Declining Moral Standards and the Role of Law

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Abstract

This paper models how moral rules form through social learning. Members of society learn morality by observing the behavior of other members. Their incentive to act morally is influenced by their expectation of other members’ moral behavior. The moral standards of a society are built on the outcomes of such interactions over time. We show that moral standards can quickly deteriorate even if the majority of a society’s members have a strong moral value individually. When they observe that there are insufficient public moral sanctions for wrongful actions, the members come to believe that their society’s moral standards are lower than what they had expected. Such a belief encourages more wrongful actions and results in less incentive for the members to act morally. As the moral standards decline, moral rules may not be able to regulate behavior. Moral standards are less likely to degenerate in the presence of legal rules as legal sanctions offer an objective and relatively time-invariant level of expectation regarding the enforcement of rules. Strong morality, in turn, enhances the effectiveness of law enforcement. We discuss a commitment value of law.

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1 Introduction

This paper develops an original model of a social learning process through which public morality forms, in order to understand how moral rules evolve dynamically. The central feature of the model is the characterization of a dynamic learning process in which individual beliefs regarding the moral standards of a society to which its members belong are updated on the basis of observed moral or immoral actions. Individual incentives for (im)moral actions depend on members’ beliefs about the society’s moral enforcement. In turn, observed moral enforcement and (im)moral actions affect their beliefs about the public morality. We investigate the role of law and its relationship to moral rules in this context.

Law and morality differ in many aspects. Posner (2004) states, "Laws are promulgated by public institutions, such as legislatures, regulatory agencies, and courts, on the basis of well-defined deliberative procedures, and are enforced by the police power of the state (p.289)," whereas morality is not necessarily promulgated. Following Cooter (2000), we define law as "an obligation backed by a state sanction," whereas morality is an obligation backed by a social sanction. In particular, moral rules are established through social interactions. Shavell (2002) states that

*The establishment of moral rules [..] comes about in part through a complex process of socialization, learning, and inculcation. When a child is raised by his or her parents, plays with peers, attends school, and the like, the child absorbs many lessons and turns out to feel guilty about certain behaviors and virtuous about others. Along with these lessons the child learns to reproach bad behavior and compliment the good. (p. 231)*

As for this process of establishing morality, an interesting question is how a society comes up with a set of rules with which most of its members are willing to comply, although every member has a different moral value. What makes this more interesting is that no one can observe other person’s moral value, given that it is private information to each person. Since moral rules are enforced on the basis of public support of the rules, a member’s decision to comply requires a belief that the rules are publicly supported and thus enforced. However, what causes the member to develop such a belief without knowing other members’ moral values and beliefs?
For example, consider a moral rule against stealing. What prevents a person from stealing is his guilt and the expected shame from public condemnation of his action. The feeling of guilt is individual and private, although learned from social education or experiences with other people. How intense these feelings are differs by person, and only he knows how guilty he would feel if he steals. Whether his guilt would be sufficient to deter him from stealing depends on how severely he expects to be morally punished by other people if he steals. This expectation of moral punishment depends on the moral values of the society to which he belongs. However, since he does not know other people’s moral values, he does not know exactly how severely he will be punished for stealing. His only resource is his belief about public moral standards and he learns whether his belief is correct by observing the actual public response to his or another person’s theft. Based on his observation, his belief on public moral standards is updated. This updated belief enters into his decision for moral actions in the next period.

This shows that, in order to explain the establishment of moral rules, it is necessary to examine a social learning process in which each member learns society’s public moral values. This paper models the social learning process. The model considers a framework in which individuals’ moral incentives are influenced by the public support or blame that they expect to receive for their actions, apart from their own conscience. With strong public moral support, an individual’s cost of acting immorally is greater and that of taking action against someone else’s immoral action (e.g., blaming him) is lower. Individuals infer public moral standards based on observing the actions of others, and estimate their cost of acting (im)morally. This inference process involves Bayesian updates in one’s belief of public morality. This paper finds that such a social learning process through Bayesian updates may not be informative as the equilibrium beliefs may deviate from true public morality.1

The model reveals several distinctive features of moral rules. First, it shows that the possibility of moral enforcement is very uncertain. What leads to social enforcement of moral rules is individual members’ beliefs of the enforcement. This indicates that the enforcement power of moral rules can be easily and substantially undermined, if the members of society doubt the possibility of enforcement. This is

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1 This Bayesian inference outcome resembles information cascades in Bikhchandani et al (1992). There have been several applications of this framework of information cascades to the study of law. For example, Daughety and Reinganum (1999) model herding behavior by appeals courts in the process of Bayesian updates about a supreme court’s interpretation of the law.
an important weakness of moral rules.

Second, we find that a society’s moral standards can easily decline, even if the members have strong moral values. This paper emphasizes that what facilitates the decline is "imperfect social learning" when the members of a society can infer the social standard based only on the "observed" actions of others. The observed actions may not be perfectly correlated with the members’ true moral values. Thus, they can misrepresent the underlying moral standards of the society. The misunderstanding will increase if there are a few more slight misrepresentations and inadequate moral sanctions.

Consider shoplifting, for example. According to the National Association for Shoplifting Prevention, "more than $13billion worth of goods are stolen from retailers each year," and "[t]he vast majority of shoplifters are "non-professionals" who steal, not out of criminal intent, [...] but as a response to social and personal pressures in their life." Although stealing is illegal and immoral, the shoplifters are caught only "once in every 48 times they steal. They are turned over to the police 50 percent of the time." Thus, as there is little enforcement of rules, more and more shoplifting occurs. Adolescents learn from each other that shoplifting is okay. This example shows how easily a person’s moral sense of a wrongful action can become hazy due to social influence when the majority of people around the shoplifters condone the act.

This paper shows that the existence of a legal sanction prevents such degeneration of morality that is produced by the social learning process. A legal sanction sets an "objective," lower bound for the expected punishment. As a result, the impact of social learning becomes less important in shaping society’s moral standards. In turn, this slows the decline of moral standards and leads to fewer offensive act and, thus, fewer instances of further misrepresentation. Therefore, when moral standards can decline quickly and the dynamic enforceability of moral rules is at risk, as in the case of shoplifting, having legal sanctions helps greatly to regulate behavior.

This finding supports the normative failure theory of law in Cooter (1997), which promotes the role of law to correct failures in social norms. Cooter (1997) states that the failures occur, "because private, informal punishment insufficiently deters wrongdoing (p.979)." This paper reveals an underlying mechanism of social interactions that frequently leads to insufficient private enforcement of desirable social norms. Law is desirable in correcting the failures of private enforcement because law

\[\text{www.shopliftingprevention.org}\]
enforcement is expected to be more certain than private enforcement, as pointed out by Locke (1690). However, a more important feature that this paper highlights is that law is more "objective" than subjective and implicit moral rules and, thus, more immune to the influence of fallible social learning.

Third, this model shows that morality is not necessarily natural. This paper uses a framework in which people inherently do not know the natural morality that embodies the moral values of members of society, and shows that the society’s equilibrium moral standards may not converge to the natural moral standards. Thus, there is no reason to believe that current moral rules have emerged because they fit or should fit the nature of people. A society’s moral rules may impose standards that differ significantly from what its members desire. For instance, although the majority of the members may be inherently prone to steal, they become educated to not steal because of strong moral rules against theft. This implies that it is untenable to advocate rules of law derived from human nature.\footnote{For an example of a reference of natural law theory, see Natural Law and Natural Rights (2011) by John Finnis.}

It also raises a question of what rules, especially laws, "should be," if they do not necessarily embody our nature. This paper argues that laws constitute our commitment to a certain way of regulating the behavior of the members of a society, in the spirit of the argument by Honoré (1993) in cases of moral conflicts.

Moral rules evolve and adapt over time in inter-generational interactions. For this reason, moral rules differ between different groups of people, across culture, and over time. In some countries, dog fighting is considered to be immoral and to violate animal rights, but it is not considered so in other countries.\footnote{Currently, dog fighting is unlawful in the US, and is a felony in many states. However, it is a legal and popular sport in Japan and Pakistan.} There is abundant evidence of time-varying morality. Adultery, pornography, and prostitution not only have been considered to be immoral, but also have been illegal for a long time. However, people are more tolerant of adultery today, and are even more lenient toward pornography and prostitution. These differences and changes may not necessarily reflect changes in underlying moral values. For this reason, morality and law can never be derived solely from the majority opinion as was evident in the case of slavery, which was legal for a prolonged period before it was banned completely. We discuss the implication of the recent repeal of adultery statutes in Colorado in this context.
There are numerous studies of social norms and morality.\textsuperscript{5} The literature on law and economics has proved the complementarity of law and morality in various aspects. Cooter (1998, 2000) analyzes how law transforms vague social norms into an explicit obligation, enhances the deterrence effects by increasing the total sanctions that people face, and promotes individuals’ incentive to acquire morality and self-control. Posner (1997) and Deffains and Fluet (2012, 2014) consider how individual moral concerns interact with legal sanctions and influence individual behavior. Deffains and Fluet (2012) show that when legal liability is perfectly enforced, legal sanctions crowd out efficient informal incentives for precaution, whereas imperfectly-enforced legal liability complements moral incentives. Deffains and Fluet (2014), on the other hand, consider how different legal designs harness moral or reputational incentives for desirable conduct in turn.

Shavell (2002) compares law and morality in social costs of enforcement and effectiveness in controlling behavior. He argues that, if the expected private gain from an undesirable action and the expected harm due to the conduct are great, it is optimal to have law supplement morality. However, if morality does not function well, law alone is optimal. In contrast, this paper emphasizes the dynamic interdependence between law and morality. Especially, this paper stresses that the social costs of moral enforcement are not fixed. The costs of law and morality are inter-dependent as moral rules evolve over time while interacting with legal rules. Thus, the efficiency of the two rules cannot be measured separately. Instead, the efficiency of rules should be measured as a whole with the consideration of the effects on the long-term trajectory of social interactions among people.

The remainder of this paper is organized as follows. Section 2 presents our basic framework of morality formation, external incentives, Bayesian inference and update of public morality, and declining moral standards. In Section 3, we analyze the role of legal sanctions. In Section 4, we discuss the implications of declining morality when law enforcement depends on people’s willingness to become involved in the enforcement. Section 5 concludes.

\textsuperscript{5}For examples of economic analyses of morality, see Frank (1987), Hirshleifer (1987), Berheim (1994), and Kaplow and Shavell (2007).
2 The Model of Moral Incentives and Rules

Suppose that at each period \( t \), nature randomly draw one individual \( I \) and \( N \) neighbors of \( I \) from the population.\(^6\) Nature presents \( I \) with an opportunity to take an offensive, immoral, action \( A \) (i.e., an action that imposes negative externalities on others). \( I \)'s motivation to refrain from taking such an action depends on how strongly he feels about the offensiveness of the immoral action \( A \) ("internal incentive" following Shavell [2002]), and the expected punishment for the action by the society to which he belongs ("external incentive" following Shavell [2002]). If \( I \) takes the action \( A \), each of \( N \) neighbors chooses either to publicly condemn the action (which is sometimes followed by an act of punishment) or to silently condone it. At \( t \), for choosing \( A \), \( I \) receives a benefit \( b \) at the cost of moral anguish \( m_{it} \), and legal sanctions, \( s \), with a probability of \( r \), if any. The moral cost \( m_{it} \) for \( I \) at \( t \) is the weighted sum of the personal moral cost, \( m_{it} \), and the external incentive \( M_t \). That is, \( m_{it} = (1-w)m_{it} + wM_t \), where \( 0 < w < 1 \) is the weight on \( M_t \). The external incentive \( M_t \) is the level of public condemnation (and punishment) that \( I \) expects to receive from his neighbors upon choosing \( A \). \( I \) estimates \( M_t \) based on his past experiences and observations of how society has responded to another similar offensive event \( A' \) in the past. Specifically, at each period \( t \), \( I \) has a learned expectation of

\[
M_t = m \cdot E(N_t|\Omega_t),
\]

where \( m > 0 \), \( \Omega_t \) represents all available information up to period \( t \) based on the observations of people's past responses, \( E(N_t|\Omega_t) = \sum \beta_{jt} \), and \( \beta_{jt} \) is the pure strategy variable of neighbor \( j \in N \) at \( t \) such that \( \beta_{jt} = 1 \) if neighbor \( j \) condemns, or is 0, otherwise. Roughly speaking, \( E(N_t|\Omega_t) \) is the expected number of neighbors who will condemn and punish \( A \) if \( I \) undertakes the action. For a given \( E(N_t|\Omega_t) \), \( I \) chooses

\(^6\)The neighbors are the boundaries of the society that matters to \( I \). Whenever \( I \) decides whether or not to take \( A \), the selection of \( N \) occurs randomly and independently across time in the society. The neighbors may be strangers to or acquaintances of \( I \) in each period. The society can be narrowly defined as a school that \( I \) attends, or widely defined as a country in which \( I \) lives. In each period, \( I \) may or may not be the same person.
\( A \) if and only if

\[
b - rs - [(1 - w)m_{it} + wM_t] \geq 0, \text{ or } \]
\[
m_{it} \leq \frac{b - rs}{1 - w} - \frac{wm}{1 - w}E(N_t|\Omega_t). \tag{2}
\]

Other things being equal, \( I \) is more likely to choose \( A \) as the legal sanction \( s \) decreases. Similarly, if the expected public moral standards \( M_t = E(N_t|\Omega_t) \) decline, \( I \) has less incentive to refrain from taking \( A \).

The public’s willingness to "get involved" in others’ situation and to take action against offensive behavior often deters crime more effectively than does the law. This is particularly true for crimes that are in progress, because public moral enforcement \( M_t \) lowers the probability of successfully committing the crimes. In contrast, law can be in effect only \textit{ex post}. For example, in the case of the murder of Kitty Genovese (1964), the perpetrator had left her wounded, but alive, after the first attack upon hearing the neighbors’ response. However, he returned and completed the murder and rape after observing that no one was coming forward to help her. If there had been a timely and adequate response by the neighbors, the perpetrator would not have been able to complete his criminal acts. Hence, sufficient moral responses by the public are often the most effective means to prevent crimes (or the completion of criminal acts).

The key element of this model is that the formation of the public moral standard \( M_t \) is \textit{social}. Each member learns the moral values from his or her parents, teachings, and observations of other persons’ responses to offensive actions. This implies that each person’s moral incentive evolves dynamically through learning.

We show that social learning may be ineffective. This is because members of a society cannot observe other members’ moral values, but only observe their "actions," which may not reveal perfectly the underlying moral values. Hence, there can be a slight misinterpretation of neighbors’ true moral standards. In the process of learning members’ moral values based on Bayesian inferences, such a misinterpretation lowers expected public moral enforcement and each member’s incentive for a moral action. This will induce more observations of immoral actions and accelerate a decline in morality by making more persons decide to silently condone the immoral actions of others. We show that this can occur even if the majority of the members of the society are strongly moral.
2.1 External Incentives

What determines the external incentives $E(N_t\mid \Omega_t) = \sum \beta_{jt}$? What makes the neighbors or members of a society interested in enforcing moral standards by becoming involved over an action taken by another person, $I$? In this section, we consider neighbor $j$’s decision to condemn $I$’s action $A$ at $t$, $\beta_{jt}$.

For $j$ chosen in period $t$, $jt$, given that there is no explicit rule that mandates one to enforce moral rules regarding someone else’s action, the benefits from criticizing $A$ are only implicit. There are two elements of a neighbor’s motivation to personally "get involved" in enforcing moral rules. These are the neighbor’s own personal moral value $m_{jt}$ and her expected cost of getting involved, which depends on the level of social support for her action. The personal cost is greater if there is strong support from other members of her society.

Upon observing an offensive action by another person, condoning it causes neighbor $jt$ to suffer moral anguish $m_{jt}$. We assume that $m_{jt}$ is independent, private to $jt$, and unobservable by others. Now, consider the cost of taking a moral "action" against $A$. It costs $C_{jt} = c(1 - \sum_{l\neq j} \beta_{lt}/N)$ for $jt$ to publicly condemn $A$ (and take the necessary action), when $\sum_{l\neq j} \beta_{lt}$ is the level of support from other neighbors $lt \neq jt$. $\sum_{l\neq j} \beta_{lt}$ represents the expected total number of $lt$ who concur. That is, a person’s decision to take a public moral action costs $c$ and it decreases if many other people concur.

Then, each neighbor $jt$ condemns the action $A$ if her own $m_{jt}$ is high or if she expects that many other people $lt$ will condemn $A$ (i.e., a high $\sum_{l\neq j} \beta_{lt}$). That is, $jt$ condemns $A$ if and only if

$$m_{jt} > c \left(1 - \frac{\sum_{l\neq j} \beta_{lt}}{N}\right)$$

$$\Leftrightarrow \sum_{l\neq j} \beta_{lt} > \beta_{jt} = \frac{(c - m_{jt})N}{c}. \quad (3)$$

Similar to $jt$’s problem, whether another neighbor $lt$ would be willing to condemn $A$ depends on his own $m_{lt}$ and the expected probability that $jt$ would condemn $A$ as

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5 In the model, we consider only two choices, condemnation or silence. Although $j$ may also choose to publicly "approve" the offensive action, such a choice is always dominated by silence, as in the end, it is an offensive action. Thus, even if $j$ expects to receive great support for not condemning the action, the dominant strategy is to be silent, rather than to actively support the action.

8 In this framework, public support can be either guilt or virtue (see, Kaplow and Shavell [2007]).
well. If $jt$ decides to condemn $A$, then any $lt$ with $m_{lt} > m_{jt}$ would make the same choice to condemn $A$.

The problem is that neighbor $jt$ observes only her own moral cost $m_{jt}$ and not those of the others, $m_{lt}$. More specifically, $jt$ does not know how many neighbors have an $m_{lt}$ that is higher than her own $m_{jt}$ and, thus, how many neighbors would take the same action if she condemns $A$. What this implies is that, for example, she may feel badly about stealing according to her own $m_{jt}$, but does not know if many others feel the same way. In addition, she does not know whether she feels badly because she has a higher moral standard than others, or because she is one of the many normal people. In the former case, she would not expect many persons to condemn $A$. In the latter case, however, she would expect many other persons to condemn.

Therefore, $jt$ needs to know the "relative standing" of her own $m_{jt}$ in the population distribution of moral values. She does not know whether her $m_{jt}$ is in the lower tail or in the upper tail of the distribution. Thus, she is unsure of the proportion of her neighbors whose moral values are higher than her own. If she knows the "distribution" of others’ $m_{lt}$, even if she does not know their individual $m_{lt}$, she can infer the proportion based on the information of her own $m_{jt}$ and the likelihood that she would receive support when she condemns $A$. Therefore, the uncertainty that each member of a society faces in determining her moral action reduces to the uncertainty in the true distribution of the population moral values.

### 2.2 Uncertain Public Moral Standards

To model the members’ uncertainty about the true population distribution, suppose that the true population cumulative density distribution (cdf) of the society’s morality $\tilde{m}$ is either $F_1$ with a mean of $\mu_1$ or $F_2$ with a mean of $\mu_2$, where $\mu_2 > \mu_1$, without loss of generality. Assume that $F_1$ and $F_2$ are continuous in $\tilde{m} \in [\underline{m}, \overline{m}]$, $\underline{m} \geq 0$.

Let $\phi(\tilde{m}) = \frac{f_2(\tilde{m})}{f_1(\tilde{m})}$ be the ratio between the two pdfs $f_2(\tilde{m})$ and $f_1(\tilde{m})$. We assume Monotone Likelihood Ratio Property for the ratio, i.e., $\phi(a_1) \geq \phi(a_2)$ for $a_1 \geq a_2$. Then, the MLRP implies that $F_2$ first order stochastically dominates $F_1$. Let $p > 0$ be the prior probability that the true distribution is $F_1$.

To see how the inferences of public moral standards develops, consider a benchmark case in which the true distribution is $F_1$ and this information is common knowledge. Suppose that there exists an $m_1$ such that at $m_1$, we can define an integer $N_1$
that satisfies $N_1/N = 1 - m_1/c = 1 - F_1(m_1)$ and, thus, $m_1 = cF_1(m_1)$. Similarly, we can define $m_2$.

**Assumption 1** $\overline{m} > c$.

Assumption 1 implies that $\overline{m} > cF_d(\overline{m})$, for $d = 1, 2$. Therefore, from (3), even if $\sum_{lt \neq jt} \beta_{lt} = 0$, there will be $jt$ whose personal moral value is too high to remain silent. Let $m_d^*$ be the largest value of $m_d$, $d = 1, 2$.

**Lemma 1** If it is known that the true distribution of population $\tilde{m}$ is $F_d$, $d = 1, 2$, then $N_d^*/N$ fraction of people condemn $A$, where $N_d^*/N$ satisfies $N_d^*/N = 1 - m_d^*/c$, and $m_d^*$ is the largest value that satisfies $m_d = cF_d(m_d)$.

**Proof.** All proofs are provided in the Appendix. 

Lemma 1 implies that, if the true distribution is $F_1$, all the neighbors $lt$ with $m_{lt} > m_1^*$ condemn $A$ in the expectation that there will be moral support from $N_1^*$ people, and that the expectation is correct because for any $m_{lt} > m_1^*$, $1 - F_d(m_{lt}) > N_1^*/N = \frac{(c-m_1^*)}{c} > \frac{(c-m_{lt})}{c}$. For any $m_{lt} < m_1^*$, $1 - F_d(m_{lt}) < \frac{(c-m_{lt})}{c}$, and the neighbor $lt$ does not want to take a moral action even if everyone else with a higher $m_{kt} > m_{lt}$ does. Knowing that a person with a lower $m_{kt} < m_{lt}$ would be even less willing, the neighbor $lt$ does not. Hence, the rational expectation is that only $N_1^*$ people condemn $A$. Thus, if everyone knows that the true distribution is $F_d$, $d = 1, 2$, when nature randomly draws $N$ neighbors from $F_d$, people expect that $N_d^*/N$ fraction of the neighbors will have moral values that belong to the range above $m_d^*$ and that they will condemn. Since $F_2$ first order stochastically dominates $F_1$, $m_1^* > m_2^*$. Thus, for the same immoral action, a larger fraction of the people are expected to take moral actions if the true population distribution is $F_2$.

Now consider the interactions among $N$ neighbors. At period 0, each neighbor’s conjecture is as follows. While each observes her own moral value only, she believes that if the neighbors are ordered by their moral values as $\{m_{10}, m_{20}, ..., m_{N0}\}$, where $m_{10} > m_{20} > ... > m_{N0}$, there should exist an $m_{N0}$ at which $m_{N0} = c(1 - E(N_0)/N)$.\(^9\) A neighbor $j0$ expects that any neighbor $k0$ with $m_{k0} \geq m_{N0}$ will condemn $A$ in

\(^9\)This critical neighbor $m_{N0}$ does not need to be actually present in the group of neighbors who are selected by nature. All that is required is the expectation of the existence of such a neighbor $m_{N0}$ that separates the group of neighbors into two groups, one group with $m_{jt} > m_{N0}$ and the other group with $m_{lt} < m_{N0}$. 

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the expectation that $E(N_0) = \sum \beta_{j0}$ people will condemn $A$. With this belief, the neighbor $j0$ condemns $A$ if her $m_{j0} > m_{N0}$. Since the fraction of people who condemn $E(N_0)/N$ must consist of those who have a moral value of $m_{j0} > m_{N0}$, the correct expectation is that

$$E(N_0)/N = p(1 - F_1(m_{N0})) + (1 - p)(1 - F_2(m_{N0})), \quad (4)$$

$$m_{N0} = c \{pF_1(m_{N0}) + (1 - p)F_2(m_{N0})\}, \quad (5)$$

and such a $m_{N0}$ exists. Then, the condition for $j0$ to condemn $A$ is

$$m_{j0} > c \{pF_1(m_{N0}) + (1 - p)F_2(m_{N0})\} = \widehat{m}_0(p). \quad (6)$$

Lemma 2 1. For any given $p \in (0, 1)$, $m_2^* < \widehat{m}_0(p) < m_1^*$ and, thus, $N_1^* < E(N_0) < N_2^*$.

2. Let $\widehat{m}_t$ be the critical value that is specified in (5) at $t$ when the prior for distribution $F_1$ is $\widehat{p}_t$. That is,

$$\widehat{m}_t = c \{\widehat{p}_t F_1(m_{Nt}(\widehat{p}_t)) + (1 - \widehat{p}_t)F_2(m_{Nt}(\widehat{p}_t))\}, \quad (7)$$

where $m_{Nt}$ is a fixed point that satisfies $m_{Nt} = \widehat{m}_t(\widehat{p}_t)$. Then, as $\widehat{p}_t$ increases $\widehat{m}_t$ increases. In particular, $\widehat{m}_t(\widehat{p}_t) \rightarrow m_1^*$ as $\widehat{p}_t \rightarrow 1$ and $\widehat{m}_t(\widehat{p}_t) \rightarrow m_2^*$ as $\widehat{p}_t \rightarrow 0$.

2.3 Social Learning Dynamics

Social learning dynamics occur in the members’ Bayesian updated beliefs of the true population distribution $\widehat{p}_t = P(F = F|\Omega_t)$. For simplicity, assume for now that the moral values of $I$ are drawn from a separate, known distribution $G(m_i)$. This allows us to focus on the social learning process that occurs among the $N$ neighbors only. This assumption will be relaxed in the next section when we consider the role of law. In order to have the posteriors remain common, we assume that neighbors do not update on the observation of their own moral value $m_{jt}$.

In the beginning of $t = 1$, each member observes how many people have condemned $A$ at $t = 0$, $\widehat{N}_0$. This may be greater or less than the expected number
From Lemma 2, then is used to update their inference of the probability, increases as \( b \) consists of observed moral enforcement by the neighbors over time. Similarly, the observation of follows. From (2), was taken or not in updating their beliefs. In this case, \( \Phi \) was condemned, as well as the neighbors’ moral enforcement. That is, \( \Omega_t = \Omega(\Phi_{t-1}, \Phi_{t-2}, \Phi_{t-3}, ..., \hat{N}_{t-1}, \hat{N}_{t-2}, \hat{N}_{t-3}...) \), where \( \Phi_t \in \{A_t, A_{t}^u\} \) shows the observed immoral action \( A_t \) or deterrence \( A_{t}^u \) at \( t \).

Since (4) and (5) are defined for any \( p \), we can define \( E(N_t|\Omega_t) \) for a general \( \hat{p}_t \) as follows.

\[
E(N_t|\Omega_t) = E \sum (\beta_j|\Omega_t) = \mathcal{N} [\hat{p}_t(1 - F_1(m_{N_t})) + (1 - \hat{p}_t)(1 - F_2(m_{N_t}))], \tag{8}
\]

From (2), \( I \) chooses \( A \) if and only if \( m_{it} \leq \frac{b - rs}{(1 - w)} - \frac{wm}{(1 - w)} E(N_t|\Omega_t) \), i.e.,

\[
m_{it} \leq \frac{b - rs}{(1 - w)} - \frac{wm}{(1 - w)} \mathcal{N} [\hat{p}_t(1 - F_1(m_{N_t})) + (1 - \hat{p}_t)(1 - F_2(m_{N_t}))] \tag{9}
\]

From Lemma 2, \( \hat{m}_t \) increases as \( \hat{p}_t \) increases. Thus, \( E(N_t|\Omega_t) = \mathcal{N} [1 - \hat{m}_t/c] \) decreases as \( \hat{p}_t \) increases. Then, (9) is more likely to be satisfied. In the following, we show the dynamics of \( \hat{p}_t \).

### 2.3.1 At \( t = 0 \)

**I’s behavior** From (7), \( I \) chooses \( A \) if

\[
m_{i0} \leq \frac{b - rs}{(1 - w)} - \frac{wm}{(1 - w)} E(N_0), \tag{10}
\]

where \( E(N_0) = \mathcal{N} [p(1 - F_1(m_{N_0})) + (1 - p)(1 - F_2(m_{N_0}))] = \mathcal{N} [1 - \hat{m}_0/c] \).

**Neighbors** Neighbor \( j \)0 condemns \( A \) if

---

\(^{10}\)In the full analysis described in Section 3, the members also use the information of whether \( A \) was taken or not in updating their beliefs. In this case, \( \Omega_t \) consists of the past observations of how often \( A \) was taken, as well as the neighbors’ moral enforcement. That is, \( \Omega_t = \Omega(\Phi_{t-1}, \Phi_{t-2}, \Phi_{t-3}, ..., \hat{N}_{t-1}, \hat{N}_{t-2}, \hat{N}_{t-3}...) \), where \( \Phi_t \in \{A_t, A_{t}^u\} \) shows the observed immoral action \( A_t \) or deterrence \( A_{t}^u \) at \( t \).
Suppose that it was observed that $c < e(0)$ at the end of the period $t = 0$. Since $N_1^* < N_2^*$, the observation of $N_0 < E(N_0)$ implies that $P(N_0|F_1) > P(N_0|F_2)$. Similarly, if $N_0 > E(N_0)$, the updated belief is $P(N_0|F_2) > P(N_0|F_1)$. For the analysis below, consider a case when $N_0 < E(N_0)$ at $t = 0$.

2.3.2 At $t = 1$

**Update of the belief** By construction,

$$
\hat{p}_1 = P(F = F_1|N_0) = \frac{p \cdot P(N_0|F_1)}{p \cdot P(N_0|F_1) + (1 - p) \cdot P(N_0|F_2)}
$$

Therefore, $\hat{p}_1 > p$ since

$$
\hat{p}_1 = \frac{p \cdot P(N_0|F_1)}{p \cdot P(N_0|F_1) + (1 - p) \cdot P(N_0|F_2)} > p \Leftrightarrow P(N_0|F_1) > P(N_0|F_2).
$$

This is consistent, given the observation of $N_0 < E(N_0)$.

**I’s behavior** In the expectation that $\hat{p}_1 > p$, I chooses $A$ if and only if

$$
m_{i1} = \min \left\{ \frac{b - rs}{1 - w} - \frac{wm}{1 - w} E(N_1|N_0) \right\}.
$$

Since $m_{i1} = c \{\tilde{p}_1 F_1(m_{N_1}(\tilde{p}_1)) + (1 - \tilde{p}_1) F_2(m_{N_1}(\tilde{p}_1))\} > \tilde{m}_0(p) = c \{p F_1(m_{N_0}(p)) + (1 - p) F_2(m_{N_0}(p))\}$

$$
E(N_1|N_0) = E(\sum \beta_i |N_0) = N \left\{ 1 - \tilde{m}_1(\tilde{p}_1)/c \right\} < E(N_0) = N \left\{ 1 - \tilde{m}_0(p)/c \right\}.
$$

Thus, based on $\hat{p}_1 > p$, I expects a lower likelihood of moral punishment for $A$. Thus, $m_i$ is more likely to satisfy the condition than it was at $t = 0$. Other things being equal, it is more likely that I chooses $A$. 

$$
m_{j0} > c - \frac{c}{N} E(N_0).
$$
Neighbor When $\widehat{N}_0 < E(N_0)$, $P[F = F_1|\widehat{N}_0] = \hat{p}_1 > p$ and $P[F = F_2|\widehat{N}_0] = 1 - \hat{p}_1 < 1 - p$. Thus, if $A$ is observed, $j1$ expects that fewer people $E(N_1|\widehat{N}_0) < E(N_0)$ will condemn $A$ than before. This makes $j1$ less likely to condemn the action, i.e.,

$$m_{j1} > c - \frac{c}{N} E(N_1|\widehat{N}_0) = \hat{m}_1 > \hat{m}_0.$$  

Thus, other things being equal, fewer people are expected to condemn $A$ at $t = 1$. This makes it more likely that people will actually observe $\widehat{N}_1 < \widehat{N}_0$ at $t = 1$.

2.3.3 At $t = 2$,

For given $\widehat{N}_1, \widehat{N}_0$ and $\widehat{N}_1 < \widehat{N}_0$, people infer the probability of true distribution being $F_1$ or $F_2$ in the following way.

$$\hat{p}_2 = P(F = F_1|\widehat{N}_0, \widehat{N}_1) = \frac{P(F = F_1 \cap \widehat{N}_0 \cap \widehat{N}_1)}{P(\widehat{N}_0 \cap \widehat{N}_1)}$$

$$= \frac{P(F_1) \cdot P(\widehat{N}_0 \cap \widehat{N}_1|F_1)}{P(F_1) \cdot P(\widehat{N}_0 \cap \widehat{N}_1|F_1) + P(F_2) \cdot P(\widehat{N}_0 \cap \widehat{N}_1|F_2)}$$

$$= \frac{p \cdot P(\widehat{N}_0|F_1) \cdot P(\widehat{N}_1|F_1)}{p \cdot P(\widehat{N}_0|F_1) \cdot P(\widehat{N}_1|F_1) + (1 - p) \cdot P(\widehat{N}_0|F_2) \cdot P(\widehat{N}_1|F_2)}$$

Suppose that $\widehat{N}_1 < \widehat{N}_0$ at $t = 1$ as it is more likely to occur as shown above. Then, since $P(\widehat{N}_1|F_1) > P(\widehat{N}_1|F_2)$, $\hat{p}_2 > \hat{p}_1$ as well, i.e.,

$$\hat{p}_2 = \frac{p \cdot P(\widehat{N}_0|F_1) \cdot P(\widehat{N}_1|F_1)}{p \cdot P(\widehat{N}_0|F_1) \cdot P(\widehat{N}_1|F_1) + (1 - p) \cdot P(\widehat{N}_0|F_2) \cdot P(\widehat{N}_1|F_2)}$$

$$> \frac{p \cdot P(\widehat{N}_0|F_1)}{p \cdot P(\widehat{N}_0|F_1) + (1 - p) \cdot P(\widehat{N}_0|F_2)} = \hat{p}_1$$

$$\Leftrightarrow P(\widehat{N}_1|F_1) > P(\widehat{N}_1|F_2)$$

Therefore, $\hat{p}_2 > \hat{p}_1 > p$, $\hat{m}_2(\hat{p}_2) > \hat{m}_1(\hat{p}_1)$, and

$$E(N_2|\widehat{N}_1, \widehat{N}_0) = N' [1 - \hat{m}_2(\hat{p}_2)/c] < E(N_1|\widehat{N}_0) < E(N_0)$$

As a result, $I$ is more likely to choose $A$, i.e.,
\[ m_{i2} \leq \frac{b - rs}{1-w} - \frac{wm}{1-w}E(N_2|\hat{N}_1, \hat{N}_0) \]

The neighbor \( j \) is even less likely to condemn \( A \) than before.

\[ m_{j2} > c - \frac{c}{N}E(N_2|\hat{N}_1, \hat{N}_0) = \hat{m}_2 > \hat{m}_1 > \hat{m}_0. \]

That is, \( A \) is more likely to be taken, and people are less likely to condemn it, i.e., \( \hat{N}_2 < \hat{N}_1 \).

Suppose that \( \hat{N}_1 > \hat{N}_0 \), instead. Even in this case, the same inference of \( \hat{p}_2 > \hat{p}_1 \) will hold as long as \( \hat{N}_1 \) is not so large that \( \hat{N}_1 < E(N_0) \). Any changes in the perspectives requires that \( \hat{N}_1 \) is large enough to induce \( \hat{N}_1 > E(N_0) > \hat{N}_0 \). In this case, the result will be \( P(\hat{N}_1|F_1) < P(\hat{N}_1|F_2) \) and, thus, \( \hat{p}_2 < p < \hat{p}_1 \). Since the information about the true distribution of morality from the two periods is conflicting, the next period’s observation would determine which of the two observations is more reliable, along with the future inference process.

### 2.3.4 At \( t = 3 \) and thereafter

For given \( \hat{N}_2, \hat{N}_1, \hat{N}_0 \), people infer the probability of the true distribution being \( F_1 \) in the following way.

\[
\hat{p}_3 = P(F = F_1|\hat{N}_2, \hat{N}_0, \hat{N}_1) = \frac{P(F = F_1 \cap \hat{N}_0 \cap \hat{N}_1 \cap \hat{N}_2)}{P(\hat{N}_0 \cap \hat{N}_1 \cap \hat{N}_2)}
\]

\[
= \frac{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1)}{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1) + (1-p) \cdot P(\hat{N}_0|F_2) \cdot P(\hat{N}_1|F_2) \cdot P(\hat{N}_2|F_2)}
\]

Then, \( \hat{p}_3 > \hat{p}_2 \) if and only if

\[
\frac{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1)}{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1) + (1-p) \cdot P(\hat{N}_0|F_2) \cdot P(\hat{N}_1|F_2) \cdot P(\hat{N}_2|F_2)} > \frac{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1)}{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) + (1-p) \cdot P(\hat{N}_0|F_2) \cdot P(\hat{N}_1|F_2)}
\]

\[
\Leftrightarrow P(\hat{N}_2|F_1) > P(\hat{N}_2|F_2)
\]
If $\hat{N}_0 < E(N_0)$, it is more likely that $\hat{N}_1 < \hat{N}_0$, and if $\hat{N}_1 < \hat{N}_0$, it is even more likely that $\hat{N}_2 < \hat{N}_1$. Suppose that $\hat{N}_2 < \hat{N}_1 < \hat{N}_0$, indeed. Since it is more likely that $P(\hat{N}_2|F_1) > P(\hat{N}_2|F_2)$, we obtain $\hat{p}_3 > \hat{p}_2 > \hat{p}_1 > p$, $\hat{m}_3(\hat{p}_3) > \hat{m}_2(\hat{p}_2)$, and

$$E(N_3|\hat{N}_2, \hat{N}_1, \hat{N}_0) = \mathcal{N}[1 - \hat{m}_3(\hat{p}_3)/c] < E(N_2|\hat{N}_1, \hat{N}_0).$$

Thus, as the past observation of moral enforcement $\hat{N}_{t-1}$ declines, the expected level of moral enforcement $E(N_t|\hat{N}_{t-1}, \hat{N}_{t-2}, \hat{N}_{t-3}...)$ declines. This makes it more likely for the next period moral enforcement $\hat{N}_t$ to decline, and more likely to encourage $A$, and so on. That is, as $E(N_t|\Omega_t)$ declines, it is more and more likely that

$$m_{jt} \leq \frac{b - rs}{(1 - w)} - \frac{wm}{(1 - w)}E(N_t|\Omega_t),$$

and the neighbor $j$ will be less and less willing to condemn $A$.\textsuperscript{11}

$$m_{jt} > c - \frac{c}{N} E(N_t|\Omega_t) = \hat{m}_t > \hat{m}_{t-1} > ... > \hat{m}_0.$$

Suppose that $\hat{N}_0 < \hat{N}_1$ and $\hat{N}_2 < \hat{N}_1$, instead, although this is less likely to occur. In this case, the observation that $\hat{N}_2 < \hat{N}_1$ leads to $\hat{p}_3 > \hat{p}_2$. Moreover, $\hat{p}_3 > \hat{p}_1$ if

$$\frac{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1)}{p \cdot P(\hat{N}_0|F_1) \cdot P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1) + (1 - p) \cdot P(\hat{N}_0|F_2) \cdot P(\hat{N}_1|F_2) \cdot P(\hat{N}_2|F_2)} > \frac{p \cdot P(\hat{N}_0|F_1)}{p \cdot P(\hat{N}_0|F_1) + (1 - p) \cdot P(\hat{N}_0|F_2)}$$

$$\Leftrightarrow P(\hat{N}_1|F_1) \cdot P(\hat{N}_2|F_1) > P(\hat{N}_1|F_2) \cdot P(\hat{N}_2|F_2)$$

That is, which of the past observations has a stronger connection to either of the two distributions will determine the inference of $\hat{p}_3$. A similar process applies to the later periods.

2.4 Declining Moral Standards

The above analysis shows that, if the first few observation of $\hat{N}_2, \hat{N}_1, \hat{N}_0$ are lower than $E(N_0)$, they will trigger an information cascade that leads to $\hat{p}_t \to 1$, i.e.,

\textsuperscript{11}The result is robust even if there are more than two possible distributions.
$P(F = F_1 | \Omega_t) \to 1$ even if the true distribution is $F_2$. If the true distribution is $F_2$, it implies that an incorrect information cascade occurs when $\hat{p}_t \to 1$ and moral standards decline quickly. Suppose that $N_1^*$ is significantly low. Declining moral standards would lead to a situation in which only a $N_1^*/N \approx 0$ fraction of the population actually enforce moral rules and thus there is no meaningful enforcement of moral rules. What facilitates such an incorrect information cascade?

**Proposition 1** The social learning process of morality makes it easy for public moral standards to decline and makes it difficult to enforce moral rules.

Because the enforcement of moral rules requires social support, the members of a society have to learn the implicit standards based only on the observed level of moral enforcement by social learning. Because the social learning is imperfect, it is easy for the public morality to decline. The decline in the standards accelerates as a result of chain reactions in the social learning process. (i) As the members observe that a lower fraction of the population actually condemn immoral actions, they form a belief that society consists of many immoral people. Then, their individual incentive to refrain from acting immorally declines. (ii) Due to the increase in immoral actions, the members are exposed more frequently to situations in which moral standards matter. More frequently they see whether other members silently condone immoral actions. More frequently their beliefs are updated. Thus, the members become more dependent on social signals. (iii) More members decide to remain silent, and more immoral actions take place. Observing fewer instances of enforcement and more instances of immoral actions serves to lower the expected moral standards even further.

3 The Role of Law

Although moral rules that are implicit and subjective, and enforcement is uncertain and occurs socially, law is explicit and objective, and its enforcement is (relatively) certain. Law creates clear expected costs of wrongdoing. Because its enforcement is not subject to a great deal of social support, law is not greatly influenced by the social learning process. These features give law a great advantage in regulating conduct. Moreover, this paper finds that the features of law help to prevent public morality from declining.
In particular, law affects the social learning process by deterring $A$. So far, we have assumed that the act of $A$ being taken does not add information to the social learning process. However, neighbors also can use the information that $A$ has (or has not) been taken in updating their beliefs regarding the true distribution of population morality. Now, we consider a full social learning process that incorporates such a channel of information update. The Appendix provides a detailed description of how the social learning process changes in this case.

**Proposition 2** Law enforcement $rs > 0$ either slows the speed of a decline of moral standards or expedites the speed of convergence for a positive information cascade, i.e., $\hat{p}_t \to 0$.

Law enforcement slows the decline of public morality by alleviating the feedback effects described in (i) and (iii) above. Even if society’s moral standards are expected to be low, imposing a legal sanction $s > 0$ reduces the incentive to undertake an immoral action (as long as $r > 0$), thereby lowering the effect of (i). As fewer immoral actions are carried out by the members, there is less need for them to update their beliefs regarding the true distribution of population morality, thus lowering (ii). Fewer immoral actions and a low degree of social influence combine to slow the speed of decline in (iii). Thus, imposing a legal sanction $s > 0$ slows the decline of moral standards as long as $r > 0$.

A positive information cascade under a legal sanction $s > 0$ is somewhat different from a negative cascade in that the positive cascade occurs by means of the experiences of "inaction," whereas the negative cascade takes place when $A$ is observed in each period. The decline of morality requires the observation of $A$. Because it is the initial wrongdoing that begets the infinite negative feedback in the social learning process that leads to the belief of low public morality, the most effective way to prevent a decline is to preempt the learning process by deterring the immoral action. In doing so, the existence of an objective and explicit punishment that lowers $I$’s incentive to take $A$ in (i) bears substantial positive externalities in the later sequences of social learning of public morality.

Suppose $A$ is deterred ($A_0^u$), and thus, no offensive action is observed at $t = 0$. People will infer that this is probably due to a higher possibility of $F_2$, forming a belief that $\hat{p}_{u1}|A_0^u < p$ ("No news is good news"). It will reduce $I$’s incentive to take
A in the next period \( t = 1 \). Thus, again, \( A \) is unlikely to be observed at \( t = 1 \). If \( A \) is taken at \( t = 0 \), however, the likelihood of a low \( \hat{N}_0 < E(N_0) \) is higher because at \( t = 0 \), \( j_0 \)'s decision to condemn \( A \) is based on the expectation of \( \hat{p}_{u1|A_0} > p \) instead of \( p \) and, thus, neighbors expect that the true distribution of morality is likely to be close to \( F_1 \). This will increase the probability that \( A \) is taken at \( t = 1 \) and \( \hat{N}_1 < \hat{N}_0 \) will lead to an expectation of \( E(N_2) < \hat{N}_1 \), raising the probability of observing \( A \) in the next period, and so on. This process shows how important it is to observe an inaction that preempts neighbors’ moral reactions to \( A \). If the rules fail to deter \( A \), the chance of a positive cascade quickly diminishes. Upon observing \( A \), the inference of \( \hat{p}_{u1|A_0} > p \) by neighbors promotes a negative cascade by reducing their immediate incentives for moral enforcement. Therefore, with the additional information gained by observing \( A \), the negative cascade will accelerate even faster than described in the previous section.

Proposition 2 implies that if the correct distribution is \( F_2 \), with the enhanced deterrence effects on \( A \), legal sanctions will expedite the social learning process of the true distribution. However, if the true distribution is \( F_1 \), legal sanctions artificially raise society’s moral standards. This result raises a question of what law ought to be. This model states that, in general, when moral enforcement tends fail, law strengthens moral enforcement by preventing a decline in moral standards and facilitating a convergence in people’s belief of a strong public morality, regardless of whether the society has a strong morality. Such an institution is beneficial only if regulating conduct according to a high public morality is desirable regardless of whether the members truly desire such regulation. Suppose that the true distribution of population moral values is \( F_1 \). Law is likely to encourage the society to depart from one that reflects the true nature of the population to another that places more restrictions on behavior. When would this be desirable?

If law needs to be independent of what the population desires, it typically is when populism fails. In such a situation, law serves as a commitment by members of the society to impose their will to organize the society according to the way that the law advocates. Such a commitment value of law has been pointed out by Honoré (1993) in the context of "moral conflicts."

For example, consider abortion. Some believe that it should be allowed. They have their own moral reasons. Similarly, there are others who believe that, for different moral reasons, it should not be allowed. In such a situation of moral conflict, Honoré
(1993) asked, "Why not resolve the abortion issue by allowing those who do not object to abortion to have abortions or perform them if they wish, while those who think abortions wrong are free to refuse to have or to perform them?" Why should there be a law? He argues that law is our "commitment" to what should be allowed or what should be discouraged. The role of law in such a situation of moral conflict is to let us coordinate our behavior in a way that the law permits. By means of laws, we decide what to discourage and what not to discourage. In this example, the two different underlying distributions $F_1$ and $F_2$ would just mean different ways of organizing behavior. A law may discourage a behavior that belongs to $F_1$, not because the unlawful behavior is immoral, but because we are convinced that it would be better to commit ourselves to discouraging the behavior.

This paper extends the question in Honoré (1993) to the case in which there is no conflict, but there is a consensus by the population against a law. Suppose that it is believed that the majority does not want to allow abortion. Should there be a law to prohibit abortion reflecting such a wish? This paper shows that law needs not be natural as the perceived public morality may not be natural, straying from its true underlying distribution in the course of social learning. Law that is not natural can work as our commitment to what we aspire. The same logic can be applied to the legal issues of abolishing laws against prostitution, adultery, and organ sales. These can be the examples in which law should take a stance that is based on its long-term implications and how it would alter the way in which people coordinate their behavior, instead of simply reflecting what the majority desires at present. Thus, whether or not law reflects a majority opinion is often irrelevant.

In this context, we can discuss the implications of the recent repeal of adultery statutes in Colorado. In 2013, Colorado decided that adultery should not be illegal on the grounds that many people believe that adultery is a personal and moral matter and, thus, an issue into which the state should not intervene.12 Denver Democratic Representative, Daniel Kagan, the proposer of the legislative change in Colorado, stated, "I see it as saying adultery is a matter between a spouse and his conscience and his God, but not his local county sheriff."13 One of the main points of this paper is that one's will to follow one's "conscience" is a moving thing, especially since it is

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12 Adultery continues to be illegal in many other states, including Georgia, Massachusetts, and Illinois.
greatly influenced by the people around him, as evidenced by the fact that adultery was once considered to be a serious crime, whereas today most people believe that it is a strictly personal matter. Moreover, the fact that adultery is a personal matter cannot be the reason that it should not be illegal. In general, a breach of contract is considered to be immoral. It is also illegal. Adultery is a breach of marital contract. Hence, one of the issues is why adultery should be treated differently than a breach of other contracts. This paper also argues that, if adultery should be illegal, this needs to be decided on the basis of its long-term implications, as it serves as our commitment to a certain way of organizing behavior.

4 Moral Involvement in Law Enforcement

Until now, we have assumed that law enforcement is independent and fixed. However, law enforcement requires the involvement of citizens in many aspects, which vary from reporting wrongful actions to providing evidence for conviction in court. That is, often \( r_t = f(N_t) \), and \( f' > 0 \). Thus, how effectively laws can be enforced also depends on the extent to which the citizens are willing to become involved in the process of law enforcement.

What causes people to be unwilling to become involved? A low \( r_t \) often occurs because people believe that there is no real reward for their involvement, but only a cost, especially since they expect to be alone with little public support in doing the right thing. Whistleblowers are often punished for doing the right thing when corruption by others makes it impossible to punish the true wrongdoers. Witnesses of crimes frequently pay a very high price when they come forward with information about a crime that they have witnessed. An awareness of such situations reinforces people’s belief that the cost of getting involved is prohibitively high. It is not surprising that, in many situations, citizens are unwilling to become involved. In 2010, a man in New York, who was attempting to defend a woman who was being mugged, was stabbed and left to die, while more than 20 persons passed by without offering to help. In Seattle, a teenager was beaten and robbed by other teenagers, "while three security guards stood by and watched." As shown in our analysis in Section 2, the

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\(^{14}\) In 1644, Mary Latham and James Britton were hanged in Massachusetts for their adultery. In Iraq, people are stoned to death for adultery even today. ("Islamic State militants stone man to death in Iraq" Reuters, August 2014)

\(^{15}\) "Good Samaritan Left for Dead on City Sidewalk" (April, 25, 2010) www.abcnews.com
decline of morality accelerates when the number of good Samaritans $\tilde{N}_t$ declines.

As $\tilde{N}_t$ decreases, so does the effectiveness of law enforcement $r_t$. As $r_t \to 0$, enforcement of the law becomes doubtful, regardless of the severity of punishment $s$. Therefore, when enforcement of the law $r_t$ also depends on the voluntary involvement of citizens, declining moral standards accompany ineffective law enforcement.

**Proposition 3** A decline in moral standards reduces the effectiveness of law enforcement.

When observing a lower $\tilde{N}_t$, people expect a lower likelihood of law enforcement and a greater probability of successful crimes. In turn, this will lead to more crimes being committed, people becoming less forthcoming, and an increase in the ineffectiveness of law enforcement, and so on. This shows the importance of maintaining the morality of society to effectively enforce the law. Without morality, law enforcement may not be possible.

Overall, our analysis highlights the interactive, mutually-enhancing relationship of morality and law. Law makes it difficult for moral rules to weaken. Maintaining moral standards, in turn, is important in causing people to become more willing to engage in the process of law enforcement. This is particularly important because not every immoral action is a legal matter. Laws are specific only to the actions that they define as being unlawful. In contrast, morality is embedded in almost all dimensions of the daily actions in people’s lives. Thus, morality based on matters that are not necessarily subject to legal judgment often engenders a sense of civic duty, which determines the effectiveness of law enforcement. Laws that maintain morality becomes easy to enforce.

## 5 Conclusions

This paper discusses the relationship between morality and law and re-examines the role of law, with a model that shows how easily moral standards can decline in the process of social learning. We find that law generally slows the decline of the standards and provides the lower bound of public morality. We argue that legal rules may be necessary when moral rules tend to be easily swayed by social influence, especially when such a commitment to regulate behavior is desirable in the long run, even if the idea of such a commitment is not popular at present. We also show that a decline
in morality can weaken the effectiveness of law when the enforcement of law depends on the morality of people, which determines their willingness to get involved.

A Appendix

A.1 Proof of Lemma 1

Consider a decision of a person who possesses the highest moral value $\frac{1}{m}$. From Assumption 1, this person would condemn an immoral action $A$ even if she believes that no one else would. Then, knowing that the first person who possesses $\frac{1}{m}$ would condemn, a person with the second highest moral value $\frac{1}{m} - \varepsilon_1$, $\varepsilon_1 > 0$, would consider whether $1 - (\frac{1}{m} - \varepsilon_1)/c < 1 - F_d(\frac{1}{m} - \varepsilon_1)$ and condemns $A$ if the condition holds. If the second person does not have an incentive to act, by construction, the third person will not have an incentive, either. However, if the second person has an incentive, knowing this, the third person who has the next highest moral value, $\frac{1}{m} - \varepsilon_2$, $\varepsilon_2 > \varepsilon_1$, would consider whether $1 - (\frac{1}{m} - \varepsilon_2)/c < 1 - F_d(\frac{1}{m} - \varepsilon_2)$, and so on. Thus, the problem for each individual with $\frac{1}{m}$ is reduced to simply considering whether his or her own moral value is high enough to encourage an act of condemnation even if all the other people with a higher moral value decide to condemn.

Since $m^*_{d}$ is the largest value of $m_d$, by construction, $N^*_d/N = 1 - F(m^*_d) = 1 - m^*_d/c$. Thus, there is a $N^*_d/N$ fraction of the population with $m_l > m^*_d$, $d = 1, 2$. Under Assumption 1, $m_l > cF_d(m_l)$ for all $m_l > m^*_d$. Therefore, $1 - m_l/c < 1 - F_d(m_l) < 1 - m^*_d/c = N^*_d/N$. Thus, from (3), a person $j$ with $m_j = m^*_d$ expects that all other neighbors $l$ with $m_l > m_j$, a $N^*_d/N$ fraction of the people, would condemn $A$. Let $m^*_{d-1}$ be the second largest $m_d$, and $m^*_{d-2}$ be the next largest $m_d$, if any. Then, it must be that $m_k < cF_d(m_k)$ for all $m_k$ in the range where $m^*_{d-1} < m_k < m^*_d$. Therefore, $1 - m_k/c > 1 - F(m_k)$. This implies that even if everyone else who has a slightly higher moral value is expected to condemn, the person with $m_k$ would not condemn. Thus, the person with $m_k$ expects that none of the $m_k$ in the range where $m^*_{d-1} < m_k < m^*_d$ would condemn. For all $m_n$ in the range where $m^*_{d-2} < m_n < m^*_{d-1}$, if any, $m_n > cF_d(m_n)$. Let $N^n_d/N$ be the fraction of people that $m_n$ would expect to condemn of all of those people with $m > m_n$. Since $m_n < m_k$, $1 - m_n/c > 1 - m_k/c > 1 - F(m_k) > N^n_d/N$, $m_n$ in this range would not condemn. Similar logic applies to the range below if any. Thus, the rational
expectation is that a $N_d^*/N$ fraction of people condemn. Q.E.D.

### A.2 Proof of Lemma 2

1. $m_2^* < \tilde{m}_0 < m_1^*$ and, thus, $N_1^* < E(N_0) < N_2^*$.

   By construction, at $m_d^*$, $N_d^*/N = 1 - F(m_d^*) = 1 - m_d^*/c$. Then, $\tilde{m}_0 = p\tilde{m}_0 + (1-p)\tilde{m}_0 = c\{pF_1(m_{N_0}(p)) + (1-p)F_2(m_{N_0}(p))\} = m_{N_0}(p)$. Define $\Gamma(\tilde{m}) := p[\tilde{m} - cF_1(\tilde{m})] + (1-p)[\tilde{m} - cF_2(\tilde{m})]$. When $\tilde{m} = m_{N_0}$, $\Gamma(\tilde{m}) = 0$. Evaluating $\Gamma(\tilde{m})$ at $\tilde{m} = m_1^*$, we get $\Gamma(m_1^*) = (1-p)c[F_1(m_1^*) - F_2(m_1^*)] > 0$ since $F_2$ FOSD $F_1$. Similarly, $\Gamma(m_2^*) = pc[F_2(m_2^*) - F_1(m_2^*)] < 0$. Since $\Gamma(\tilde{m})$ is increasing in $\tilde{m}$ and continuous in the range where $\tilde{m} \geq m_2^*$, $\Gamma(m_1^*) > 0$, $\Gamma(m_2^*) < 0$, and $m_2^* < m_1^*$ imply that $\tilde{m}_0$ satisfying $\Gamma(\tilde{m} = \tilde{m}_0) = 0$ must be that $m_2^* < \tilde{m}_0 < m_1^*$.

2. $\tilde{m}_t \to m_1^*$ as $\tilde{p}_t \to 1$ and $\tilde{m}_t \to m_2^*$ as $\tilde{p}_t \to 0$.

   By construction, $\tilde{m}_t = c\{\tilde{p}_tF_1(m_{N_t}(\tilde{p}_t)) + (1-p)F_2(m_{N_t}(\tilde{p}_t))\}$ and $m_{N_t}(\tilde{p}_t)$ are $\tilde{m}_0$ and $m_{N_0}(p)$ defined at $p = \tilde{p}_t$. Since $F_1$ and $F_2$ are continuous, $\tilde{m}_0$ is continuous for any given $p \in (0,1)$. From (5), $m_{N_0}$ is a fixed point that satisfies $m_{N_0} = c\{pF_1(m_{N_0}) + (1-p)F_2(m_{N_0})\}$ for a given $p$. As $p$ increases to $p'$, the right-hand-side of (5) increases since $F_2$ stochastically dominates $F_1$. Then, the corresponding fixed point $m_{N_0}$ at $p' = \tilde{p}_t$ must be higher. Thus, $m_{N_t}(\tilde{p}_t)$ is an increasing function of $\tilde{p}_t$. When $p' \to 1$, $\tilde{m}_t \to cF_1(m_{N_0}(p'))$, thus, $m_{N_0}(p') \to m_1^*$. Similarly, when $p' \to 0$, $m_{N_0}(p') \to m_2^*$. Q.E.D.

### A.3 When I’s actions are included in the inference

In this case, social learning must involve I’s choices as well. Neighbors update their beliefs about population distribution based on the observed $A$ or inaction. Social learning occurs through two channels: (i) updates in the probability of observing $A$ and (ii) updates in the probability that the observed $A$ is associated with $F_1$ distribution. Neighbors’ expectations differ when $A$ is observed and when it is not. We define Perfect Bayesian Nash Equilibrium in the following.

#### A.3.1 At $t = 0$

Let $E(N_0|A_0)$ be the expected level of neighbors’ condemnation upon observing $A_0$ by $I$ at $t$. Then, at $t = 0$, $I$ takes $A$ if
\[ m_{i0} < \frac{b - rs}{(1 - w)} - \frac{wm}{(1 - w)} E(N_0|A_0) = \tilde{M}_0, \quad (14) \]

Thus, the probability of observing any offensive action at \( t = 0, A_0 \), is

\[ P(A_0) = P(m < \tilde{M}_0) = pF_1(\tilde{M}_0) + (1 - p)F_2(\tilde{M}_0). \quad (15) \]

Let \( \Phi_0 \) be the observation of action of \( I \)'s choice at \( t = 0, \Phi_0 \in \{A_0, A_0^\nu\} \), where \( A_0 \) and \( A_0^\nu \) refer to action and inaction, respectively. If \( A \) is observed at \( t = 0 \) (\( \Phi_0 = A_0 \)), neighbors expect that there is a \( \tilde{p}_0 \) probability that the true distribution is \( F_1 \).

\[ \tilde{p}_0 = P(F = F_1|\Phi_0 = A_0) = \frac{pF_1(\tilde{M}_0)}{P(A_0)}. \quad (16) \]

Note that \( \tilde{p}_0 > p \) since \( F_2 \) FOSD \( F_1 \).

Then, in figuring out \( m_{N_0} \) and \( E(N_0|A_0) = \sum \beta_j0 \), the rational expectation of \( E(N_0|A_0)/N \) is that

\[ E(N_0|A_0)/N = \tilde{p}_0(1 - F_1(m_{N_0|A_0})) + [1 - \tilde{p}_0](1 - F_2(m_{N_0|A_0})) \quad (17) \]

\[ m_{N_0|A_0} = c \{ \tilde{p}_0 F_1(m_{N_0|A_0}) + (1 - \tilde{p}_0) F_2(m_{N_0|A_0}) \}, \quad (18) \]

and such a \( m_{N_0|A_0} \) exists. Then, neighbor \( j0 \) condemns \( A \) if

\[ m_{j0} > c \{ \tilde{p}_0 F_1(m_{N_0|A_0}) + (1 - \tilde{p}_0) F_2(m_{N_0|A_0}) \} = \tilde{m}_0(\tilde{p}_0). \quad (19) \]

Since \( \tilde{p}_0 > p \), \( \tilde{m}_0(\tilde{p}_0) > \tilde{m}_0(p) \). Thus, \( E(N_0|A_0) = N [1 - \tilde{m}_0(\tilde{p}_0)/c] < E(N_0) = N[1 - \tilde{m}_0(p)/c] \). Thus, compared to (10), (14) is more likely to be satisfied. That is, \( A_0 \) is more likely to be observed when neighbors update their beliefs from \( I \)'s action.

Suppose that \( A \) was taken and it was observed that \( N_0|A_0 < E(N_0|A_0) \) at \( t = 0 \). This would generate the information that \( P(A_0 \cap \tilde{N}_0|F_1) > P(A_0 \cap \tilde{N}_0|F_2) \). Then,

\[ p_1 = \frac{P(F = F_1|A_0, \tilde{N}_0)}{P(F = F_1 \cap A_0 \cap \tilde{N}_0)} = \frac{p \cdot P(A_0 \cap \tilde{N}_0|F_1)}{p \cdot P(A_0 \cap \tilde{N}_0|F_1) + (1 - p) \cdot P(A_0 \cap \tilde{N}_0|F_2)} \quad (20) \]
Therefore, at \( t = 1 \), the probability of \( F_1 \) is higher.

**A.3.2 At \( t = 1 \)**

Let \( E(N_1|A_1, A_0, \widehat{N}_0) \) be the expected level of neighbors’ condemnation upon observing \( A_1 \) by \( I \) at \( t = 1 \). Then, at \( t = 1 \), \( I \) takes \( A \) if

\[
m_{i1} < \frac{b - rs}{(1 - w)} - \frac{wm}{(1 - w)} E(N_1|A_1, A_0, \widehat{N}_0) = \widehat{M}_1, \quad \text{where} \quad (21)
\]

The probability of observing \( A_1 \) is

\[
P(A_1|A_0, \widehat{N}_0) &= P(F_1|A_0, \widehat{N}_0) \cdot F_1(\widehat{M}_1) + P(F_2|A_0, \widehat{N}_0) \cdot F_2(\widehat{M}_1) \tag{22} \\
&= p_1 \cdot F_1(\widehat{M}_1) + (1 - p_1) \cdot F_2(\widehat{M}_1).
\]

Since \( p_1 > p \), comparing (15) and (22), we can show that \( P(A_1|A_0, \widehat{N}_0) > P(A_0) \), other things being equal. If \( A \) is observed at \( t = 1 \) (\( \Phi_1 = A_1 \)), neighbors expect that there is a \( \tilde{p}_1 \) probability that the true distribution is \( F_1 \).

\[
\tilde{p}_1 = P(F = F_1|\Phi_1 = A_1, A_0, \widehat{N}_0) \\
= \frac{P(F_1 \cap A_1|A_0, \widehat{N}_0) \cdot P(A_0 \cap \widehat{N}_0)}{P(A_1|A_0, \widehat{N}_0) \cdot P(A_0 \cap \widehat{N}_0)} \\
= \frac{P(F_1|A_0, \widehat{N}_0) \cdot F_1(\widehat{M}_1))}{P(A_1|A_0, \widehat{N}_0)} \\
= \frac{p_1 \cdot F_1(\widehat{M}_1)}{p_1 \cdot F_1(\widehat{M}_1) + (1 - p_1) \cdot F_2(\widehat{M}_1)}. \quad (23)
\]

Note that \( \tilde{p}_1 > \tilde{p}_0 = \frac{p^*}{p^*F_1(M_0) + (1 - p^*)F_2(M_0)} \) if \( \widehat{M}_0 \leq \widehat{M}_1 \) under the MLRP.

\( \widehat{M}_1 \), however, depends on the neighbors’ expectation of \( E(N_1|\Omega_1)/N \), which depends on \( \tilde{p}_1 \), in turn. For a given \( \Omega_1 = \{A_1, A_0, \widehat{N}_0\} \), the neighbors’ rational expectation of \( E(N_1|\Omega_1)/N \) is that

\[
E(N_1|\Omega_1)/N = \tilde{p}_1(1 - F_1(m_{N_1|\Omega_1})) + [1 - \tilde{p}_1](1 - F_2(m_{N_1|\Omega_1})), \tag{24} \\
m_{N_1|\Omega_1} = c \{\tilde{p}_1F_1(m_{N_1|\Omega_1}) + (1 - \tilde{p}_1)F_2(m_{N_1|\Omega_1})\}. \tag{25}
\]
Neighboring j insults A if

$$m_{j1} > c \{ \tilde{p}_1 F_1(m_{N_1|\Omega_1}) + (1 - \tilde{p}_1) F_2(m_{N_1|\Omega_1}) \} = \tilde{m}_1(\tilde{p}_1). \tag{26}$$

To show $E(N_1|\Omega_1)/\mathcal{N} < E(N_0|A_0)/\mathcal{N}$, suppose that neighbors initially expect $\tilde{M}_0 = \tilde{M}_1$ and $I$ faces the same incentives for $A$ at $t = 0$ and at $t = 1$. In this case, the $t = 0$ observation of $\tilde{N}_0|A_0 < E(N_0|A_0)$ leads to $p_1 > p$, $P(A_1|A_0, \tilde{N}_0) > P(A_0)$ and $\tilde{p}_1$ increases above $\tilde{p}_0$. Then, this increases $m_{N_1|\Omega_1}$ and lowers $E(N_1|\Omega_1)/\mathcal{N}$, increasing $\tilde{M}_1$ above $\tilde{M}_0$. An increase in $\tilde{M}_1$ further increases $P(A_1|A_0, \tilde{N}_0, \tilde{p}_1)$, and lowers $E(N_1|\Omega_1)/\mathcal{N}$, and so on. Thus, $\tilde{N}_0|A_0 < E(N_0|A_0)$ leads to $\tilde{p}_1 > \tilde{p}_0$, $\tilde{m}_1(\tilde{p}_1) > \tilde{m}_0(\tilde{p}_0)$, and $E(N_1|\Omega_1) = \mathcal{N}[1 - \tilde{m}_1(\tilde{p}_1)/c] < E(N_0|A_0) = \mathcal{N}[1 - \tilde{m}_0(\tilde{p}_0)/c] < E(N_0) = \mathcal{N}[1 - \tilde{m}_0(p)/c]$. Thus, $A_1$ is more likely to be observed and neighbors are less likely to condemn it.

Thus, when Is incentives are incorporated and the observation of $A$ is taken into account for the information update, the speed of information cascades accelerates.

### A.4 Proof of Proposition 2.

When the social learning process incorporates information from an observation of $A$, it generates a different social learning process conditional on whether $A$ is observed or not. If $A$ is observed, the path follows the one that is described above. In the following, we describe a case in which $A$ is deterred. Since neighbors’ reactions become available only after $A$ is undertaken, deterrence leads to an off-the-equilibrium path. We characterize Perfect Bayesian Nash Equilibrium with sequentially rational beliefs off the equilibrium path.

At $t = 0$, $I$ is deterred from taking $A$ if

$$m_i > \frac{b - rs}{1 - w} - \frac{wm}{1 - w} E(N_0|A_0^u) = \tilde{M}_0,$$

which is based on a sequentially rational belief $E(N_0|A_0^u)$ that is consistent with the choice of inaction. The probability of not observing any offensive action at $t = 0$, $A_0^u$, is $P(A_0^u) = P(m > \tilde{M}_0) = 1 - p F_1(\tilde{M}_0) - (1 - p) F_2(\tilde{M}_0)$. If $rs > 0$ increases, $\tilde{M}_0$ decreases and, thus, $P(A_0^u)$ increases.

Suppose $A$ is deterred at $t = 0$ due to a legal sanction $rs > 0$. Then, the
sequentially rational belief \( E(N_0|A_0^u)/N \) must satisfy the following:

\[
E(N_0|A_0^u)/N = \tilde{p}_u(1 - F_1(m_{N_0|A_0^u})) + [1 - \tilde{p}_u](1 - F_2(m_{N_0|A_0^u})),
\]

\[
m_{N_0|A_0^u} = c \{ \tilde{p}_u F_1(m_{N_0|A_0^u}) + (1 - \tilde{p}_u) F_2(m_{N_0|A_0^u}) \},
\]

where \( \tilde{p}_u \), the belief that true distribution is \( F_1 \) given inaction, is defined as

\[
\tilde{p}_u = \frac{p \cdot P(A_0^u|F_1)}{p \cdot P(A_0^u|F_1) + (1 - p) \cdot P(A_0^u|F_2)}.
\]

Then, \( \tilde{p}_u < p \) since

\[
\tilde{p}_u = \frac{p \cdot P(A_0^u|F_1)}{p \cdot P(A_0^u|F_1) + (1 - p) \cdot P(A_0^u|F_2)} < p
\]

\[
\Leftrightarrow P(A_0^u|F_1) < P(A_0^u|F_2).
\]

At \( t = 1 \), another \( I \) and \( N \) are independently drawn from the population. Based on the information that there was no first period observation of \( A \), \( I \) refrains from \( A \) if

\[
m_{i1} > \frac{b - rs}{1 - w} - \frac{wm}{1 - w} E(N_1|A_1^u, A_0^u) = \tilde{M}_{u1},
\]

where \( E(N_1|A_1^u, A_0^u) \) is a sequentially rational belief that is consistent with inaction \( A_1^u \) given \( A_0^u \). Since \( \tilde{p}_u < p \), if \( \tilde{M}_{u1} \approx \tilde{M}_{u0} \), it is likely that \( P(A_1^u|A_0^u) > P(A_0^u) \), i.e.,

\[
P(A_1^u|A_0^u) = 1 - P(F_1|A_0^u) \cdot F_1(\tilde{M}_{u1}) - P(F_2|A_0^u) \cdot F_2(\tilde{M}_{u1}) = 1 - \tilde{p}_u \cdot F_1(\tilde{M}_{u1}) - (1 - \tilde{p}_u) \cdot F_2(\tilde{M}_{u1}) > P(A_0^u).
\]

That is, at \( t = 1 \), other things being equal, based on inaction from the previous period, \( I \) expects a higher probability of condemnation than before. Thus, it is more likely that \( I \) refrains from \( A \).

Suppose that \( A_1^u \) is observed. The sequentially rational belief of \( \tilde{M}_{u1} \) must satisfy the requirement that \( E(N_1|A_1^u, A_0^u) \) is consistent with \( \tilde{p}_u = P(F = F_1|A_0^u, A_1^u) \). When
$A_1^u$ is observed at $t = 1$, it implies that $P(A_1^u|F_1) < P(A_1^u|F_2)$ and, thus,

$$
\hat{p}_{u2} = \frac{P(F = F_1 \cap A_0^u \cap A_1^u)}{P(A_0^u \cap A_1^u)} = \frac{p \cdot P(A_0^u|F_1) \cdot P(A_1^u|F_1)}{p \cdot P(A_0^u|F_1) \cdot P(A_1^u|F_1) + (1 - p) \cdot P(A_0^u|F_2) \cdot P(A_1^u|F_2)} < \frac{p \cdot P(A_0^u|F_1)}{p \cdot P(A_0^u|F_1) + (1 - p) \cdot P(A_0^u|F_2)} = \hat{p}_{u1}
$$

and so on. Thus, it is more likely that a positive cascade occurs, $\hat{p}_{ut} \to 0$.

Suppose, instead, that $m_{i1}$ was so low that the action was taken at $t = 1$ and thus $\hat{N}_1$ was observed. Note that, even if $A$ is taken, since each neighbor remembers that at $t = 0$, $A$ was not taken, with $\hat{p}_{u1} < p$, they still believe the distribution to be closer to $F_2$, while the observation of $A$ would be considered shocking, neighbors expect many people would condemn, $E(N_1|A_0^u) > E(N_0)$. Therefore, it is more likely that $\hat{N}_1 > E(N_0)$. Then, at $t = 2$,

$$
\hat{p}_2 = P(F = F_1|A_0^u, A_1, \hat{N}_1) = \frac{P(F = F_1 \cap A_0^u \cap A_1 \cap \hat{N}_1)}{P(A_0^u \cap A_1 \cap \hat{N}_1)} = \frac{P(F_1) \cdot P(A_0^u \cap A_1 \cap \hat{N}_1|F_1)}{P(F_1) \cdot P(A_0^u \cap A_1 \cap \hat{N}_1|F_1) + P(F_2) \cdot P(A_0^u \cap A_1 \cap \hat{N}_1|F_2)} \frac{P(A_1|F_1) \cdot P(\hat{N}_1|F_1)}{p \cdot P(A_0^u|F_1) \cdot P(A_1|F_1) \cdot P(\hat{N}_1|F_1)} \frac{P(\hat{N}_1|F_1) + (1 - p) \cdot P(A_0^u|F_2) \cdot P(A_1|F_2) \cdot P(\hat{N}_1|F_2)} {p \cdot P(A_0^u|F_1) \cdot P(A_1|F_1) \cdot P(\hat{N}_1|F_1) + (1 - p) \cdot P(A_0^u|F_2) \cdot P(A_1|F_2) \cdot P(\hat{N}_1|F_2)}
$$

(28)

(29)

The observation of $A$ in period 1 may not be strong enough to overturn the belief that the true distribution is likely to be $F_2$ as long as the following holds.

$$
\hat{p}_2 < \frac{p \cdot P(A_0^u|F_1)}{p \cdot P(A_0^u|F_1) + (1 - p) \cdot P(A_0^u|F_2)} = \hat{p}_{u1}
$$

$$
\Rightarrow \frac{P(A_1|F_1)}{P(A_1|F_2)} < \frac{P(\hat{N}_1|F_2)}{P(\hat{N}_1|F_1)}
$$

$$
\Rightarrow \frac{P(A_1|F_1) \cdot P(\hat{N}_1|F_1)}{P(A_1|F_2) \cdot P(\hat{N}_1|F_2)} < 1
$$

Given that $\hat{p}_{u1} < p$, in the expectation of $E(N_1|A_0^u) > E(N_0)$, it is more likely that $\hat{N}_1 > E(N_0)$, and $\frac{P(\hat{N}_1|F_2)}{P(\hat{N}_1|F_1)} > 1$, whereas people may expect that $P(A_1|F_1)$ is not much larger than $P(A_1|F_2)$ since $A$ did not occur at $t = 0$. In that case, $\hat{p}_2 < \hat{p}_{u1}$. In order
to overturn this expectation, it must be that $\hat{N}_1 < E(N_0) < E(N_1|A_0^i)$ in addition to the occurrence of $A$ by an $I$ with a very low $m_i$. Thus, a legal sanction $s$ that increases a chance of inaction slows the negative cascade. Q.E.D.

References


