i(\alpha_i, \alpha_j) \equiv [\lambda_i \beta(\alpha_i, \alpha_i) + (1 - \lambda_i) \beta(\alpha_i, \alpha_j)].
\]

The worker invests as long as \( \pi_i(\alpha_i, \alpha_j)v \geq c_i \). Thus, \( k_i = \pi_i(\alpha_i, \alpha_j)v \) becomes the “cut-off” that matters for group \( i \) workers in determining their investment decision. Since \( 0 < \pi_i(\alpha_i, \alpha_j) < 1 \) for a given \( \lambda_i \in [0, 1] \) and \( q,u \in (0,1) \), the cutoff \( k_i \) is always interior, i.e., \( 0 < \alpha_i < 1 \).

Then, an equilibrium is defined as \( (\alpha^*_A, \alpha^*_B) \in [0, 1]^2 \) such that for each \( i \in \{A, B\} \),

\[
\pi_i(\alpha^*_i, \alpha^*_j)v = k_i \text{ and } F(k_i) = \alpha^*_i.
\]

\(^6\)For detailed proof of the externality, see Lemma 1 in the Appendix.
Since $F(\cdot)$ is strictly increasing in $k$, the equilibrium in (6) can be rewritten in terms of $(k_A^*, k_B^*) \in [0, \varpi]^2$ so that for each $i \in \{A, B\}$,

$$G_i(k_i^*, k_j^*) = k_i^*,$$

(7)

where $G_i(k_i, k_j) \equiv \pi_i(F(k_i), F(k_j))$. Similarly, we define $k_s$ as a level at which $\alpha_s = F(k_s)$ and $P_S(k) = p_S(F(k))$ for all $s$. In the following two subsections, using (7), we find an equilibrium in terms of $k$ and examine the existence of equilibria defined as $(k_A^*, k_B^*)$ at which $k_A^* = k_B^*$ if symmetric, or $k_A^* \neq k_B^*$, if asymmetric.

### 2.2 Symmetric Equilibrium

A symmetric equilibrium arises only if $\varphi(\alpha_i, \alpha_j) = 1/2$. A symmetric equilibrium is defined by $k^* \in (0, \varpi)$ so that

$$G_i(k^*, k^*) = k^* \text{ for } i \in \{A, B\}.$$ 

For any symmetric level of $k$ from both groups, define $\beta_1: [0, \varpi] \rightarrow [0, 1]$ and $\beta_h: [0, \varpi] \rightarrow [0, 1]$ for $k \geq k_s$ and for $k < k_s$, respectively. That is, for $k \geq k_s$,

$$\beta_h(k) \equiv (1 - q)[P_H(k) \frac{1}{2} + P_M(k) + P_L(k)] + (q - u)[P_M(k) \frac{1}{2} + P_L(k)],$$

(8)

and for $k < k_s$,

$$\beta_l(k) \equiv (1 - q)[P_H(k) \frac{1}{2} + P_M(k) + P_L(k)],$$

(9)

Note that for any given $k$, $\beta_h(k) > \beta_l(k)$, and both are continuous in the relevant range. More importantly, each of them is a strictly decreasing function of $k$, implying that competition reduces a worker’s incentive to invest.

Let $k_l$ and $k_h$ be the levels of cutoff at which $\beta_l(k_l) = k_l$ and $\beta_h(k_h) = k_h$. Then, $k_l$, $k_h \in (0, \varpi)$ are unique and $k_h > k_l$. Depending on the level of $k_s$, which is determined by $\rho$, there are three possible cases of symmetric equilibria.

**Proposition 1**

(i) If $k_s \leq k_l$, the unique symmetric equilibrium is $\text{HS} = (k_h, k_h)$.

(ii) If $k_l < k_s \leq k_h$, there are multiple symmetric equilibria. The equilibrium occurs either at $\text{LS} = (k_l, k_l)$ or $\text{HS} = (k_h, k_h)$.

(iii) If $k_s > k_h$, the unique symmetric equilibrium is at $\text{LS} = (k_l, k_l)$.

$k_s$ represents a minimum group standard that makes hiring a worker of that group worthwhile for the employer when the signal is $M$. Suppose the wage $v$ is sufficiently low in comparison to the employer’s return from selecting a qualified worker, $R$. In this case, a low $\rho = v/R$ results in a low $k_s$ so that $\text{HS}$ is the only symmetric equilibrium. Intuitively, a low wage reduces
the employer’s expected risk of hiring a worker of signal $M$, and thus, increases the employer’s willingness to hire. This, in turn, increases the worker’s incentive to invest by generating a higher payoff from the investment ($R > 0$) than otherwise. Instead, if $\rho$ and $k_s$ are sufficiently high, the employer faces a greater risk from hiring a worker with $M$, and the symmetric equilibrium exhibits a low level of investment. Thus, the equilibrium investment levels are higher when $\rho$ and $k_s$ are lower. Figure 1 illustrates the symmetric equilibria when $F$ is uniform.

Suppose that $(1 - \lambda_i) = 0$. This implies that a worker from group $i$ always expects to be competing against another worker from the same group. In this case, the only possible equilibria are symmetric. Thus, any symmetric equilibrium outcome is equivalent to a case in which the worker from each group faces no competition from another group member. In the next section, we derive equilibrium when there is a strictly positive probability of facing a worker from a different group $j$ and the equilibrium outcome is not symmetric.

### 2.3 Asymmetric Equilibrium

Deriving asymmetric equilibria is similar to deriving symmetric equilibrium. For details, see the Appendix. However, there is an important difference in how one group gains a competitive advantage in the employer’s hiring decision, i.e., $\varphi(\alpha_i, \alpha_j)$, with regard to asymmetric equilibrium. When workers face competition with others from the same group, $\varphi(\alpha_i, \alpha_j) = 1/2$, and there is no competitive advantage, whereas the advantage can be $\varphi(\alpha_i, \alpha_j) = 1$ if $\alpha_i > \alpha_j$, $i \neq j$. Since facing competition from workers in the same group is equivalent to the case of symmetric equilibrium, this implies that the investment of winning group workers will be much greater under asymmetric equilibrium than under symmetric equilibrium. This becomes an important motive for discrimination. In the following, we prove this property of asymmetric equilibria.

Without loss of generality, we derive the equilibria for the case where $k_i > k_j$. First, we
find that in asymmetric equilibrium, the cutoffs for two groups, $k_i$ and $k_j$, may be extremely asymmetric, $\text{EA}$, or moderately asymmetric, $\text{MA}$.

**Proposition 2** Let $\text{EA}$ and $\text{MA}$ be the set of all asymmetric equilibrium cutoffs $(k_i^e, k_j^e)$ and $(k_i^m, k_j^m)$ satisfying $k_i^e > k_s > k_j^e$ and $k_i^m > k_j^m > k_s$, respectively. Then,

(i) $(k_i^e, k_j^e)$ and $(k_i^m, k_j^m)$ exist, and

(ii) $0 < k_j^e < k_j^m < k_i^m < k_i^e < \sigma$.

Proposition 3 shows that asymmetric equilibrium, either $\text{EA}$ or $\text{MA}$, exists as long as $k_s$ is not too high.

**Proposition 3** Let $\underline{k} \equiv \min k_j^e$, $\overline{k} \equiv \max k_j^m$, and $\overline{\sigma} \equiv \max k_i^e$, where $0 < \underline{k} < \overline{k} < \overline{\sigma}$. Then,

(i) For $k_s \leq \underline{k}$, $\text{MA}$ is the unique set of asymmetric equilibria.

(ii) For $\underline{k} < k_s \leq \overline{k}$, equilibrium occurs either at $\text{EA}$ or $\text{MA}$.

(iii) For $\overline{k} < k_s \leq \overline{\sigma}$, $\text{EA}$ is the unique set of asymmetric equilibria.

(iv) For $k_s > \overline{\sigma}$, there is no asymmetric equilibrium.

If $\rho$ is so low that $k_s$ is sufficiently low, workers in each group will have a high probability of competing with another qualified worker. Since one group’s investment imposes a negative externality on the other group’s investment, this implies that workers in an advantaged group $i$ do not see much competitive advantage from investing in qualification given a favorable condition in selection. Thus, when there is a high probability that both groups will invest, the resulting equilibrium exhibits only a moderate difference in their equilibrium investments ($\text{MA}$). As the level of $\rho$ grows, workers expect that a disadvantaged group will have a low probability of qualification below the cutoff $k_s$. Such an expectation enhances an advantaged group’s incentive to invest further. Thus, significantly asymmetric investments between two groups can occur in equilibrium ($\text{EA}$).

### 2.4 Synthesis

Let us formally define the trade-off between two groups’ expected qualification levels in terms of a group $i$– specific dominance.

**Definition 1** An asymmetric allocation $x = (x_i, x_j)$ $i$-dominates $y = (y_i, y_j)$, which is denoted by $x >_D y$, if $x_i > y_i$ and $x_j < y_j$, for $x, y \in \mathbb{R}^2$.
Proposition 4 shows that for each \( x \in \text{MA} \), \( x \) \( i \)-dominates the symmetric equilibrium \( \text{HS} \), and for each \( x \in \text{EA} \), \( x \) \( i \)-dominates the symmetric equilibrium \( \text{LS} \). Combining Proposition 3 and 4, we find that for each \( x \in \text{EA} \), \( x \) \( i \)-dominates the symmetric equilibrium \( \text{HS} \).

**Proposition 4**  
(i) For each \( x \in \text{MA} \), \( x \geq_{D} \text{HS} \).

(ii) For each \( x \in \text{EA} \), \( x \geq_{D} \text{LS} \).

This dominance relationship is useful in explaining the trade-off between discriminatory equilibrium and symmetric equilibrium and the motivation for discrimination in the following.

**Proposition 5** There are three levels of wage/output ratios \( \rho_h, \rho_m, \) and \( \rho_l \), where \( \rho_h > \rho_m > \rho_l > 0 \), for which

(i) if \( \rho > \rho_h \), the unique equilibrium is \( \text{LS} \),

(ii) if \( \rho_m < \rho \leq \rho_h \), equilibrium occurs at either \( \text{EA} \) or \( \text{LS} \),

(iii) if \( \rho_l < \rho \leq \rho_m \), any one of the four types of equilibria among \( \text{EA}, \text{MA}, \text{HS}, \) or \( \text{LS} \) is possible.

(iv) if \( \rho \leq \rho_l \), equilibrium occurs at either \( \text{MA} \) or \( \text{HS} \).

In this equilibrium configuration, there are two considerations. They are: (i) whether the workers’ investment incentives are high enough to warrant the risk of considering a candidate with a signal \( M \) for the employer, i.e., whether \( k_i > k_s \) and (ii) whether a group gains a competitive advantage over the other group, i.e., whether \( k_i > k_j \).

In symmetric equilibrium, only (i) matters. In asymmetric equilibrium, an additional benefit of competitive advantage from (ii) further enhances the investment incentives of workers from an advantaged group at the expense of the incentives of workers from a disadvantaged group. If the disadvantaged group’s investment incentive decreases to a level below \( k_s \) from (18), resulting in \( \text{EA} \), the second effect (ii) is much greater than the case in which it remains above \( k_s \) and results in \( \text{MA} \). Naturally, it is \( \rho \) that determines the sizes of (i) and (ii) through its impact on \( k_s \). Figure 2 (b) and (c) characterize a set of feasible equilibria for this model, whereas Figure 1 (a) is the case for typical statistical discrimination models.

## 3 Discrimination

In this section, we analyze what discrimination implies to the employer in this framework. We explain why an employer may prefer discrimination toward \( \text{ex ante} \) identical groups. In
the comparison of an employer’s expected payoff from symmetric equilibrium with that from asymmetric equilibrium, we show how discrimination spreads the risks across groups. We show how such a spread can lower the overall risks to the employer and thus promotes the employer’s preference for discrimination.

As for the reason that engenders discrimination over ex ante identical groups, statistical discrimination literature has been quiet about anything beyond the mechanism of self-fulfilling expectation. The literature explains that, because workers of two groups expect unequal treatment, they make different levels of investment in human capital. Thus, their expectation fulfills a discriminatory outcome, although there is no ex ante reason why they should have such an expectation of unequal treatment. In this process, employers do not play a role in determining the discriminatory outcomes. Instead, the employers’ decisions to discriminate against one group is simply a rational and fair response to workers’ asymmetric investment choices, because one group has a higher probability of qualification than the other due to the self-fulfilling expectation of unequal treatment.

In contrast, our paper explains an active role of employers’ preference for discrimination in fulfilling the expectation. We offer several channels through which employers can implement their preferred discriminatory outcome.

3.1 Employer’s Expected Payoff and Risk

Let $U(\alpha_i, \alpha_j)$ be the expected payoff of the employer who anticipates being matched with two workers drawn from groups $i$ and $j$. Denote by $P(H \lor H)$ and $P(S_i, S_j)$ the probability that at least one of the two workers has a signal $H$ and the probability that group $i$’s worker’s signal is
In particular, the weight depends on the size of representation of a group in the population:

$$S_i \text{ and group } j \text{'s worker's signal is } S_j, \text{ respectively. Then, }$$

$$U(\alpha_i, \alpha_j) = P(H \lor H)(R - v) + P(M, M)\Gamma(\alpha_i, \alpha_j)$$

$$+ 1_{\{\alpha_i \geq \alpha_s\}}P(M, L)[\mu(\alpha_i)R - v] + 1_{\{\alpha_j \geq \alpha_s\}}P(L, M)[\mu(\alpha_j)R - v],$$

where

$$\Gamma(\alpha_i, \alpha_j) \equiv \begin{cases} \{\mu(\alpha_j) + [\mu(\alpha_i) - \mu(\alpha_j)]\varphi(\alpha_i, \alpha_j)\}R - v & \text{if } \alpha_i > \alpha_s \text{ or } \alpha_j > \alpha_s, \\ 0 & \text{otherwise}. \end{cases}$$

Let $U_H(\alpha_i, \alpha_j), U_{HL}(\alpha_i, \alpha_j),$ and $U_L(\alpha_i, \alpha_j)$ be the levels of $U(\alpha_i, \alpha_j)$ when both $\alpha_i$ and $\alpha_j$ are greater than or equal to $\alpha_s$, $\alpha_i \geq \alpha_s > \alpha_j$, and both are lower than $\alpha_s$, respectively. Since the employer exercises an option to not hire whenever $\alpha_i < \alpha_s,$ we can see that $U_H(\alpha_i, \alpha_j) > U_{HL}(\alpha_i, \alpha_j) > U_L(\alpha_i, \alpha_j)$ for any given $\alpha_i$ and $\alpha_j$. Similarly, we can define $U(\alpha_i)$ as the payoff when both workers have the same $\alpha_i$, and let $U_H(\alpha_i)$, and $U_L(\alpha_i)$ be the levels of $U(\alpha_i)$ when $\alpha_i$ is greater than $\alpha_s$, and $\alpha_i$ is lower than $\alpha_s$.

Given that a $\lambda_i$ fraction of the population belongs to group $i$, if the two workers are selected in proportion to the size of their representation in the population, the employer’s total expected payoff can be described as follows.

$$W(\alpha_i, \alpha_j) = \lambda_i^2U(\alpha_i) + (1 - \lambda_i)^2U(\alpha_j) + 2\lambda_i(1 - \lambda_i)U(\alpha_i, \alpha_j).$$

(11) shows that $W(\alpha_i, \alpha_j)$ is comprised of a couple of different risk components depending on whether the interactions occur within the same group members (within-group) or between the members of two different groups (inter-group). In a symmetric equilibrium, an employer expects the same levels of risks, represented by the same levels of $\alpha$, for all workers, regardless of which group they belong to. Moreover, at a symmetric equilibrium, $U(\alpha_i) = U(\alpha_i, \alpha_j) = U(\alpha_j)$, and thus, the employer’s expected payoff is $W(\alpha, \alpha) = U(\alpha)$. That is, the symmetric equilibrium removes any potential impact from the only available distinction between the groups, $\lambda_i$. Consequently, the risks at symmetric equilibrium are equivalent to the risks of having competition within only one group.

Introducing asymmetry between the two groups affects $W(\alpha_i, \alpha_j)$ in three ways. First, it spreads the risks across the groups, thereby lowering the risk of an advantaged group below the symmetric equilibrium level while increasing the risk of a disadvantaged group above the level. This implies a different level of risk for within-group interactions, depending on whether the group is favored or not. Second, the weight of each group’s within-group interactions differs. In particular, the weight depends on the size of representation of a group in the population:
a weight of $\lambda_i^2$ is given to the interactions within group $i$ candidates and $(1 - \lambda_i)^2$ to the interactions within group $j$ candidates. Therefore, a greater weight is given to the interactions within majority group members. Third, in the case of inter-group interactions, discrimination is more likely to enhance the quality of signaling as the workers from an advantaged group have a higher quality of signals and the employer is less likely to select a low quality worker from the group. In the next, we show how these factors motivate the employer’s preference for discrimination.

3.2 Preference for Discrimination

The employer may prefer discriminatory equilibrium to symmetric equilibrium. One of the main reasons is that each group’s size of representation in the population $\lambda_i$ differs. Suppose that a group has a significantly larger $\lambda_i$. Spreading the investment incentives across groups improves the quality of signals from the majority group. Since the employer expects more frequent interactions with the majority group, the decrease in the majority group’s risk can easily outweigh the increase in the minority group risk if $\lambda_i$ is large enough. In this way, discrimination can lower the overall risks that the employer faces. The following analysis derives the necessary conditions for this result.

From Proposition 5, both symmetric and asymmetric equilibria are feasible in all ranges, except in the case when $\rho > \rho_h$. Thus, as long as $\rho$ is not too high, there is a potential trade-off between the two types of equilibria at every level of $\rho$. To simplify our analysis, let us focus on cases (ii) and (iv) of Proposition 5, where only one type of asymmetric equilibrium is available as an alternative to a symmetric equilibrium.

When $\rho \leq \rho_l$, MA is the only alternative to HS and the transition from HS to MA results in $k^m_1 > k_h > k^m_2 > k_s$ (Proposition 4). Hence, the change involves an asymmetric perturbation around HS. On the other hand, when $\rho_m < \rho \leq \rho_h$, EA is the only alternative to LS and the transition from LS to EA involves a shift of $\alpha_i$ from a range where $\alpha_i < \alpha_s$ to the range where $\alpha_i \geq \alpha_s$ for the winning group in addition to an asymmetric perturbation. (Proposition 3).

First, to see the effect of asymmetry on the employer’s payoff, consider a small asymmetric perturbation around the symmetric equilibrium. Although there are two types of symmetric equilibria, because the level of qualification for the advantaged group at asymmetric equilibrium is higher in both ranges than the level at HS, we can consider an asymmetric perturbation around HS.\(^7\) Let $W_E$ be the total expected payoff at $E$, $E \in \{HS, LS, MA, EA\}$. At HS,
$W_{HS} = U_H(\alpha)$, where $U_H(\alpha)$ is $U(\alpha)$ in the range where $\alpha > \alpha_s$. Let $\alpha_i = \alpha + \varepsilon$ and $\alpha_j = \alpha - \varepsilon$, $\varepsilon > 0$. Then,

$$\frac{dW_{HS}}{d\varepsilon}_{\varepsilon \to 0} = [\lambda_i^2 - (1 - \lambda_i)^2] \frac{dU_H(\alpha)}{d\alpha} + 2\lambda_i(1 - \lambda_i)[-qu + qu(2\varphi)].$$

(12)

If the employer is fair in hiring $\varphi = 1/2$ around the symmetric case, the second term is zero.\(^8\)

Then, the employer’s preference for discrimination, the sign of (12), will depend on the sign of $\frac{dU_H(\alpha)}{d\alpha}$ and if $\lambda_i > 1/2$. For $\lambda_i > 1/2$, $[\lambda_i^2 - (1 - \lambda_i)^2] > 0$ and thus, as long as a higher $\alpha$ increases $U_H(\alpha)$, $\frac{dW_{HS}}{d\varepsilon}_{\varepsilon \to 0} > 0$, and thus, the employer will prefer such a spread.\(^9\)

**Proposition 6** Suppose that the employer’s $U_H(\alpha)$ increases as the expected probability of qualification $\alpha_i$ increases. Then, the employer benefits from spreading the qualification incentives favorably to the majority group.

To see how the total equilibrium payoff for the employer varies between the two types of equilibria, first consider the case where $\rho \leq \rho_i$. In this range, there is a trade-off between MS and HS. The symmetric equilibrium induces $\alpha_h = F(k_h)$, whereas in asymmetric equilibrium, $\alpha_i^m = F(k_i^m) > \alpha_h = F(k_h) > \alpha_j^m = F(k_j^m) > \alpha_s = F(k_s)$. The effect of moving from a symmetric equilibrium HS to an asymmetric equilibrium MA on the employer’s payoff is

$$W_{MA} - W_{HS} = \lambda_i^2 [U_H(\alpha_i^m) - U_H(\alpha_h)]$$

$$+ (1 - \lambda_i)^2 [U_H(\alpha_j^m) - U_H(\alpha_h)] + 2\lambda_i(1 - \lambda_i)[U_H(\alpha_i^m, \alpha_j^m) - U_H(\alpha_h)].$$

(13)

The first and second terms refer to the effect created by introducing a spread in the symmetric qualification level $\alpha_h$ within the same group interactions (hence, within-group effect). The last term shows the effect on the payoff from inter-group interactions (inter-group effect). If $\frac{dU_H(\alpha)}{d\alpha} > 0$, the first term in (13) $[U_H(\alpha_i^m) - U_H(\alpha_h)]$ is positive, whereas the second term whether $\frac{dU_H(\alpha)}{d\alpha} > 0$ for the advantaged group’s increased probability of qualification in the range where $\alpha_i \geq \alpha_s$. Therefore, it is more pertinent to consider the effect of asymmetric perturbation around HS.

\(^8\)Regardless of $\varphi$, the second term is always non-negative because at asymmetric allocation, the employer always chooses the group that is most qualified. If the employer chooses a group $\alpha_i \geq \alpha_j$, the second term is positive. In that case, the preference for discrimination is even greater. We discuss the impact of unfair rule $\varphi > 1/2$ in Section 4.

\(^9\) $U_H(\alpha)$ may not be an increasing function of $\alpha$ always. In the Appendix, we show that the expected payoff begins to decrease in $\alpha$, $\frac{dU_H(\alpha)}{d\alpha} < 0$, if $\alpha_i$ is sufficiently high. The highest level of $U_H(\alpha)$ is achieved at $\tilde{\alpha} = \frac{2R - qu - 2q(1 - u)}{2R - 2qu - 2(1 - u)\varphi}$. If $q$ is large, $\tilde{\alpha}$ may not be too high, inducing a considerable range where $\frac{dU_H(\alpha)}{d\alpha} < 0$. As $q$ increases, the likelihood of facing a signal $M$ increases. Then, asymmetric equilibria enhancing one group’s qualification may not be a great idea as most of the highly qualified workers will have a signal $M$ pooled with unqualified workers. In such a case, the employer may prefer symmetric equilibrium to asymmetric equilibrium.
\[ U_H(\alpha_h^m) - U_H(\alpha_h) \] is negative. In Proposition 7, we show that if \( \lambda \) is sufficiently large, the employer’s equilibrium payoff is higher at the discriminatory equilibrium.

**Proposition 7** Suppose \( \rho \leq \rho_l \) and \( \frac{dU_H}{d\alpha} > 0 \). Then, there exists a \( \lambda^m \in (0, 1) \) such that for a \( \lambda > \lambda^m \), \( \lambda \in (1/2, 1) \), \( W_{MA} > W_{HS} \), and therefore, the employer prefers discrimination.

For an exogenous \( \lambda_i \), discrimination not only affects the inter-group effect, but also assigns a different weight to the within-group effect of each group. The employer can benefit from discrimination because, by means of it, the employer can reduce the within-group risks for the majority group as they represent a substantial fraction of the expected interactions.

Similarly, for the case of a move from \( \text{LS} \) to \( \text{EA} \), the effect of discrimination on the employer’s payoff is as follows.

\[
W_{EA} - W_{LS} = \lambda_i^2 [U_H(\alpha_i^e) - U_H(\alpha_i) + U_L(\alpha_i) - U_L(\alpha_i)] + (1 - \lambda_i)^2 [U_L(\alpha_i^e) - U_L(\alpha_i)] + 2\lambda_i(1 - \lambda_i)[U_{HL}(\alpha_i^e, \alpha_i^e) - U_L(\alpha_i)]
\]  

(14)

The first term in (14) shows an extra benefit of introducing asymmetry in this case as it raises the qualification of the advantaged group beyond \( \alpha_s \). Since \( U_H(\alpha) > U_L(\alpha) \) for any given \( \alpha \), if \( \frac{dU_H}{d\alpha} > 0 \), the benefit from asymmetry for the advantaged group is much greater than that obtained by moving from \( \text{HS} \) to \( \text{MA} \). For the same reason, the last term from inter-group interactions will have a larger positive effect. This implies that the employer is more likely to prefer discrimination if the symmetric equilibrium is at \( \text{LS} \).

**Proposition 8** Suppose \( \rho_m < \rho \leq \rho_h \) and \( \frac{dU_H}{d\alpha} > 0 \). Then, there is a \( \lambda^e \in (0, 1) \) such that for a \( \lambda \geq \lambda^e \), \( \lambda \in (1/2, 1) \), \( W_{EA} > W_{LS} \), and the employer prefers discrimination.

Propositions 7 and 8 underline the importance of \( \lambda_i \), the group representation in the labor market. If the size of \( \lambda_i \) is naturally determined, it implies that one group’s majority standing in the population naturally advocates the employer’s preference for favoring the group.

However, in the current framework of a self-fulfilling equilibrium, the employer’s preference for discrimination matters only if he has a means of influencing the selection of asymmetric equilibrium or symmetric equilibrium. In the following section, we discuss how the employer can influence the workers’ equilibrium beliefs and the type of equilibrium, and thus, show why the preference matters.

### 3.3 Channels of Equilibrium Selection

We find two channels through which employers can influence equilibrium selection when they prefer discriminatory equilibrium. These are: pre-selection of the pool of job candidates and the unfair tie-breaking rule.
Until now, we have assumed that nature selects the workers who are in competition in proportion to the size of two groups in the population, i.e., \( \lambda_i \). However, an employer does have some degree of control of the selection of pools, especially in the final stage of recruiting, when determining whether the finalists are more likely to be from one group or both groups. Indeed, the equal opportunity protection guaranteed by laws in the U.S. mainly concerns guaranteeing the fairness of opportunity in this pre-selection stage. Suppose in that stage, the employer favors a group by selecting finalists more frequently from one group than the other regardless of \( \lambda_i \). This implies that there may be an inconsistency between the population \( \lambda_i \) and the employer-chosen group representation rate, \( \hat{\lambda}_i \), in the pool of finalists. What matters in determining the equilibrium dynamics is \( \hat{\lambda}_i \). Then, the employer can select its preferred equilibrium type by optimally pre-selecting the pool of candidates \( \hat{\lambda}_i \). Propositions 7 and 8 show that the optimal \( \hat{\lambda}_i \) exists.

More importantly, this implies that discriminatory equilibrium can persist even if \( \lambda_i \) itself is not greatly inclined toward one group, as in the case of gender distribution. Even if \( \lambda_i = 1/2 \), as the employer optimally makes the choice of an appropriate \( \hat{\lambda}_i \) biased toward one group in the labor market, the workers of the unfavored group are discouraged from investing in qualification since they correctly anticipate that they are unlikely to be chosen as the finalists. In this way, discriminatory equilibrium can become salient based on the workers’ rational beliefs of a biased \( \hat{\lambda}_i \).

Another means by which the employer may be able to determine the equilibrium path is in the choice of \( \varphi \). In Section 2, we have shown that symmetric equilibrium exists only when \( \varphi = 1/2 \) for \( \alpha_i = \alpha_j \). However, the strategy \( \varphi \neq 1/2 \) is also sequentially rational for the employer at that decision node when \( \alpha_i = \alpha_j \). If \( \varphi \neq 1/2 \), the resulting equilibrium is asymmetric. That is, symmetric equilibrium requires that workers expect the employer to use a “fair” decision rule even when the employer is indifferent between being fair and not. Although being unfair to workers from group \( j \) does not affect the employer’s payoff at the moment given that \( \alpha_i = \alpha_j \), it would make a difference by reducing the group \( j \) workers’ expected payoffs for being equally qualified as group \( i \) workers. Thus, a seemingly harmless deviation from the fair rule will alter the entire equilibrium path by altering the workers’ beliefs.

4 Policy and Welfare Implications

4.1 Equal Opportunity Laws

Our discussion in the previous section sheds light on the important roles of equal opportunity laws in shaping workers’ expectation of the opportunities to obtain returns from their investment. If \( \lambda_i \) is biased toward one group, the employer may prefer discrimination. Moreover, even if the
true population parameter is $\lambda_i \approx 1/2$, if asymmetric equilibrium is preferable to the employer, the employer may choose $\tilde{\lambda}_i$ so that $\tilde{\lambda}_i \gg 1/2$, were it not for policy intervention. Also, a seemingly harmless unfair tie-breaking rule $\varphi \neq 1/2$ can alter the entire dynamics of the game by changing the workers’ beliefs of the expected return from investment.

In such situations, Equal Opportunity laws help to secure a level playing field for workers from all groups, thus ensuring that the equilibrium is more likely to be non-discriminatory. Equal Opportunity laws declare it illegal to make “employment decisions based on stereotypes or assumptions of the abilities, traits, or performance of individuals of a certain sex, race, age, religion, or ethnic group, or individuals with disabilities, [...].”\(^{10}\) The interpretation of the laws differs among states. In some states, affirmative action programs are often used to restore a balance between minority and majority representations in the workplace by encouraging employers to treat minority workers more favorably, which induces $\tilde{\lambda}_i \approx 1/2$. In other states, equal opportunity protection simply means fair judging, when other things are equal, ensuring a fair tie-breaking rule $\varphi = 1/2$ when appropriate.

In any event, it is clear that, as a result of the laws, an expectation of favorable conditions or fair opportunities will enhance the minority workers’ incentive for investment. This makes symmetric equilibrium more preferable to the employer as well. In this way, the laws guarantee fair opportunities for capable minority workers who, without such laws, would have had few prospects, due to their self-fulfilling expectation of unfavorable returns from investment based on their low level of representation in the population.

## 4.2 Fairness vs. Efficiency

Although the Equal Opportunity laws enhance fairness, whether or not they also improve efficiency is another question. Since only one position is available for two competing workers in the current framework, only the selected worker will obtain a return from qualification. The investment of unselected worker will be wasted. Thus, it is efficient if only one worker becomes qualified. Besides, since the winning group’s expected probability of qualification is higher in asymmetric equilibrium, asymmetric equilibrium is more efficient than symmetric equilibrium in increasing the quality of selected workers.\(^{11}\)

\(^{10}\)http://www.eeoc.gov/

\(^{11}\)For the same reason, a higher level of qualification does not necessarily mean higher welfare for the workers. For example, suppose either $\text{HS}$ or $\text{LS}$ is feasible in equilibrium as in the case in the range where $p_l \leq p < p_m$. Although both groups have a higher qualification level at $\text{HS}$ than at $\text{LS}$, this does not necessarily imply that workers are better off under $\text{HS}$. Since only one position is available for two competing workers, having a high probability of qualification does not necessarily enhance one group’s probability of being selected, especially if the other group also has a high probability of qualification. Moreover, if both groups invest a lot, it means a greater loss of efficiency since a higher level of investment by unselected workers will be wasted. Thus, such a waste is greater at $\text{HS}$ than at $\text{LS}$.  

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Then, what is the cost of discrimination? Discrimination obviously lowers the welfare of the minority group. However, even the majority workers may not benefit from discrimination. This is due to more severe competition within their own group. With the advantage of unequal treatment, more workers from the majority group decide to invest in qualification. However, since the majority group workers are more frequently competing with another from the same group than with minority workers, they will not enjoy a significant competitive advantage. In fact, a substantial proportion of their increased investment will be wasted because only half of them can be employed in competition with another majority worker. Hence, the increased competition within the majority group may actually lower their welfare.

More importantly, the problem with asymmetric equilibria in this paper is that the mechanism of selecting winners is not random or individual, but systematic and group-oriented. Although, in principle, either group should be eligible for a favorable treatment, the employer prefers asymmetric equilibrium only if the allocation is favorable to the majority. Thus, if the employer’s preference is the main driving force of discrimination, it means systematically imposing disadvantages only on the minority group.

In this framework, the problem of wasteful investment fundamentally stems from a shortage of available positions, which is independent of workers’ qualification. Therefore, discrimination cannot be the solution to the problem of wasteful human capital investment. Instead, creating more positions in the economy would be a more effective solution for both discrimination and wasteful investment. More jobs would encourage workers’ investment and engage more qualifying individuals. In a similar framework, Mialon (2013) shows that prejudice is less likely to occur as the shortage of positions declines.

5 Concluding Remarks

We provide a model of discrimination in employment when there is competition between two groups of workers due to a shortage in available positions. We show that, when facing competition among workers, an employer may seek discriminatory equilibrium.

A comparison of the employer’s payoffs under symmetric equilibrium and asymmetric equilibrium shows the role of discrimination in this framework. Given that equal treatment of the two groups is equivalent to having within-group interactions only, discrimination would be preferred only when it helps to divide the population into groups that act differently. In order to divide the population into groups, group-specific traits are necessary, no matter how irrelevant they are, since it is impossible without them to systematically differentiate within the same

Nevertheless, high levels of qualification of both groups are beneficial to the employer as it lowers the chance of encountering a signal $M$.  

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group. Thus, group-specific traits are used as a means to facilitate such a division.

Dividing the population into groups helps the employer because it can lower overall risk. The employer faces a risk when hiring a worker with a noisy signal $M$. The risk when the two group workers have different levels of expected probability of qualification differs from the risk encountered when the levels are the same. The employer prefers discrimination when the risk is lower as a result of creating different expected probabilities for the two groups.

What makes unequal treatment of potentially identical groups profitable is the curvature of the employer’s total expected payoff $W$ and the different size of each group $\lambda_i$. In this framework, $W$ becomes non-linear because two workers are competing for one position. As the degree of competition grows, $W$ is expected to become more non-linear, indicating that the employer’s payoffs become more sensitive to the spread of risks across groups in competitive signaling. Therefore, the preference for discrimination due to the benefit of a spread of risk is somewhat intrinsic in the case of competitive signaling.

While we consider only two groups in population for simplicity, the qualitative results of the current model will hold even when the model is extended to the case of many groups. One of the main reasons is that, although more than two types of group indices are available, employers may not want to utilize all of them for the purpose of discrimination. Given that treating all groups equally is an option that is always feasible for employers, it is essentially up to them to determine how many of the available indices to use for differentiation and discrimination. When discrimination is preferred, the optimal number of group indices to use will be chosen so as to maximize the effect of introducing differential investment incentives on the "effectively" differentiated groups in the population. If the current observation of discrimination (as in the case of gender or race discrimination) reflects such an optimization process, it implies that the optimal number of effectively differentiated groups is often two. This enhances the generality of the results of our paper.

We show how employers can influence the equilibrium selection by their choice of finalist and tie-breaking rules. Thus, the employers’ preference for discrimination is of significant importance in determining the equilibrium. We argue that the Equal Opportunity laws play important roles in securing a level playing field and a fair incentive mechanism for workers.

Appendix: Proofs

5.1 Symmetric equilibria

We first show that $\beta$ is an increasing function of $k_i$ and a strictly decreasing function of $k_j$.

Lemma 1 $\beta$ is an increasing function of $k_i$ given a fixed $k_j$ and a strictly decreasing function of $k_j$ given a fixed $k_i$. 
Proof. Consider (4). Since $F(k)$ is strictly increasing, it is sufficient to show that $\beta$ is an increasing function of $\alpha_i$ given a fixed $\alpha_j$ and a strictly decreasing function of $\alpha_j$ given a fixed $\alpha_i$. For any pair $\alpha'_i > \alpha_i, (q-u)1_{\{\alpha'_i \geq \alpha_s\}} \geq (q-u)1_{\{\alpha_i \geq \alpha_s\}}$ since $q > u$, and $\varphi(\alpha'_i, \alpha_j) \geq \varphi(\alpha_i, \alpha_j)$ given $\alpha_j$. The first term is a strictly decreasing function of $\alpha_j$:

$$p_H(\alpha_j) \frac{1}{2} + p_M(\alpha_j) + p_L(\alpha_j) = -\frac{1}{2} \alpha_j (1 - q) + 1.$$

For any pair $\alpha'_j > \alpha_j$, we have $\varphi(\alpha_i, \alpha'_j) \leq \varphi(\alpha_i, \alpha_j)$ given $\alpha_i$, and

$$p_M(\alpha_j) \varphi(\alpha_i, \alpha_j) + p_L(\alpha_j)$$

$$= -\alpha_j [(1-u) - (q-u) \varphi(\alpha_i, \alpha_j)] + u \varphi(\alpha_i, \alpha_j) + (1-u),$$

where $(1-u) - (q-u) \varphi(\alpha_i, \alpha_j) > 0$ for all $(\alpha_i, \alpha_j)$, which shows the latter. ■

Proof of Proposition 1. First, $\beta_1(0) v = (1-q) [1 - \frac{1}{2} F(0) (1-q)] > 0$ and $\beta_1(\mathfrak{r}) v = (1-q) [1 - \frac{1}{2} F(\mathfrak{r}) (1-q)] v < \mathfrak{r}$ since $v < \mathfrak{r}$. Then, from Lemma 1, there exists $k_l \in (0, \mathfrak{r})$ such that $\beta_1(k_l) v = k_l$. Similarly, for $\beta_h$, we have $\beta_h(0) v > \beta_1(0) v > 0$ and $\beta_h(\mathfrak{r}) v < \mathfrak{r}$, so there exists $k_h \in (0, \mathfrak{r})$ such that $\beta_h(k_h) v = k_h$. Furthermore, $k_h$ and $k_l$ are unique, and $k_h > k_l$ because $\beta_h(k) > \beta_1(k)$, and $\beta_1$ and $\beta_h$ are decreasing functions of $k$.

If $k_s \leq k_l$, the unique fixed point of $\beta_1$, $k_l$, cannot be attained, and since $k_s \leq k_l < k_h$, $k_h$ can be attained. If $k_s > k_h$, the unique fixed point of $\beta_h$, $k_h$, cannot be attained, and since $k_s > k_h > k_l$, $k_l$ can be attained. If $k_l < k_s \leq k_h$, since $k_l < k_s$ and $k_s \leq k_h$, both can be attained. ■

5.2 Asymmetric equilibria

By construction, in asymmetric equilibrium, $k_A \neq k_B$, and thus, group $i$ workers’ net increase in the expected payoff from the investment defined in (4) now depends on whether $k_i > k_j$ or $k_i < k_j$, through $\varphi(\alpha_i, \alpha_j)$ in the term $S_i = M$. However, if $k_i < k_s$, the term with $S_i = M$ disappears. So, only for $k_i \geq k_s$, we need to consider two cases, $k_i > k_j$ or not. Hence, for $k_i \geq k_s$, we define $G_{iu} : [0, \mathfrak{r}]^2 \rightarrow [0, 1]$ and $G_{id} : [0, \mathfrak{r}]^2 \rightarrow [0, 1]$ as the expected payoff of worker $i$ when $k_i > k_j$ and when $k_i < k_j$, respectively. That is,

$$G_{iu}(k_i, k_j) \equiv \lambda_i \beta_h(k_i) + (1-\lambda_i) \beta_u(k_j); \quad (15)$$

$$G_{id}(k_i, k_j) \equiv \lambda_i \beta_h(k_i) + (1-\lambda_i) \beta_d(k_j), \quad (16)$$

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where for a given \( k \), \( \beta_u : [0, 1] \rightarrow [0, 1] \) and \( \beta_d : [0, 1] \rightarrow [0, 1] \) are defined as

\[
\begin{align*}
\beta_u (k) & \equiv (1 - q) \left[ P_H (k) \frac{1}{2} + P_M (k) + P_L (k) \right] + (q - u) \left[ P_M (k) + P_L (k) \right]; \\
\beta_d (k) & \equiv (1 - q) \left[ P_H (k) \frac{1}{2} + P_M (k) + P_L (k) \right] + (q - u) P_L (k).
\end{align*}
\]

Note that \( \beta_h \) is a function of \( i \) with respect to \( k_i \) as it represents group \( i \) worker’s payoff from competition against a same group worker, whereas \( \beta_u \) and \( \beta_d \) are the functions with respect to \( k_j \) as they represent group \( i \) worker’s payoff from competition against a worker from a different group. For \( k_i < k_s \), we define \( G_{id} (k_i, k_j) \) as follows.

\[
G_{id} (k_i, k_j) \equiv \lambda_i \beta_i (k_i) + (1 - \lambda_i) \beta_l (k_j),
\]

(17)

When a group \( i \) worker is paired with a worker from the same group with a probability \( \lambda_i \), it necessarily induces a symmetric equilibrium. Thus, \( \beta_h (k_i) \) and \( \beta_l (k_i) \) coincide with those defined in (8) or (9).

Note that for each \( k \), the following relationship holds:

\[
\beta_u (k) > \beta_h (k) > \beta_d (k) > \beta_l (k).
\]

This relationship in (18) shows the effect of inter-group competition \( \varphi \) on group \( i \) workers’ incentive to invest. Winning a competition against group \( j \) workers enhances the incentive to invest. In the following, we show that this property has an important role in asymmetric equilibria.

First, for each possible case of \( \gamma \in \{ u, d, l \} \), consider an implicit function \( g_{i\gamma} : [0, \bar{v}] \rightarrow (0, \bar{v}) \) such that \( k_i = g_{i\gamma} (k_j) \) for any given \( k_j \).

**Lemma 2** For each \( \gamma \in \{ u, d, l \} \), there exists a unique continuous and strictly decreasing function \( g_{i\gamma} \) that satisfies

\[
G_{i\gamma} (g_{i\gamma} (k_j), k_j) \nu = g_{i\gamma} (k_j).
\]

(19)

**Proof of Lemma 2.** We only prove for \( g_{id} \) since the other proofs can be obtained in a similar way. Since \( G_i (0, \bar{v}) = G_{id} (0, \bar{v}) \),

\[
0 < G_{id} (0, \bar{v}) \nu = [\lambda_i \beta_l (0) + (1 - \lambda_i) \beta_l (\bar{v})] \nu < [\lambda_i \beta_h (0) + (1 - \lambda_i) \beta_d (\bar{v})] \nu = G_{id} (0, \bar{v}) \nu.
\]

Since \( \beta_d \) is decreasing, for each \( c \in [0, \bar{v}] \),

\[
G_{id} (0, c) \nu > 0.
\]

(20)
Since $G_i(\overline{c}, 0) = G_{iu}(\overline{c}, 0)$,

$$\overline{c} > G_{iu}(\overline{c}, 0) v = [\lambda_i \beta_h(\overline{c}) + (1 - \lambda_i) \beta_u(0)] v$$

$$\overline{c} > [\lambda_i \beta_h(\overline{c}) + (1 - \lambda_i) \beta_d(0)] v = G_{id}(\overline{c}, 0) v.$$

Since $\beta_d$ is decreasing, for each $c \in [0, \overline{c}]$,

$$G_{id}(\overline{c}, c) v < \overline{c}. \tag{21}$$

Note that $G_{id}(k_i, k_j) v = k_i$ can be rewritten as

$$\beta_d(k_j) = b_1 k_i - b_2 \beta_h(k_i),$$

where $b_1 = \frac{1}{(1 - \lambda_i)} v$, $b_2 = \frac{\lambda_i}{(1 - \lambda_i)}$. (20) and (21) imply that for each $c \in [0, \overline{c}]$,

$$\beta_d(c) > 0 - b_2 \beta_h(0) \text{ and } \beta_d(c) < b_1 \overline{c} - b_2 \beta_h(\overline{c}).$$

Since $b_1 k_i - b_2 \beta_h(k_i)$ is a continuous and strictly increasing function of $k_i$, there exists a unique continuous functions $g_{id} : [0, \overline{c}] \rightarrow (0, \overline{c})$ such that

$$\beta_d(k_j) = b_1 g_{id}(k_j) - b_2 \beta_h(g_{id}(k_j)),$$

Moreover, $\beta_d$ is decreasing, and $b_1 k_i - b_2 \beta_h(k_i)$ is strictly increasing. So, $g_{id}$ is strictly decreasing.

To show how an asymmetric treatment of two groups affects their investment incentives differently, first let us define several reference points that resembles the symmetric cutoffs under different incentives, $\gamma = \{u, d, l\}$. These reference points will be used later to establish the location of asymmetric equilibrium allocations.

**Lemma 3** Let $k_{iu}$, $k_{jd}$, and $k_{jl}$ be the fixed points of $G_{id}(k_i, k_i) v = k_i$, $G_{jd}(k_j, k_j) v = k_j$, and $G_{jl}(k_j, k_j) v = k_j$, respectively. Then,

(i) $k_{iu} > k_{jd} > k_{jl} = k_l$

(ii) For any given $k \in [0, \overline{c}]$, $g_{jd}(k) > g_{jl}(k)$

**Proof of Lemma 3.** (i) Suppose $k_{jd} \geq k_{iu}$. By construction, this implies

$$\lambda_j \beta_h(k_{jd}) + (1 - \lambda_j) \beta_d(k_{jd}) \geq \lambda_i \beta_h(k_{iu}) + (1 - \lambda_i) \beta_u(k_{iu})$$

$$\Leftrightarrow (1 - \lambda_i) \beta_h(k_{jd}) + \lambda_i \beta_d(k_{jd}) \geq \lambda_i \beta_h(k_{iu}) + (1 - \lambda_i) \beta_u(k_{iu}),$$

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and since $\beta_h$ and $\beta_d$ are decreasing,

$$(1 - \lambda_i) \beta_h(k_{iu}) + \lambda_i \beta_d(k_{iu}) \geq \lambda_i \beta_h(k_{iu}) + (1 - \lambda_i) \beta_u(k_{iu})$$

$$\Leftrightarrow (1 - \lambda_i) [\beta_h(k_{iu}) - \beta_u(k_{iu})] + \lambda_i [\beta_d(k_{iu}) - \beta_h(k_{iu})] \geq 0,$$ 

which contradicts $\beta_h(k) < \beta_u(k)$ and $\beta_d(k) < \beta_h(k)$. Similarly, we can prove that $k_{jd} > k_{jl}$.

(ii) Since for each $k \in [0, \overline{\gamma}]$, $\beta_h(k) > \beta_d(k) > \beta_l(k)$,

$$\beta_d(k_i) = \frac{g_{jd}(k_i)}{1 - \lambda_j} - \frac{\lambda_j}{(1 - \lambda_j)} \beta_h(g_{jd}(k_i))$$

implies

$$\beta_l(k_i) < \frac{g_{jd}(k_i)}{1 - \lambda_j} - \frac{\lambda_j}{(1 - \lambda_j)} \beta_l(g_{jd}(k_i)).$$

Since $\frac{k_j}{(1 - \lambda_j)} - \frac{\lambda_j}{(1 - \lambda_j)} \beta_h(k_j)$ is a strictly increasing function of $k_j$, given each $k_i$, we must have $g_{jl}(k_i) < g_{jd}(k_i)$. ■

**Proof of Proposition 2.** Step 1. Show that there exist $(k^e_i, k^e_j)$ and $(k^m_i, k^m_j)$.

It follows from Lemma 2 and Lemma 3 that $k_{iu} > k_{jd} = g_{jd}(k_{ja}) > g_{jd}(k_{iu})$, which in turn implies

$$g_{iu}^{-1}(k_{iu}) - g_{jd}(k_{iu}) > 0.$$ 

On the other hand, since $g_{iu}$ is strictly decreasing, $g_{iu}(0)$ is the maximum of $g_{iu}$, and $g_{iu}(0) < \overline{\gamma}$. By Lemma 2,

$$g_{iu}^{-1}(g_{iu}(0)) - g_{jd}(g_{iu}(0)) < 0.$$ 

The continuity of $g_{iu}$ and $g_{jd}$ entails that there exists $k^m_i \in (k_{iu}, \overline{\gamma})$ such that

$$g_{iu}^{-1}(k^m_i) - g_{jd}(k^m_i) = 0.$$ 

Given $k^m_i$, the value of $g_{iu}^{-1}(k^m_i)$ is $k^m_j$, which must be in $(0, \overline{\gamma})$. Hence, given $(k^m_i, k^m_j)$,

$$G_{iu}(k^m_i, k^m_j)v = k^m_i \text{ and } G_{jd}(k^m_j, k^m_i)v = k^m_j.$$ 

Since $g_{iu}^{-1}$ is strictly decreasing, and $k_{iu} = g_{iu}^{-1}(k_{iu})$, given $k^m_i > k_{iu}$, we have $k^m_i > k^m_j$.

By Lemma 3,

$$0 = g_{iu}^{-1}(k^m_i) - g_{jd}(k^m_i) < g_{iu}^{-1}(k^m_i) - g_{jl}(k^m_i).$$ 

On the other hand, $g_{iu}^{-1}(g_{iu}(0)) - g_{jl}(g_{iu}(0)) < 0$. The continuity of $g_{iu}$ and $g_{jl}$ implies that
Hence, given \((k^e_i, k^e_j)\),

\[
g_{iu}^{-1}(k^e_i) - g_{ji}(k^e_i) = 0.
\]

Hence, given \((k^e_i, k^e_j)\),

\[
G_{iu}(k^e_i, k^e_j)u = k^e_i \quad \text{and} \quad G_{ji}(k^e_j, k^e_i)v = k^e_j.
\]

Since \(g_{iu}^{-1}\) is strictly decreasing, and \(k_{iu} = g_{iu}^{-1}(k_{iu})\), given \(k^e_i > k_{iu}\), we have \(k^e_i > k^e_j\).

**Step 2.** Show the characterization.

Note that both \((k^m_i, k^m_j)\) and \((k^e_i, k^e_j)\) are on the graph \(k_i = g_{iu}(k_j)\) where \(g_{iu}\) is strictly decreasing. Thus, \(k^e_i > k^m_i\) implies \(k^e_j > k^m_j\). Thus, we have

\[
0 < k^e_j < k^m_j < k^m_i < k^e_i < \bar{c}.
\]

**Proof of Proposition 3.** Consider (15), (16) and (17). If \(k_s = k^e_j\), the fixed point \((k^e_i, k^e_j)\) cannot be attained, and since \(k_s \leq k^e_j < k^m_i < k^m_f, (k^m_i, k^m_j)\) can be attained. If \(k^m_j < k_s \leq k^e_i\), the fixed point \((k_i^e, k^m_j)\) cannot be attained, and since \(k^e_i < k^m_i < k_s \leq k^m_i\), \((k_i^e, k^m_j)\) can be attained. If \(k^e_j < k_s \leq k^m_j\), since \(k^e_j < k_s < k^e_i\) and \(k_s \leq k^m_i < k^m_j\), both can be attained. Lastly, if \(k_s > k^e_i\), neither can be attained.

**Proof of Proposition 4.** (i) First, we show \(k_{iu} > k_h\). Suppose \(k_{iu} \leq k_h\). This implies that \(\lambda_i\beta_h(k_{iu}) + (1 - \lambda_i)\beta_u(k_{iu}) \leq \lambda_i\beta_h(k_h) + (1 - \lambda_i)\beta_u(k_h)\), since \(\beta_h\) is decreasing, we have \(\lambda_i\beta_h(k_{iu}) + (1 - \lambda_i)\beta_u(k_{iu}) \leq \lambda_i\beta_h(k_{iu}) + (1 - \lambda_i)\beta_u(k_{iu})\), so that \(\beta_u(k_{iu}) \leq \beta_h(k_{iu})\), which contradicts \(\beta_u(k_{iu}) > \beta_h(k_{iu})\). Then, from the proof of Proposition 3, \(k_{iu} < k_h < k^m_i\). Similarly, we show \(k_{jd} < k_h\). Suppose \(k_{jd} \geq k_h\). This implies \((1 - \lambda_j)\beta_h(k_{jd}) + \lambda_j\beta_d(k_{jd}) \geq (1 - \lambda_j)\beta_h(k_j) + \lambda_j\beta_h(k_j)\), since \(\beta_h\) is decreasing, we have \((1 - \lambda_j)\beta_h(k_{jd}) + \lambda_j\beta_d(k_{jd}) \geq (1 - \lambda_j)\beta_h(k_{jd}) + \lambda_j\beta_h(k_{jd})\), so that \(\beta_d(k_{jd}) \geq \beta_h(k_{jd})\), which contradicts \(\beta_d(k_{jd}) < \beta_h(k_{jd})\). Since \(g_{jd}\) is strictly decreasing, and \(g_{jd}(k_{jd}) = k_{jd}, k_h > k_{jd} > k^m_j\).

(ii) Similarly, the proof of Proposition 3 and the property of \(g_{jd}\) can show the result.

**Proof of Proposition 5.** Proposition 4 entails that \(\underline{k} < k_l < k_h < \bar{k}\). Then, from (2), \(\rho_h\) can be defined as a value at which

\[
\rho_h \equiv \frac{uF(\bar{k})}{uF(\underline{k}) + q(1 - F(\bar{k}))}, \quad (22)
\]
and similarly, at \(\rho_m\) and \(\rho_l\),
\[
\rho_m \equiv \frac{uF(k_h)}{uF(k_h) + q(1 - F(k_h))}, \quad \text{and} \quad \rho_l \equiv \frac{uF(k)}{uF(k) + q(1 - F(k))}.
\]

Proposition 1 and Proposition 3 establish the results. ■

5.3 Discrimination

**Properties of** \(U_H(\alpha)\) **and** \(U_L(\alpha)\) **at the Symmetric Equilibrium.** If both workers are from the same group \(i\), because the two workers can no longer be differentiated by group, it must be that \(\varphi = 1/2\). Then, when \(\alpha \geq \alpha_s\),
\[
U_H(\alpha) = R \left\{ (1 - q) \left(2\alpha - \alpha^2(1 - q)\right) + \alpha q(\alpha q + (1 - \alpha)u) \right\} + 2\alpha(1 - \alpha)q(1 - u) - v(1 - (1 - \alpha)^2)(1 - u)^2), \tag{23}
\]
Similarly, when \(\alpha \leq \alpha_s\),
\[
U_L(\alpha) = (R - v)(1 - q) \left(2\alpha - \alpha^2(1 - q)\right), \tag{24}
\]
Thus, \(U_H(\alpha) > U_L(\alpha)\) for any given \(\alpha\).

From (23) and (24), \(U(\alpha)\) is discontinuous when \(\alpha\) is around \(\alpha_s\). However, \(U_H(\alpha)\) is twice differentiable with respect to \(\alpha\) in the range where \(\alpha > \alpha_s = \frac{u\rho}{u\rho + q(1 - \rho)}\) and so is \(U_L(\alpha)\) in the range where \(\alpha \leq \alpha_s\). When \(\alpha \geq \alpha_s\), with respect to \(\alpha\),
\[
\frac{dU_H(\alpha)}{d\alpha} = -2\alpha[R(1 - qu) - v(1 - u)^2] + R(2 - qu) - 2v(1 - u)^2, \quad \text{and} \tag{25}
\]
\[
\frac{d^2U_H(\alpha)}{d\alpha^2} = -2[R(1 - qu) - v(1 - u)^2] < 0
\]
since \(R > v, (1 - qu) > (1 - u) > (1 - u)^2\). The highest level of \(U_H(\alpha)\) is achieved at \(\hat{\alpha} = \frac{2R - quR - 2v(1 - u)^2}{2R - 2quR - 2v(1 - u)^2}\). Similarly, for \(\alpha < \alpha_s\),
\[
\frac{dU_L(\alpha)}{d\alpha} = (R - v)(1 - q)[2 - 2\alpha_s(1 - q)] > 0, \quad \text{and} \tag{26}
\]
\[
\frac{d^2U_L(\alpha)}{d\alpha^2} = -2(R - v)(1 - q)^2 < 0.
\]

Proof of Proposition 7. From Propositions 2 and 4, \(\alpha_i^m > \alpha_h > \alpha_j^m > \alpha_s\). Thus, as
\[ dU_H(\alpha) / d\alpha > 0, \quad U_H(\alpha^m_i) > U_H(\alpha_h) > U_H(\alpha^m_j). \] Then, we can rewrite \( U_H(\alpha^m_j) = bU_H(\alpha_h), \) \( b > 1 \) and \( U_H(\alpha^m_j) = sU_H(\alpha_h), s < 1. \) Also, let \( U_H(\alpha^m_i, \alpha^m_j) = \delta U_H(\alpha^m_i). \) Then,

\[
W_{MA} = \lambda^2 U_H(\alpha^m_i) + (1 - \lambda)^2 U_H(\alpha^m_j) + 2\lambda(1 - \lambda)U_H(\alpha^m_i, \alpha^m_j)
\]

\[ = \left\{ \begin{array}{l}
(\lambda^2 + 2\lambda(1 - \lambda)\delta)U_H(\alpha^m_i) + (1 - \lambda)^2 U_H(\alpha^m_j) \\
\{[\lambda^2 + 2\lambda(1 - \lambda)\delta]b + (1 - \lambda)^2 s\}U_H(\alpha_h).
\end{array} \right. \]

Suppose there is a \( \lambda^m \in (0, 1) \) at which \( W_{MA}(\lambda^m) = W_{HS} = U_H(\alpha_h). \) Then, at \( \lambda^m \)

\[ 1 = \{[(\lambda^m)^2 + 2\lambda(1 - \lambda^m)\delta]b + (1 - \lambda^m)^2 s \} \]

**Step 1.** First, we show that \( \delta < 1 \) and \( \delta b > s. \)

In this range, when \( \alpha_i > \alpha_j > \alpha_s, \)

\[
U_H(\alpha^m_i, \alpha^m_j) = R \left\{ \begin{array}{l}
(1 - q)[\alpha^m_i + \alpha^m_j - \alpha^m_i \alpha^m_j (1 - q)] + \alpha^m_j q[\alpha^m_i q + (1 - \alpha^m_i)u] + qu(\alpha^m_i - \alpha^m_j) \\
-v(1 - (1 - \alpha^m_i)(1 - \alpha^m_j)(1 - w)^2).
\end{array} \right.
\]

Since \( U_H(\alpha^m_i) \) has the same \( \alpha^m_i \) but a higher \( \alpha_j = \alpha^m_j \) than \( U_H(\alpha^m_i, \alpha^m_j), \)

\[
U_H(\alpha^m_i) - U_H(\alpha^m_i, \alpha^m_j) = (1 - \delta)U_H(\alpha^m_i)
\]

\[ = \Delta \alpha_j \{(1 - \alpha^m_i)(R - vu^2) + R(1 + \alpha^m_i)qu \} > 0 \]

The symmetry between \( \alpha_i \) and \( \alpha_j \) implies that \( U_H(\alpha^m_i, \alpha^m_j) - U_H(\alpha^m_j) = (\delta b - s)U_H(\alpha_h) > 0. \) Thus, \( \delta < 1 \) and \( \delta b > s. \)

**Step 2.** Solving for (28), we obtain

\[
\lambda^m = \left\{ \begin{array}{l}
\frac{-\delta b - s + \sqrt{(\delta b - s)^2 + (1-s)A}}{A} \quad \text{if} \quad A = b + s - 2\delta b > 0, \\
\frac{\delta b - s - \sqrt{(\delta b - s)^2 - (1-s)A}}{A'} \quad \text{if} \quad A' = -A > 0,
\end{array} \right.
\]

and \( \lambda^m \in (0, 1). \)

\[ \lambda^m > 1/2 \quad \text{if} \quad 4 - 2\delta b > b + s \quad \text{when} \quad A > 0, \]

\[ \text{if} \quad 4 - 2s > 2\delta b \quad \text{when} \quad A' = -A > 0. \]

**Step 3.** By construction, \( W_{MA} \) increases as \( \lambda \) increases. Thus, for each \( \lambda > \lambda^m, \) \( W_{MA} > W_{HS}, \) and therefore, the employer is better off at the discriminatory equilibrium.
Proof of Proposition 8. Let $\lambda^e$ be the level at which

$$U_L(\alpha_l) = (\lambda^e)^2 U_H(\alpha_i^e) + (1 - \lambda^e)^2 U_L(\alpha_j^e) + 2\lambda^e(1 - \lambda^e)U_{HL}(\alpha_i^e, \alpha_j^e).$$

The proof is almost the same as that for Proposition 7, except that if the jump from $U_L$ to $U_H$ is large, there may not be a $\lambda^e$ such that $W_{EA}(\lambda^e) = W_{LS} = U_L(\alpha_l)$ in the range $[0,1]$, in which case, for all $\lambda \in [0,1]$, $W_{EA} > W_{LS}$, and the employer prefers discrimination. This case occurs if $\frac{U_H(\alpha_i^e) - U_L(\alpha_j^e)}{\alpha_i^e - \alpha_j^e} > \frac{dU_H(\alpha)}{d\alpha}|_{\alpha = \alpha_l}$. In this case, any convex combination of $U_H(\alpha_i^e)$ and $U_L(\alpha_j^e)$ is greater than $U_L(\alpha_l)$. Thus, for each $\lambda \in [0,1]$, $W_{EA} > W_{LS}$. If $\lambda^e$ exists, the same logic as that shown in the proof of Proposition 7 applies.

References


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